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THE BANK BALANCE PROBLEM

Kamil Msefer, System Dynamics Education Project, System Dynamics Group, Sloan School of Management
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Why is the bank balance model important? Simple models discuss basic ideas of system dynamics, and a profound mastery of these concepts is indispensable to mastering more subtle and complex notions in any field. From the point of view of teaching mathematics, exponential growth could be taught using this model. This model can also serve as an introduction to the concept of limits and differential equations. From a systems point of view, many issues such as modeling, and positive feedback loops can all be dealt with just using this simple model.

The Bank Balance Problem
by
Kamil Msefer[1]

Introduction

Seventy-two years ago, a wise man named Ralph put his entire fortune, two hundred dollars, in a secure bank. He never told anyone, not even his family, and all his descendants lived miserably all their lives. A few days ago, Joe, his grandson, found an old bank document stating when and where the money was deposited. Interestingly, the bank document affirmed that the money was to accrue at a constant interest rate of 6% during the entire duration of time that the money would be deposited. This bank was not affected by the great depression of the 1930's and the account remained open the entire 72 years. Joe, who's formal schooling was limited to elementary school, was unable to calculate how much money was presently in that account. Consequently, he decided to go and see his friend Alphonso, who was very good in math.

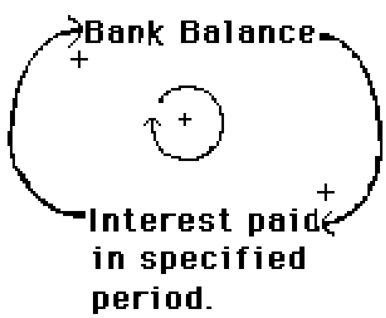

Figure 1. Causal Structure

Ralph told Alphonso about the bank account and how he was puzzled about how to determine how much money it contained. Alphonso's immediate reaction was to draw the diagram above. He called it a Causal Structure. The idea behind it was very simple. When there is an increase in the bank balance, there is an increase in the amount of interest paid per year. This is because, the interest paid per year is always going to be 6% of the bank balance. Because there is a linear[2] relationship between interest and the bank balance, an increase in the bank balance leads to an increase in the flow of interest. The interest is added to the bank balance. So when the interest paid increases, the bank balance increases even faster. This cycle continues causing the bank balance to get bigger and bigger. This behavior is called exponential growth. Exponential growth will be discussed later.

Because it was Sunday, the bank was closed, and they could not find out how often every year, interest was calculated. Alphonso, who had just bought an Apple Macintosh computer and the STELLA[3] computer package, decided to use them to help Joe visualize the process by which money in a bank account accumulates.

Illustration of the Model on STELLA

Transferring the causal loop connections into a STELLA model structure, Alphonso created the model in fibure 2.

The model has one stock where the money (bank balance) is accumulating. Alphonso reminded Ralph that a stock does not change instantly; it is either raised or lowered by a flow. The flow, represented by a pipe with a valve hanging down from it, is in this case the

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UP-DATES...

Ridgewood Public Schools

At the first networking meeting of the Greater New York and New Jersey area, hosted by Tim Lucas and Rich Langheim at Ridgewood, a group of educators from throughout the state started a dialogue. Various forums for continuing discussions and support were aired and a fall meeting was tentatively set for the end of September or the beginning of October. Anyone who is interested in joining a systems thinking network in that area should contact Rich Langheim at (201) 670-2679.

System Dynamics in Education Project

Road Maps II and III are finished and available through the Creative Learning Exchange. The summer projects for the MIT students will be to continue on with the Road Maps. The fourth in the series is well along and the fifth and sixth are in the planning phases. Creation of generic models for systems is another project in the works. If any of you have ideas about helpful projects for these capable students which key into their talents, please let Nan Lux know. They are always interested in feedback about their materials and suggestions for future work. Nan Lux, SDEP, Bldg. E40-294, M.I.T., Cambridge, MA 02139. (617) 253-1574.

FROM THE EDITOR. . .

You will notice that this newsletter is accompanied by a list of materials available through the Creative Learning Exchange. Please only order those materials you really will use. If you wish quite a number and have not sent in an investment, we would appreciate that to help defray the costs to us.

In any case, please fill out the questionnaire if you have not done so. We would like to continue sending you newsletters and information, but only if you wish to receive them. I am still looking for feedback. What are you doing? What works and what doesn't?? I would love to hear from you.

Have a rejuvenating summer. We will be in touch in the fall with our next newsletter.

Lees Stuntz, Editor

'93 NETWORK FOR IMPROVING EDUCATIONAL THINKING CONFERENCE

HOTEL PARK TUCSON - TUCSON, ARIZONA JUNE 28-30, 1993

The Waters Grant Project, Orange Grove Middle School, Catalina Foothills School District, Tucson, AZ would like to extend an invitation to educators or interested supporters of education currently involved with innovative systemic processes/change in education to attend the Network for Improving Educational Thinking Conference scheduled for June 28-30, 1993 in Tucson, AZ.

The goal of the conference is to increase the ability of educators and students to understand and deal with dynamic, complex problems. Some of the concepts to be included are: System Dynamics, Organizational Learning, Learning in Context, Use of Simulations, Total Quality, Dimensions of Learning.

Donella Meadows and H. Stephen Glenn will be the keynote speakers. For additional information on the conference and/or the presentations planned for the three day conference, please call Sam DeVore at (602) 575-1243.

Peter Büttner, a good friend to the introduction of systems education into K-12 schools, died after a long illness in January of this year. His efforts in Brattleboro, Vermont were discussed in the Fall, 1992 *Exchange*. It is people like Peter with love, drive and mission who make a difference in this world. His encouragement of the use of system dynamics in Brattleboro paved the way for its continued use there and, in fact, set the stage for the STACI project.

Our sincerest sympathies to his family. They have asked that any contribution that people would wish to make in his memory be sent to the Creative Learning Exchange.

THE WATERS GRANT PROJECT - ORANGE GROVE MIDDLE SCHOOL

Developing Productive Thinking and Problem Solving Skills

The 1992-93 school year marks the fifth year of middle school development at Orange Grove. The resulting program is a combination of components from the past thirty years of middle school study and components that were previously in place at Orange Grove Junior High and components based on research in other levels of education.

During the past five years, we have become more familiar with the concept and the use of System Dynamics as a tool for dealing with dynamic, complex problems. Because most problems in today's world are both dynamic and complex, increased skill in the use of this tool has been beneficial. It was our interest in the use of the tools of System Dynamics that led us to the Waters Grant Project. As the work to constantly improve our program has continued, we have expanded the project to include a great variety of strategies that have increased our ability to teach and to use productive problem solving skills. Although we have gradually expanded use of the System Dynamics tools in the classroom, the most rapid and evident implementation of increased systemic thinking has been in the organization. Having adults practice the types of communication, collaboration, problem solving and decision making skills that are desirable for students to learn has resulted in a greater sense of community and a greater effectiveness in dealing with challenging situations.

Citizen Champion

A unique aspect of our work in the past five years has been the efforts of Dr. Gordon Brown, professor emeritus and former dean of the engineering school at MIT. Dr. Brown is a resident of the Catalina Foothills School District. He brought the concept of System Dynamics to the attention of the Orange Grove staff. He has made a variety of resources available to district staff members to be used to increase our problem solving skills and teaching skills. In addition he has helped us to maintain an awareness of the realities of today's world that will face the students graduating from our schools. In the community, Dr. Brown has been a supporter of our school district and has influenced others to be supportive also.

In The Classroom

System Dynamics entered Orange Grove through the use of STELLA simulations in science. For many years research has shown and educators have realized that students must develop greater skills in the area of problem solving. Training in instructional strategies related to critical thinking, higher order thinking skills, decision making, problem solving, information literacy and creative thinking has been highly encouraged and is evident in successful schools. The Orange Grove science teachers recognized System Dynamics as a necessary element in the teaching of problem solving skills. Because students will be facing extremely complex problems in their future, it is necessary that we extend their learning through understanding dynamic complexity. Through the use of the System Dynamics tools, content knowledge can be applied in a "real world" context, interrelationships between the parts of the whole can be analyzed and long term consequences of actions can be studied.

STELLA simulations are a powerful tool for application of knowledge. Over the past five years, six 8th grade science simulations, four 7th grade science simulations, one 6th grade science simulation, one 7th grade social studies and one 8th grade literature simulation have been developed and implemented. Students demonstrate a high level of understanding of content and concepts following a STELLA simulation activity. The length of the simulations varies from 2-5 days.

Other System Dynamics tools can be used for the purpose of identifying causal relationship (causal loops) and studying both patterns of behavior and long term consequences of decisions (behavior over time graphs). Both of these concepts are essential to the development of productive thinking and problem solving skills. Traditional subject content can be analyzed, compared and synthesized through the use of these tools. Introduced to these concepts in the 6th grade, 8th grade students often use them to support conclusions or ideas during debates or presentations. Students also report using their newly acquired problem solving skills in daily, personal interactions.

In The Organization

Traditional school organizational structures have not been conducive to communication or collaboration between staff members. The lack of communication and collaboration has often resulted in omissions, overlaps and a general lack of connectivity between various aspects of instruction and other functions of schools. For these reasons, the concept of teaming is very desirable to educators. The challenge of designing the right combinations of groupings, developing the skills necessary for successful team decision making and planning and managing the

WATERS GRANT continued on page 4

NETWORKS...

As more people make contact with the Exchange, they give us information about what has been helpful to them. Many people feel that networking opportunities with other teachers, schools, and professionals are an integral part of their lifelong learning process. We here at the Exchange obviously do also. As we hear about other networking opportunities, we will be sharing them with you. This column will be a regular feature whenever we have successful ideas and implementations. Any inclusion in this column does not endorse the institution or group mentioned. It is merely a way of passing on information which has been given to us and has proved helpful to someone else.

The K-12 Transforming Schools Consortium

The goal of The K-12 Transforming Schools Consortium is the full implementation of a comprehensive, personalized, technology-based approach to the learning process in order to achieve the necessary improvement in the academic performance of students and to level off the rising costs of education.

A wide spectrum of organizations and individuals have participated in the design of the Transforming Schools Program, which is based on the findings and recommendations of many national and state commissions. Emerging from these collective works is the clear call for restructuring by "transforming" from traditional group learning to a more personalized approach. Long favored by educators, personalized learning is, for the first time, feasible through the utilization and ongoing development of computer-based technology, which is the primary method of managing and delivering instruction.

The Transforming School concept is being implemented by the Consortium in collaboration with The William C. Norris Institute, a mot-for-profit organization chartered to help address critical unmet or poorly met social needs. The Consortium's membership includes schools, professional organizations and commissions. For more information about the Consortium call 612/225-1433 or write to The K-12 Transforming Schools Consortium, 245 East Sixth Street, Suite 815, St.. Paul, Minnesota, 55101.

WATERS GRANT PROJECT, *continued. from page 3*

new dynamics produced must be handled carefully. Other entities in the world outside of education are struggling with this same challenge. Stimulated by the major concepts of systems thinking, the Orange Grove staff has consistently sought and used resources that will increase the ability of the OG organization to function as an effective system.

The tools of System Dynamics have increased staff awareness of the interrelationships between the roles that they play and the actions that they take. They understand the importance of both attention to the parts and attention to the whole. When making decisions, they are now much more likely to consider effects of those decisions on other parts of the organization and to compare both short and long term effects. Graphs, causal loops and models are often used to discuss a problem. Although time does not allow the construction of a STELLA model for most problems, the awareness and understanding created by STELLA training has benefited many staff members.

Study and application of the disciplines of the "Learning Organization" have helped Orange Grove staff members to increase their dedication to continual personal growth and to continued improvement of the system. Other resources have been identified and utilized as needed. The entire staff has played a part in reinforcing the importance of personal responsibility, willingness to optimize the whole system rather than maximizing one's subsystem, seeking to understand and accept other mental models and many other systemic concepts. The evident enthusiasm for lifelong learning is a powerful model for students. Students and guests often comment on the amount of collaboration and mutual support among staff members as well as a sense of joy in the work to be done.

Summary

The Orange Grove program is not perfect. Blending the old with the new, combining the visions of students, staff, parents and community and utilizing available skills, facilities and other resources to the greatest extent possible is a never ending challenge. As one parent so aptly described the situation - "it's a system" - complex and ever changing. The good news is that by using the skills of collaboration and communication, of seeing both the details and the big picture, analyzing both short and long term consequences and examining a variety of mental models - we can continue to increase our capacity to produce the results we desire — meeting the needs of today's students who will live in tomorrow's world.

BANK BALANCE, *continued from page 1*

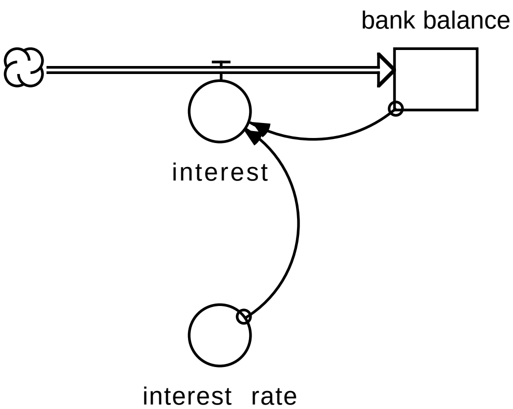


Figure 2. Stock and Flow Diagram of a Bank Account Accruing Interest.

interest, and represents an inflow of money pouring into the bank balance. The inflow is regulated by the equation in the flow which states that interest is equal to the present bank balance times the interest rate. (**`interest=interest rate * bank balance`**) Converters are mainly used to hold constant values, to do algebraic operations, or to make conversions from one unit of measure to another. In this model, the converter is used to hold a constant value, the interest rate. The next question is how does the model work?

DT Solution Interval

DT (from Delta Time) is the interval of time between calculations in a model. It therefore answers the question of how many times in a time period the numerical values in a model are recalculated. If DT is set to 1.0, a round of calculations will be done once every month. But if DT is changed to .5, the round of calculations will be carried out every 1/2 of a month. Similarly, if DT is changed to .25, the round of calculations will be carried out every 1/4 of a month.. In STELLA, DT is found in Simulation Time...under the specs menu. At every increment DT, STELLA carries out the necessary calculations. It first estimates the change in the stocks over the interval DT, and calculates the new values for these stocks using the previous values for the flows and converters. Then, using the new values for the stocks, it calculates new values for flows and converters. Finally it updates the simulation time by an increment of DT. In the bank balance problem, DT is initially equal to 1.0. The time unit is years.

INFLOW EQUATION
Interest Compounded Annually

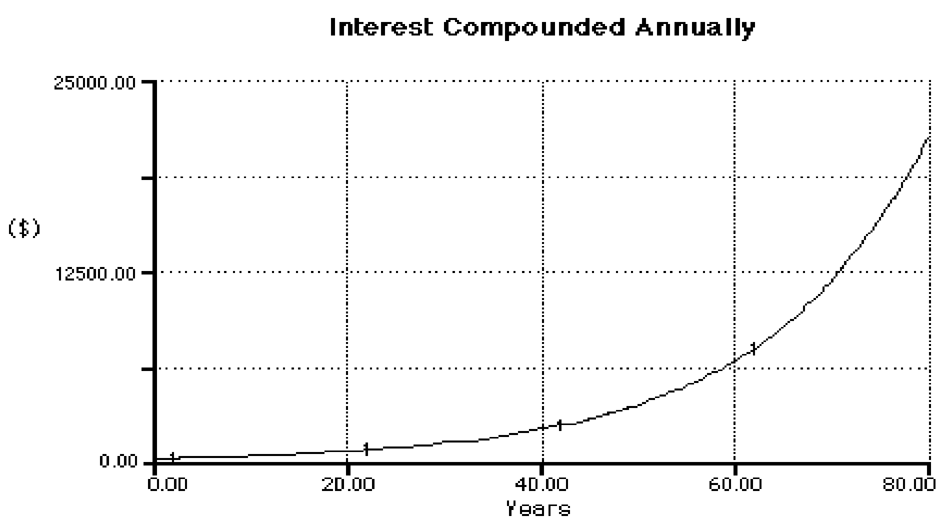
The compounding frequency is how often per year interest is compounded. The next few pages show examples of differing compounding frequencies. DT under Simulation Time in the Specs menu should be set to 1.0. The only stock is the bank balance, and the only flow is interest[4]. STELLA first estimates the change in the bank balance stock over the time interval DT, by multiplying together the net flow and DT. The net flow in general is equal to the inputs to the stock minus the outputs from the stock. In this case, the net flow is equal to the interest rate multiplied by the bank balance. This net flow is the input to the bank balance stock. There is no output from the stock.

In the bank balance model, since DT is equal to 1.0, the change in the bank balance stock is the interest rate multiplied by the bank balance. STELLA adds the bank balance to the change in the bank balance, and makes this the new value for the bank balance stock. Next, STELLA calculates the new value for interest by multiplying the updated bank balance and the interest rate. Finally it increments the simulation time by DT, and repeats the whole procedure. It keeps repeating this procedure until the simulation time reaches the desired ending time. In real life, if interest is to be compounded annually, for example on December 31, it is calculated by multiplying the interest rate and the current bank balance. That interest is then immediately added on to the current bank balance, and this number becomes the new bank balance. Every year, at the same time, the process is repeated. The figure below shows the progression of the bank account as interest compounded annually is reinvested into the bank account. The equations governing the graph are:

interest = interest rate * bank balance

bank balance(t) = bank balance(t-dt) + dt * interest

where dt is 1.0 in the case of interest compounded annually.



(figure 3)

continued on page 6

BANK BALANCE, *continued from page 5*

Interest Compounded Quarterly

When interest is compounded quarterly, the only difference is that the procedure discussed above will be executed four times every year instead of once. Accordingly, DT will have to be changed to .25. This would mean that when the simulation time would have reached 1 year, STELLA would have carried out the procedure four times, or once every quarter. The figure at right shows the progression of the bank account as interest compounded quarterly is reinvested into the bank account.

The equations governing the graph below are:

interest = interest rate * bank balance

bank balance(t) = bank balance(t-dt) + dt*interest

where dt is .25 in the case of interest compounded quarterly.

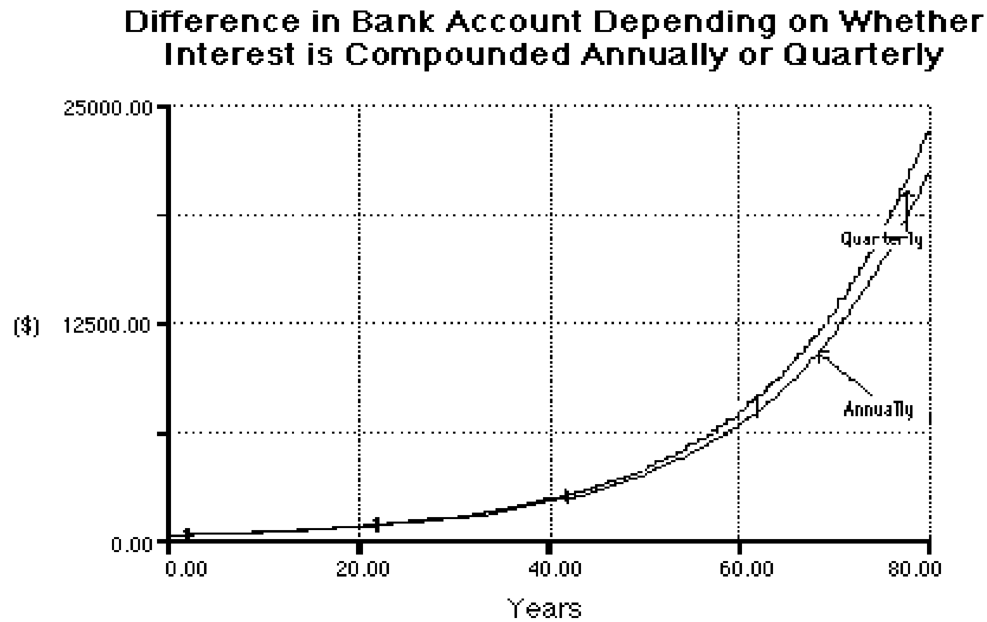


Figure 4.

Interest Compounded Monthly

If interest is compounded monthly, once again the only change will be DT which will be set to 0.083333 (approximately 1/12). In this case, when the simulation time is equal to 1 year, the interest procedure will have been executed 12 times, or once every month. The figure below shows the progression of the bank account as interest compounded monthly is reinvested into the bank account.

Again the equations governing the graph below are:

interest = interest rate * bank balance

bank balance(t) = bank balance(t-dt) + dt*interest

where dt is .083333 in the case of interest compounded monthly.

Difference in Bank Account Depending on Whether Interest is Compounded Annually, Quarterly, or Monthly.

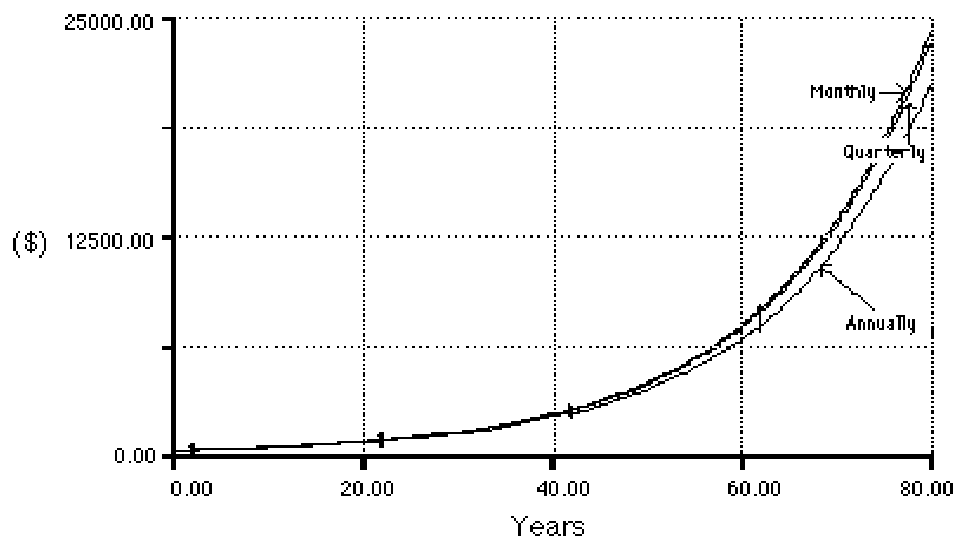


Figure 5.

Numerical Illustration of Ralph's Problem

After explaining the procedure by which interest accumulates in a bank account, Alphonso then proposes to explicitly solve Ralph's problem. Plugging in $200 for the initial bank balance, assuming the interest rate is 6% compounded annually, 72 years later, can you find out how much money the account would contain? What about 8 years later? Try determining how much money the account would contain after 72 years and after 80 years, for the three compounding frequency cases, annually, quarterly, and monthly.

	Annually	Quarterly	Monthly
72 yrs	_______	_______	_______
80 yrs	_______	_______	_______

When interest is compounded annually, 72 years later the account would contain $13,279.52. Better yet 8 years later that amount will have reached a level of $21,159.20. Ralph cannot believe his eyes! If interest is compounded quarterly, changing DT to .25 is the only necessary modification. When interest is compounded quarterly, Joe finds out that 72 years later he would have $14,562.98 in the bank. If he decides to leave the money in the bank just eight more years he would have $23,451.12. Joe now really hopes that interest is compounded quarterly.

But Alphonso, who is quite familiar with his math, tells him that he would get even more money if the interest is compounded monthly. In this case, DT will be set to .083333 (12 months in a year). Ralph would have $14,876.73 72 years after the bank account was first opened. He would have $24,013.16 8 years later. Joe leaves Alphonso happier than he had ever been, thinking about what he would buy tomorrow when he gets his money.

	Annually	Quarterly	Monthly
72 yrs.	$13,279	$14,562	$14,876
80 yrs.	$21,158	$23,451	$24,013

Model Visualized as a Positive Feedback Loop

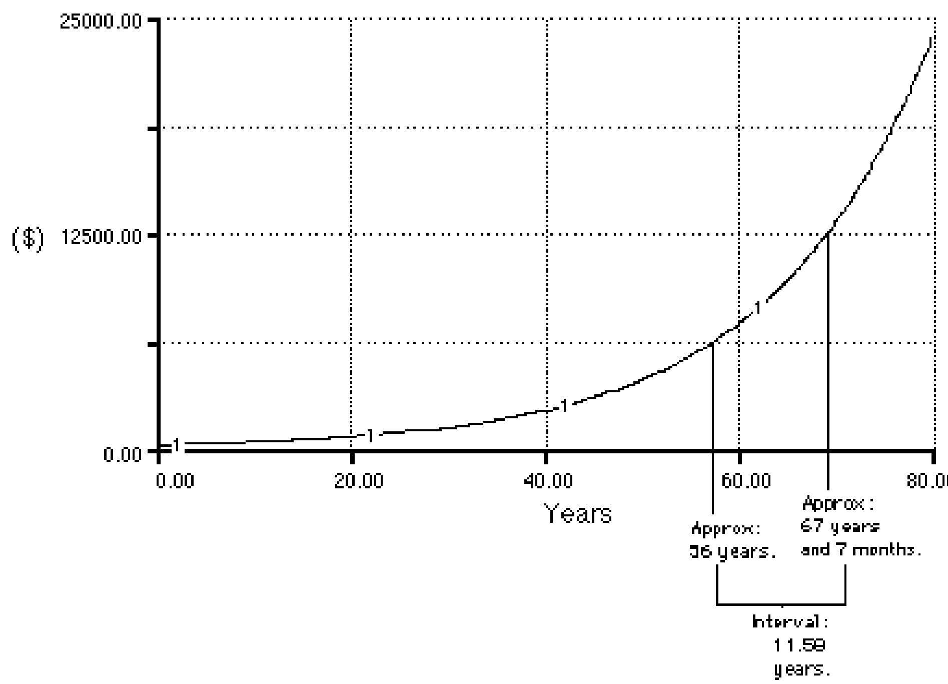
The bank balance model is a representation of a positive feedback loop. The bank balance continually feeds itself through the flow. After each time increment DT, the bank balance increases by the compounded interest. The populations of rabbits, bacteria, and humans also are examples of positive feedback loops because the bigger the population, the more births there are, the bigger the population becomes. The loops self-reinforce.

Introduction to Limits

As interest is calculated more frequently in a specified period, the total interest accrued over the year approaches a limiting value. This concept can be experimented with using a STELLA Computer package and a Macintosh. You could make DT smaller and see the effect on the bank balance after a certain period of time. Using this model, you could try changing DT to .00274 (1/365, interest compounded daily).

The idea of limits can also be visualized from the perspective of exponential growth. Mathematically, exponential growth is characterized by unbounded growth at a faster and faster rate. The quantity under study, in this case the quantity of money in the bank account, doubles repeatedly, after a specific time interval, called the "doubling time". As some stock in a positive feedback loop begins to increase, a snowball effect takes place and that quantity continues to increase at a faster and faster rate. In the case of the bank balance, when interest was compounded annually, the "doubling time" for the quantity of money in the bank account was approximately 11.95 years. In the case where interest was compounded quarterly, the "doubling time" was approximately 11.74 years, and in the case where interest was compounded monthly, the "doubling time" was approximately 11.58 years. As interest is compounded more often, the differences in doubling time get smaller and smaller, until the doubling time is very close to 11.55 years. Any amount of money that accrues at an interest rate of 6% annually, cannot be doubled in less than 11.55 years, independently of how often interest is compounded[5]. Other examples where exponential growth occurs is in the growth in population of bacteria or rabbits in an unlimited area, and in the growth of the world population in the last several centuries.

Illustration of the Notion of Doubling Time when Interest is Compounded Monthly



In approximately 11.58 years, the bank balance doubled from $6250 to $12500.

Figure 6.

BANK BALANCE cont. on page 8

BANK BALANCE, *contínted from page 7*

Appendix to The Bank Balance Problem

Mathematical Approach to Limits

The idea of limits is a very important concept in math, and it shall be briefly introduced in this paragraph. Assume that interest is calculated X times in a year. Let p_k be the amount of money in the bank at an arbitrary time. Time X later, the amount of money in the bank will be p_{k+1}. In other words:

$$p_{k+1}= p_k + 0.06(1/X)p_k.$$

Moving p_k to the left and dividing by (1/X) we get:

$$(p_{k+1} - p_k)/(1/X) = 0.06p_k.$$

Let

$$dp = p_{k+1} - p_k$$

and

$$dt = 1/X.$$

The purpose of this exercise is to calculate interest an infinite amount of times in a given time, so as to determine the maximum (limit) amount of money that the bank account can contain after a specified period of time. As interest is calculated more and more often in one year, X gets bigger and bigger, until it approaches infinity. When this happens, dt approaches 0, because 1 divided by infinity is so small that it is very close to 0. Expressed as a differential equation, the equation is:

$$dp/dt= 0.06p.$$

The solution to this differential equation is:

$$p(t)= e^{0.06t}p_0.$$

In Joe's case, as discussed earlier, over a period of 72 years, even if interest were calculated infinitely many times, presently, he would have $15,037.73, not a great difference from when interest was compounded monthly.

[1] *Kamil Msefer is a Sophomore at MIT, majoring in Math with Computer Science. He is presently working part-time in the System Dynamics Education Project.*

[2] *Linear meaning that in the relationship, an increase in one element is proportional to an increase in the other element. The increase in the bank balance is proportional to the increase in the interest paid.*

[3] *STELLA is a simulation Software package designed by High Performance Systems.*

[4] *It is important to differentiate between interest and interest rate. Interest is a flow that is equal to the interest rate multiplied by the bank balance. The interest rate is a constant equal to 6% annually.*

[5] *See section of appendix dealing with limits for a better understanding of why this is so.*

The complete text of this article, including all appendixes, can be obtained from The Creative Learning Exchange, *1 Keefe Road, Acton, MA 01720.*

INTERESTED IN INVESTING?

All of us are interested in promoting the use of systems education in our schools. A number of you have asked if there is a charge for the services of the Creative Learning Exchange, or what you can send to help defray the costs of printing and mailing to you.

The Creative Learning Exchange will continue to send out materials free of charge to all those on the mailing list, regardless of their desire to invest at this time. However, if you would like to invest in our effort here at the Creative Learning Exchange, your contribution would be appreciated. You may donate any amount you wish; perhaps $25 is a reasonable amount for a year. All contributions are tax-deductible.

I am sending ________ to *Trust in Diversity* to help invest in the future of systems education .

Name____

Address____

Thank you!!

The Creative Learning Exchange, 1 Keefe Road, Acton, MA 01720

The Creative Learning Exchange
1 Keefe Road
Acton, MA 01720
Phone (508) 287-0070 Fax (508) 287-0080

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