Guided Study Program in System Dynamics
System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management[1]

Solutions to Assignment #28
Tuesday, August 17, 1999

Reading Assignment:

Please refer to Road Maps 6: A Guide to Learning System Dynamics (D-4506-3) and read the following paper from Road Maps 6:

- *Economics Supply and Demand, by Jody Whelan and Kamil Msefer (D-4388)*

Please refer to Road Maps 7: A Guide to Learning System Dynamics (D-4508-1) and read the following paper from Road Maps 7:

- *Modeling Exercises: Section 2, by Joseph Whelan (D-4451-1)*

Exercises:

1. Economics Supply and Demand

A. How does the system dynamics model presented in this paper resemble or differ from your own mental model of supply and demand? Do you think the model accurately and realistically captures consumer behavior? Why or why not?

Before reading the paper my mental model of supply and demand was the classical economic theory, and I never considered the effect of inventory. After reading the paper and stopping to think about consumer behavior, I can see how the market price is affected by the inventory of goods rather than the rate at which goods are supplied. Just consider the sales at the department stores at the end of winter. Prices were slashed up to 70%, and suppliers were not shipping more winter clothing. The department stores are just trying to rid themselves of the winter inventory. If the rate at which manufacturers are supplying goods was what drives the price, then the new spring and summer clothes would be marked way down and the price for winter clothes would be unaffected.

B. Build the supply and demand model in Figure 10 on page 20. In your assignment solutions document, include the model diagram and documented equations.

In building the supply and demand model in Vensim, you should have non-dimensionalized the input and output to all three lookup functions, as below:

Model diagram:
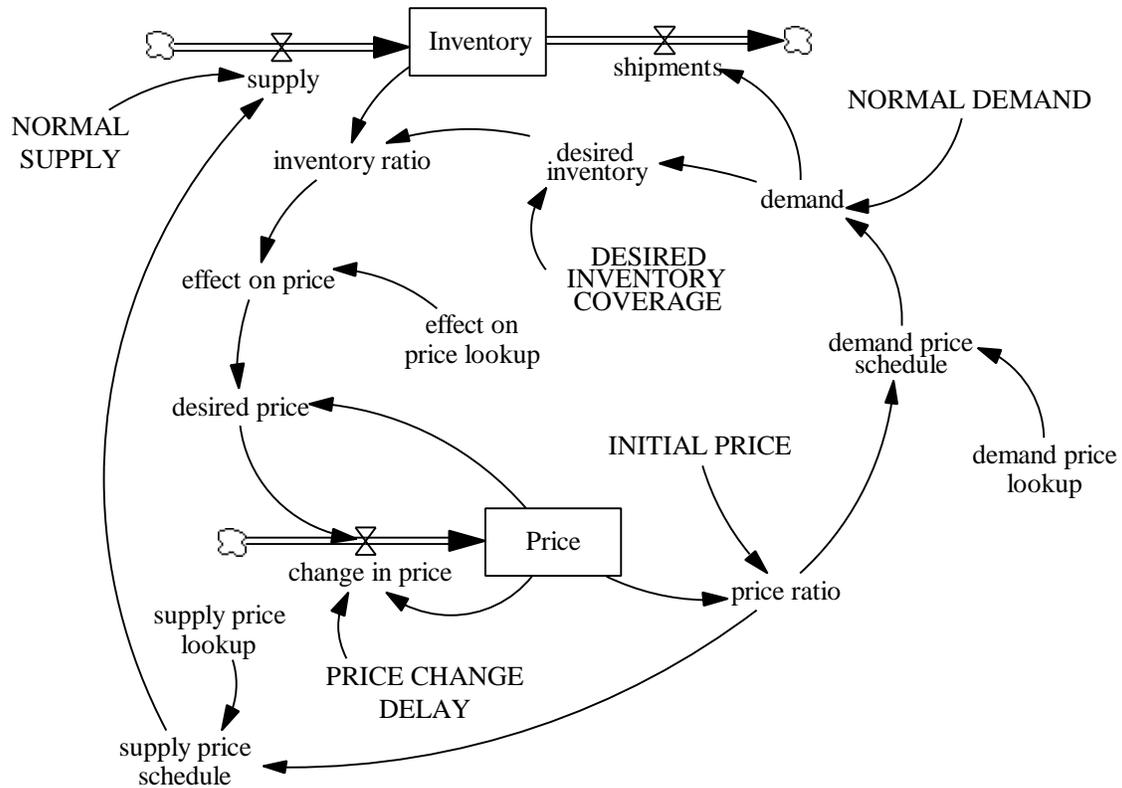
Model equations:

change in price = (desired price – Price) / PRICE CHANGE DELAY
    Units: (dollars/shirt)/week
    Change in price can be either positive or negative depending on the effect on price. If the effect is greater than 1, then price will increase. If the effect is less than 1, the price decreases. If effect equals 1, price remains the same. Price changes slowly, so we divide the change by price change delay.

demand = demand price schedule * NORMAL DEMAND + STEP(10,10)
    Units: shirts/week
    The rate at which consumers wish to purchase clothing from the company. The step function is used to push the system out of equilibrium.

demand price lookup ([(0,0) - (4,2)], (0,2), (1,1), (2,0.5), (3,0.25), (4,0.2))

Units: dimensionless
The nonlinear relationship between price ratio and demand.

demand price schedule = demand price lookup (price ratio)
Units: dimensionless
Based on the simple demand curve. At some particular price, the consumers are willing and able to purchase clothing at a certain rate; the lower the price, the higher the demand.

desired inventory = demand * DESIRED INVENTORY COVERAGE
Units: shirts
Desired inventory is the number of shirts the suppliers would like to have in inventory. It is calculated as the number of weeks worth of demand they would like to store in inventory.

DESIRED INVENTORY COVERAGE = 4
Units: week
The number of weeks of demand the suppliers would like to keep in inventory.

desired price = effect on price * Price
Units: dollars/shirt
The equilibrium price as set by the inventory ratio. The actual price will reach this value after a delay specified by the price change delay.

effect on price = effect on price lookup (inventory ratio)
Units: dimensionless
The effect regulating price change. When inventory is greater than desired inventory, the inventory ratio is greater than 1 and the price must be reduced. When the inventory ratio is less than 1, the price must be increased.

effect on price lookup ([(0.5,0) - (1.5,2)], (0.5,2), (0.8,1.35), (1,1), (1.2,0.75), (1.5,0.5))
Units: dimensionless
The nonlinear relationship between inventory ratio and price.

INITIAL PRICE = 15
Units: dollars/shirt
The initial price of each shirt.

Inventory = INTEG (supply – shipments, desired inventory)
Units: shirts
Inventory is the stock of produced clothing in the company's warehouse.

inventory ratio = Inventory / desired inventory
Units: dimensionless
The ratio of inventory to desired inventory.

NORMAL DEMAND = 57
>Units: shirts/week
>The normal number of shirts customers demand per week.

NORMAL SUPPLY = 57
>Units: shirts/week
>The normal number of shirts per week that can be supplied.

Price = INTEG (change in price, 15)
>Units: dollars/shirt
>Price is modeled as a stock in order to model the delays inherent in changes in price.

PRICE CHANGE DELAY = 15
>Units: week
>Prices do not change instantaneously. This value determines how quickly price can change.

price ratio = Price / INITIAL PRICE
>Units: dimensionless
>The ratio of price over initial price.

shipments = demand
>Units: shirts/week
>The shipments equal the demand. The shipments deplete the inventory.

supply = NORMAL SUPPLY * supply price schedule
>Units: shirts/week
>The price supply schedule is based on a classical short-term supply curve. The company uses this algorithm to set a desirable supply rate for a given price. This curve is being used in lieu of more complicated dynamics structure.

supply price lookup ([(0,0) - (4,2)], (0,0), (1,1), (2,1.5), (3,1.75), (4,2))
>Units: dimensionless
>The relationship between price ratio and supply.

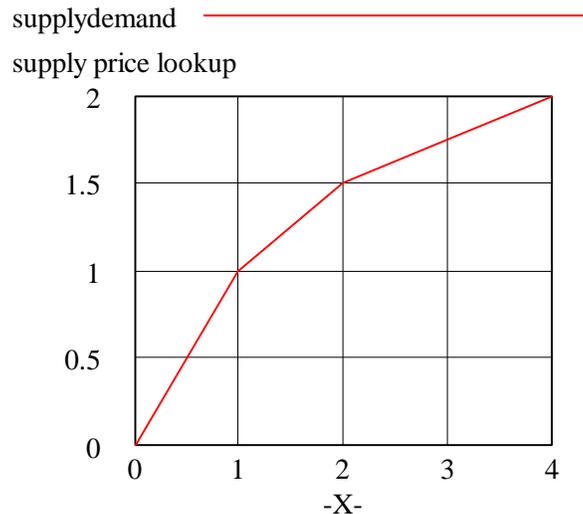supply price schedule = supply price lookup (price ratio)
>Units: dimensionless
>This is a short-term supply curve. At higher prices there is more incentive to produce, more producers can enter the market, etc. This is why this curve points upward as price increases.
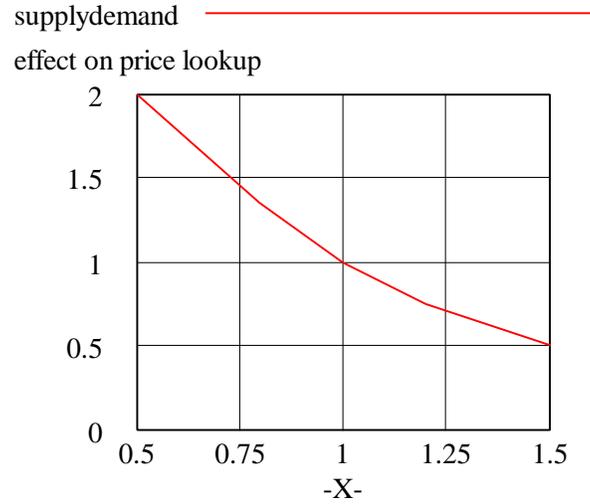
The lookup functions were non-dimensionalized as follows:

supplydemand ─────────────
demand price lookup
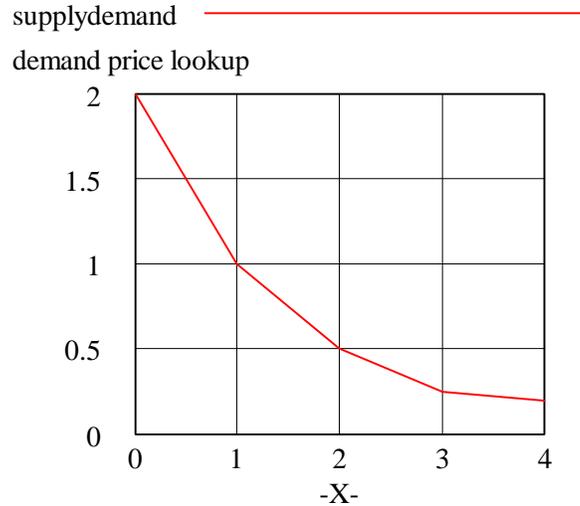
The "demand price lookup" takes in a ratio of price and initial price, then outputs a corresponding value that multiplies "NORMAL DEMAND" to get actual "demand." Initially the price and initial price are both 15, yielding a ratio of 1. In order to make sure that the lookup function replicates the old non-dimensionalized one, we would like a price of 15 to correspond to a demand of 57. We set the "NORMAL DEMAND" to a value of 57, and alter the lookup function so that with an input ratio of 1, the lookup outputs 1 as well. Adjust the remaining points accordingly. The new lookup ranges from 0 to 4 on the X-axis and are rounded slightly to make things look nice. A few data points were removed because not so many points are needed to attain the same general degree of smoothness.

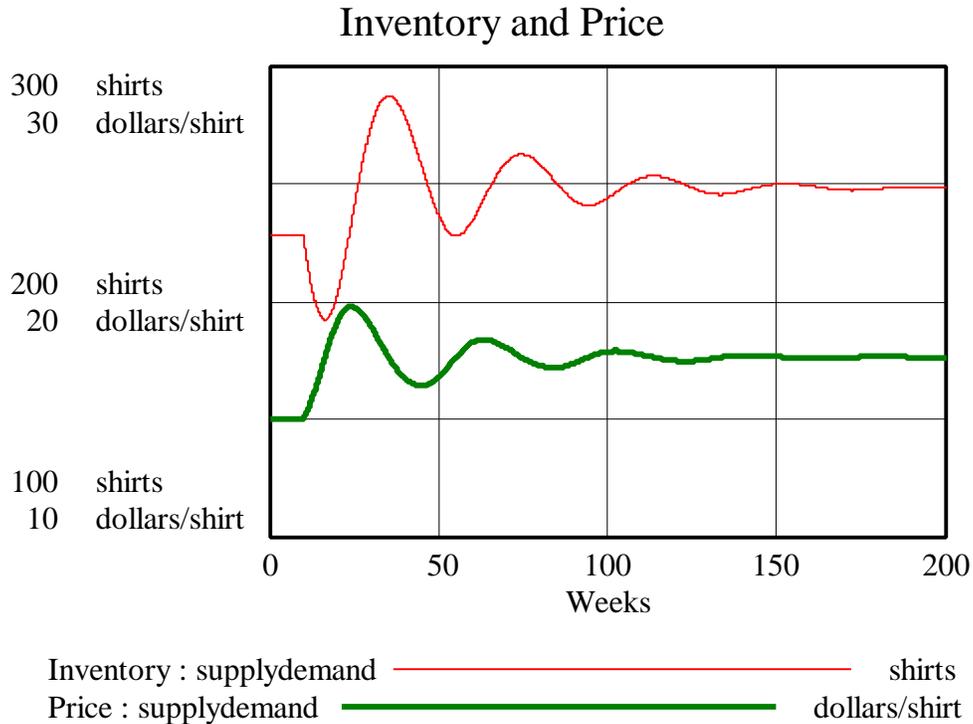supplydemand ─────────────
supply price lookup

The same sort of manipulation was done on the "supply price lookup." When the price is 15 and therefore the "price ratio" is 1, the output of the lookup, 1, multiplies the "NORMAL SUPPLY" of 57 to get an actual "supply" rate of 57 shirts/week. Again, some extraneous data points were removed.

supplydemand
effect on price lookup

The "effect on price lookup" did not need to be changed because it was already non-dimensionalized in the original paper. However, the lookup function in the paper contains 11 data points. The large number of data points is unnecessary. The same shape can be replicated with much fewer points, such as the five seen in the modified lookup function above.

Model behavior:

The new model with the non-dimensionalized lookup functions generates the following damped oscillatory behavior:

Inventory and Price

Inventory : supplydemand	—————————————	shirts
Price : supplydemand	▬▬▬▬▬▬▬▬	dollars/shirt

C. Do exercises 5.1 through 5.3 from the paper, using the model you built in part B. Did you get the answers that you expected?

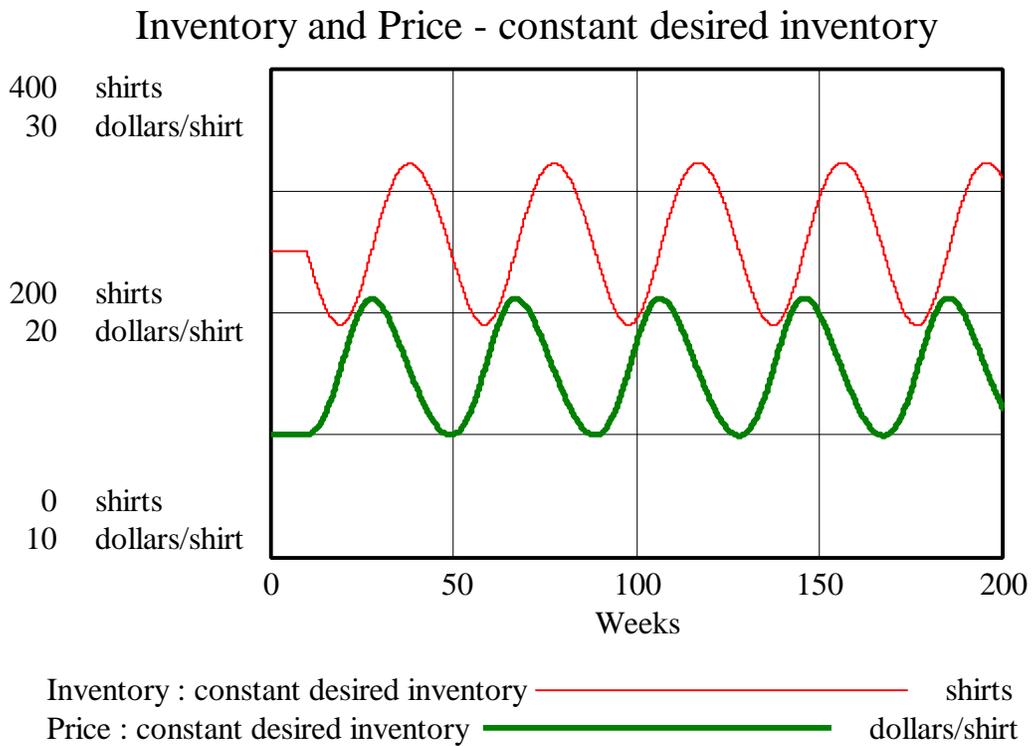The answers to these exercises are included in the paper on page 27.[2]

D. The supply and demand model produces damped oscillations, as we can see from the graphs in Section 7.2. Trace through the causal relationships and feedback loops. Explain what causes the damped oscillations. What structural elements make this model different from the generic structure for sustained oscillations?

A second-order system generates sustained oscillations when an increase in one stock drives up the flow to the other stock; but as the second stock increases, it drives down the flow to the first stock. Each flow in the system is directly affected by the other stock. When the system contains an additional negative feedback loop, however, the loop detracts from one of the two stocks but has no effect on the value of the other stock. As the affected stock decreases, it no longer drives the flow to the second stock up as much as before, and consequently the second stock decreases in amplitude. Furthermore, the second stock then drives the flow to the first stock less, and so on. As a result, the oscillations become damped. In other words, to dampen a system that gives sustained oscillations, add negative feedback loop(s).

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[2] Please refer to the **Economics Supply and Demand Corrections** section located at the end of these solutions.

In this model, as price increases, supply goes up and demand goes down simultaneously, both increasing the value of the inventory stock. As the inventory gets larger and larger, however, suppliers try to get rid of excess inventory by decreasing prices, hoping to drive up demand. Consequently, the price and inventory fluctuate, creating oscillatory behavior. The loop from "Price" to "supply" to "Inventory" to "Price" and the loop from "Price" to "demand" to "Inventory" to "Price" both contribute to the oscillatory behavior in the supply-demand model.

For the system to generate only sustained oscillations, the model would contain only the above two loops. The supply and demand model, on the other hand, contains a number of additional loops that affect its dynamics.

First, a positive feedback loop is present from "Price" to "desired price" to "change in price" and back to "price." This positive feedback loop generates exponential behavior. Despite this positive feedback loop, however, the model still exhibits the damped oscillatory behavior associated with a negative feedback loop in addition to the loops causing oscillations. Why? Because a dominant negative feedback loop exists in the system.

The highlighted loop in the following diagram is responsible for changing the system from a sustained oscillations to a damped oscillations system (causal relations marked with a + or a -):

The above loop contains three negative causal relations ("price ratio" to "demand price schedule," "desired inventory" to "inventory ratio," and "inventory ratio" to "effect on price"), so it is a negative feedback loop. As the price increases, the demand decreases and inventory increases. The negative causal relation between "desired inventory" and "inventory ratio" (which makes "inventory ratio" greater than what it would be without the negative feedback loop) decreases the "effect on price."

To prove that the highlighted loop is the cause of the damping behavior, set the "desired inventory" variable to a constant and simulate the model. If, for example, "desired inventory" is set to a constant value of 250 shirts, the highlighted negative feedback loop is no longer intact, and the model behaves in the following manner (no more damped oscillations!):

Inventory and Price - constant desired inventory

| 400 | shirts |
| 30 | dollars/shirt |

| 200 | shirts |
| 20 | dollars/shirt |

| 0 | shirts |
| 10 | dollars/shirt |

Weeks

Inventory : constant desired inventory ———————————— shirts
Price : constant desired inventory ━━━━━━━━━━ dollars/shirt

We can therefore conclude that the highlighted negative feedback loop is responsible for the damping in the oscillating system.

2. Modeling Exercises: Section 2

A. *Build the urban growth model. In your assignment solutions document, include the model diagram, documented equations, graphs of lookup functions, and a graph of the base run behavior.*

Model diagram:

Model equations:

AVERAGE LIFETIME = 66.7
> Units: Year
> The average lifetime of a person living in the urban area is approximately 67 years.

AVERAGE STRUCTURE LIFETIME = 50
> Units: Year
> Average lifetime of a business structure.

BIRTH FRACTION = 0.015
> Units: 1/Year
> The fraction of the population that reproduces each year.

births = Population * BIRTH FRACTION

Units: people/Year
The number of people born in the area per year.

Business Structures = INTEG (construction - demolition, 1000)
Units: structure
Number of business structures in the city.

construction = Business Structures * CONSTRUCTION FRACTION * labor availability
multiplier * land availability multiplier
Units: structure/Year
Rate of construction of business structures. It is affected by the number of already existing business structures, a normal construction fraction, and the availability of land and labor.

CONSTRUCTION FRACTION = 0.02
Units: 1/Year
Normal rate of construction of business structures per existing business structure.

deaths = Population / AVERAGE LIFETIME
Units: people/Year
The number of people who die in the area each year.

demolition = Business Structures / AVERAGE STRUCTURE LIFETIME
Units: structure/Year
Rate of demolition of business structures.

inmigration = Population * INMIGRATION NORMAL * job attractiveness multiplier
Units: people/Year
The number of people who move into the urban area each year. It is affected by the current population, a normal fraction of inmigration, and the availability of jobs.

INMIGRATION NORMAL = 0.08
Units: 1/Year
The fraction of the population that moves into the area each year under normal conditions.

job attractiveness lookup ([(0,0) - (2,4)], (0,4), (0.2,3.95), (0.4,3.82), (0.6,3.56),
(0.8,2.86), (1,1.24), (1.2,0.64), (1.4,0.32), (1.6,0.18), (1.8,0.105), (2,0.075))
Units: dmnl
The job attractiveness lookup function shows the effect of labor availability on inmigration. When there are many available jobs (labor availability is less than 1), people are inclined to move to the city. When there are not enough available jobs (labor availability greater than 1), people tend not to move to the urban area.

job attractiveness multiplier = job attractiveness lookup (labor availability * jobs per
person)
Units: dmnl
The job attractiveness multiplier is a function of the labor availability.

jobs = Business Structures * JOBS PER STRUCTURE
Units: job
Number of jobs provided by the existing business structures. It is the product of
the number of business structures and the average number of jobs per structure.

jobs per person = 1
Units: job/people
Number of jobs per worker.

JOBS PER STRUCTURE = 20
Units: job/structure
The number of jobs provided by each structure.

labor availability = labor force / jobs
Units: people/job
The ratio between the labor force and the number of available jobs.

labor availability lookup ([(0,0) - (2,4)], (0,0.05), (0.2,0.105), (0.4,0.225), (0.6,0.36),
(0.8,0.54), (1,0.84), (1.2,1.24), (1.4,2.36), (1.6,3.34), (1.8,3.86), (2,4))
Units: dmnl
The labor availability multiplier shows the effect of labor availability on
construction. When labor availability is greater than 1, businesses are more likely
to construct new structures.

labor availability multiplier = labor availability lookup (labor availability * jobs per
person)
Units: dmnl
The labor availability multiplier is a function of the labor availability ratio.

labor force = Population * LABOR PARTICIPATION FRACTION
Units: people
The number of people who are willing and able to work. It is a constant fraction
of the population.

LABOR PARTICIPATION FRACTION = 0.35
Units: dmnl
The fraction of the total population that is willing and able to work.

LAND AREA = 5000
Units: acre
Total land area available for commercial development.

land availability lookup ([(0,0) - (1,4)], (0,1), (0.1,2.3), (0.2,2.98), (0.3,3.34), (0.4,3.48), (0.5,3.5), (0.6,3.44), (0.7,3.12), (0.8,2.3), (0.9,1), (1,0))
    Units: dmnl
    The land availability multiplier shows the effect of availability of land on business construction. When nearly all the land is available, construction is slow because there are very few businesses around to provide capital. As more of the land area is occupied, the effect on construction increases. As the available land area is used up, construction drops until it is zero at maximum capacity.

land availability multiplier = land availability lookup (land fraction occupied)
    Units: dmnl
    The land availability multiplier is a function of the land fraction occupied.

land fraction occupied = Business Structures * LAND PER STRUCTURE/LAND AREA
    Units: dmnl
    The fraction of the commercial land that has already been developed.

LAND PER STRUCTURE = 1
    Units: acre/structure
    The amount of land required by each business structure.

outmigration = Population * OUTMIGRATION FRACTION
    Units: people/Year
    The number of people who leave the urban area each year.

OUTMIGRATION FRACTION = 0.08
    Units: 1/Year
    The fraction of the population that leaves the area each year.
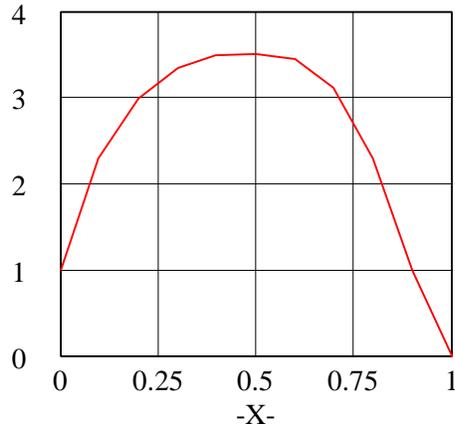
Population = INTEG (births + inmigration - deaths - outmigration, 50000)
    Units: people
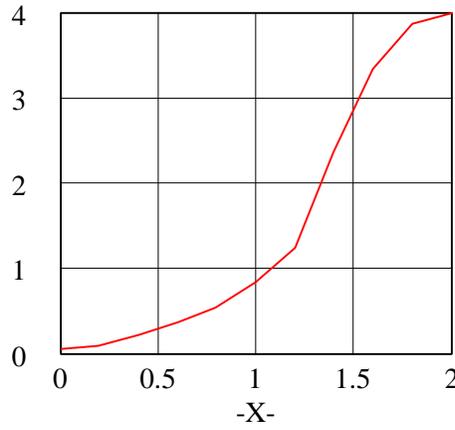   The number of people living in the urban area.
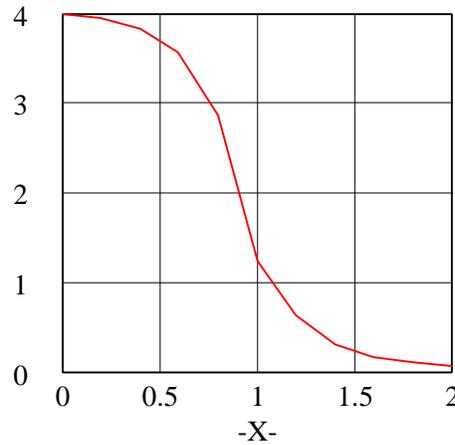
urban

land availability lookup

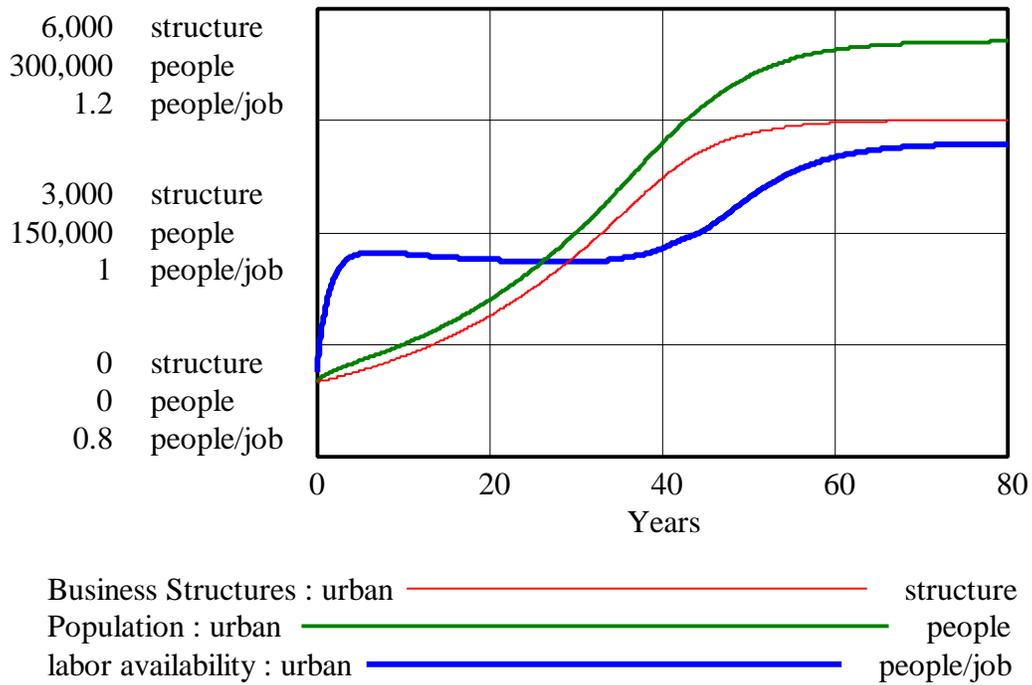
urban

labor availability lookup

urban

job attractiveness lookup

Base run behavior:

Base run - Business Structures, Population, and labor availability

6,000	structure
300,000	people
1.2	people/job
3,000	structure
150,000	people
1	people/job
0	structure
0	people
0.8	people/job

0   20   40   60   80

Years

Business Structures : urban ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯ structure
Population : urban ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯ people
labor availability : urban ▬▬▬▬▬▬▬▬▬▬ people/job

B. Examine the rate equation for "In Migration." What factors does the "In Migration Normal" represent? What factors does the "Job Attractiveness Multiplier" represent? What additional factors influence migration in the real world?

The "In Migration Normal" represents normal in migration due to the inherent attractiveness of the city. The rate, "inmigration," represents the 8% "In Migration Normal" plus the effect that the number of jobs available has on migration patterns. It is assumed that if a city has plenty of job openings, people will move to that city in order to get a job. This model does not take into consideration that the number of people who are willing to work in a certain community may suddenly increase, or that people will sometimes hold two jobs, or even that not all jobs are year-round. All of these factors would change the jobs available and thus change the migration patterns of the community.

The "Job Attractiveness Multiplier" represents how appealing it is for one to move to a particular city. A value of 1.00 means that those who would move to the city under normal job conditions will continue to do so. A value of 2.00 would indicate that there is a seriously healthy job market and that twice as many people would be moving into the city.

Additional factors influencing migration are, for example:

- availability and cost of housing
- attainability of the area (airports, highways, absence of traffic jams)
- attractiveness of surrounding areas (woods, water, mountains, etc.)
- opportunities for recreation, amusement, etc.
- average wage level
- absence of (air) pollution
- good social security regulations
- absence of violence, low crime rate

C. Equilibrium employment conditions in the urban growth model are insensitive to most parameter changes. What parameter changes can alter the equilibrium value of the "Labor Availability" ratio? Are the parameter changes reasonable? Why or why not? What do such changes represent in the real world? Explain how the parameter changes that you made change the "Labor Availability" and substantiate your explanations with graphs of the behavior produced by the different scenarios.

Factors such as "labor participation fraction" and "jobs per structure," which are intuitively thought to influence employment conditions in equilibrium, in fact have no effect on the equilibrium employment conditions. These parameters do, however, change the equilibrium values of "Population" and "Business Structures." On the other hand, parameters that influence the inflows and outflows to "Population" and the equilibrium value of "Population" do have an effect on the equilibrium employment conditions. These parameters are the "INMIGRATION NORMAL," "BIRTH FRACTION," "OUTMIGRATION FRACTION," and "AVERAGE LIFETIME." If, for example, the "INMIGRATION NORMAL" increases, the "job attractiveness multiplier" should decrease in order to keep the "Population" in equilibrium. The "job attractiveness multiplier" decreases only if "labor availability" increases. While some of these parameters are more or less fixed values (for example, "AVERAGE LIFETIME"), others can be influenced in the real world by various policy changes.

If the "INMIGRATION NORMAL" were to double to 16% while outmigration remained the same, the equilibrium value of "labor availability" would increase. This could occur if, for example, the environmental conditions in the city were better than in other cities:

D-5012-1

INMIGRATION NORMAL 16 %
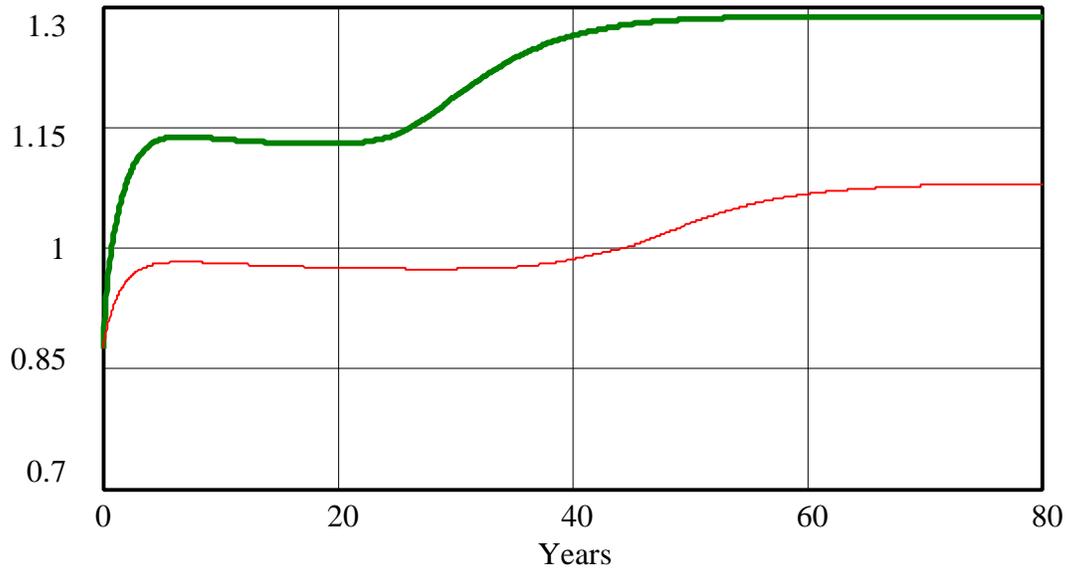
|	
labor availability : urban	people/job
labor availability : inmigration 16%	people/job

If the "INMIGRATION NORMAL" was reduced to 5%, the equilibrium value of "labor availability would decrease to a value lower than 1. Inmigration could be discouraged by governmental policies, taxes, etc.:

INMIGRATION NORMAL 5%

| labor availability : urban | people/job |
| labor availability : inmigration 5% | people/job |
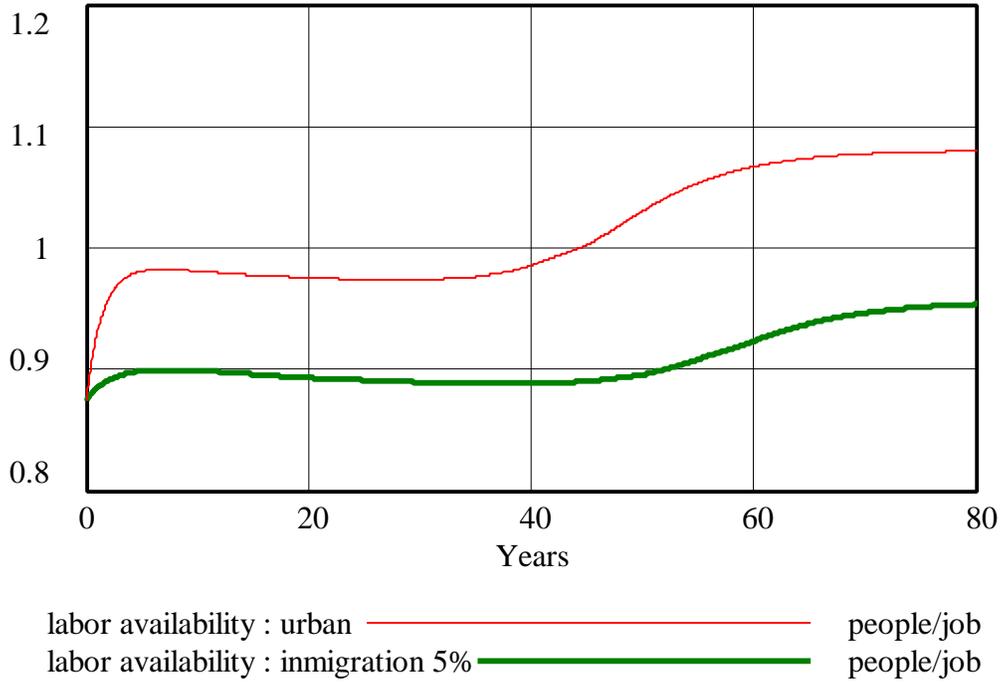
A reduction in the "BIRTH FRACTION" to 0.01 through a birth control program will only have a limited effect on "labor availability," whose equilibrium value would still be higher than 1.

BIRTH FRACTION 1%
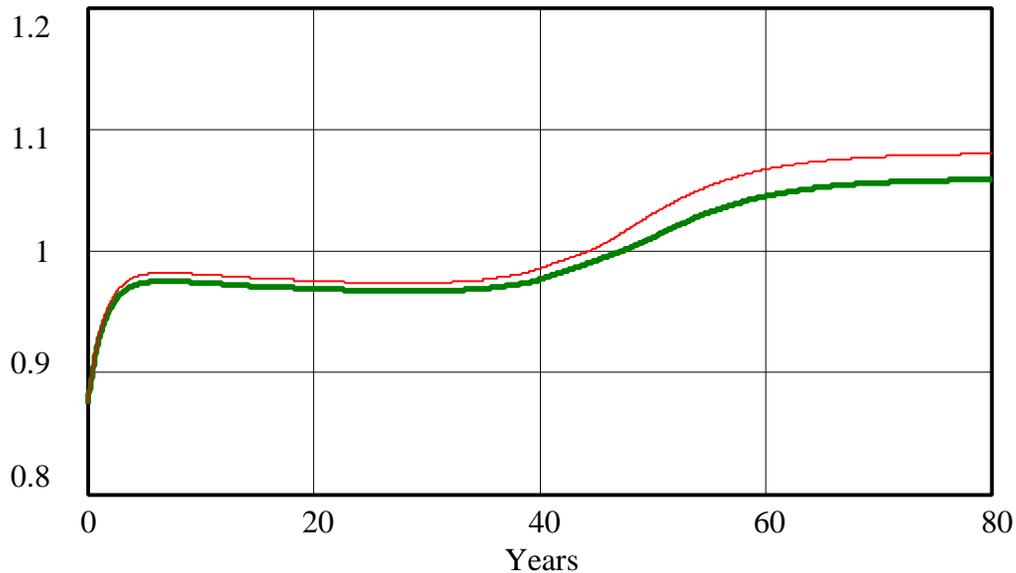
| | labor availability : urban | people/job |
| | labor availability : birth fraction 1% | people/job |

D. Under conditions of job shortage or low wages, some people may choose not to participate in the labor force. Add equations to the urban growth model to permit the "Labor Participation Fraction" to change according to fluctuations in the "Labor Availability" ratio. What dynamic behavior do you expect? Simulate the model. Does the model behave as you expected? Why or why not? In your assignment solutions document, include the modified model diagram, documented equations, graphs of lookup functions, and a graph of the model behavior.

<u>Modified model diagram:</u>
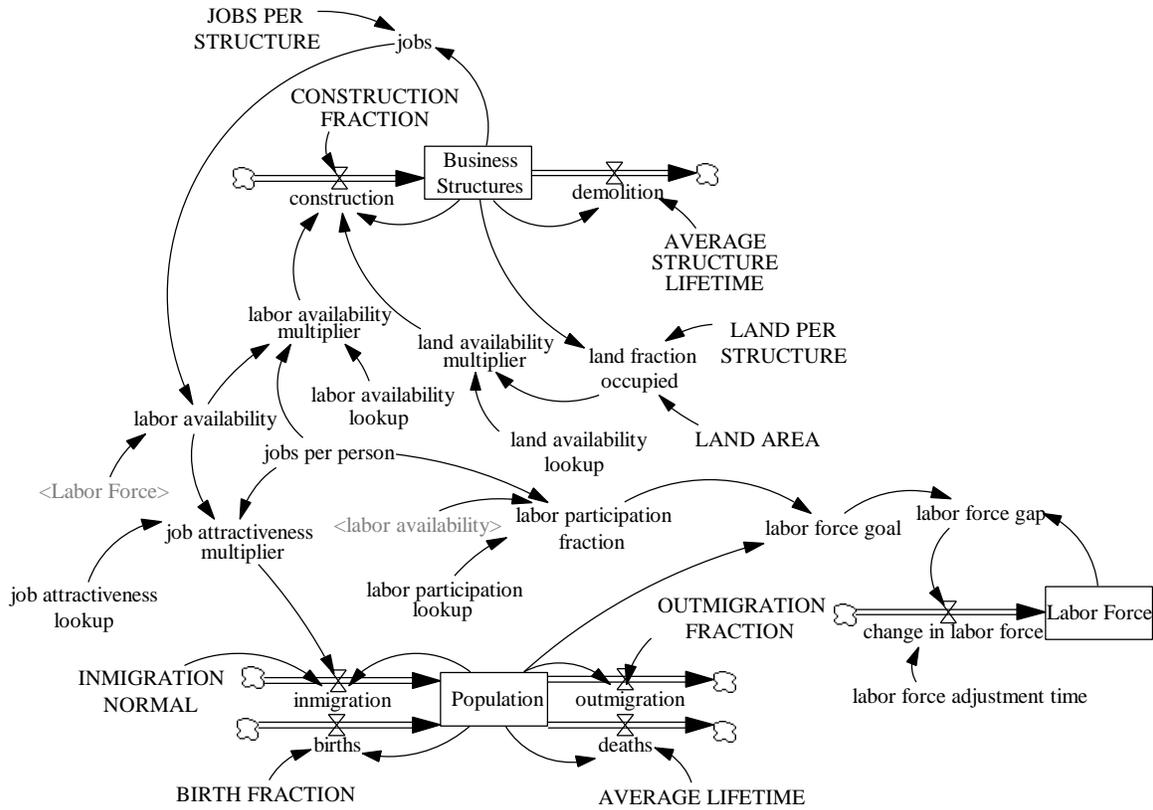
Modified model equations:

change in labor force = labor force gap / labor force adjustment time
> Units: people/Year
> The annual change in the labor force.

Labor Force = INTEG (change in labor force, 0.35*Population)
> Units: people
> The number of people who are in the labor force.

labor force adjustment time = 1
> Units: Year
> The time it takes to adjust the labor force to the goal value.

labor force gap = labor force goal - Labor Force
> Units: people
> The difference between the number of people who would like to be in the labor force and those who actually are in the labor force.

labor force goal = Population * labor participation fraction
> Units: people
> The number of people who are willing and able to work.

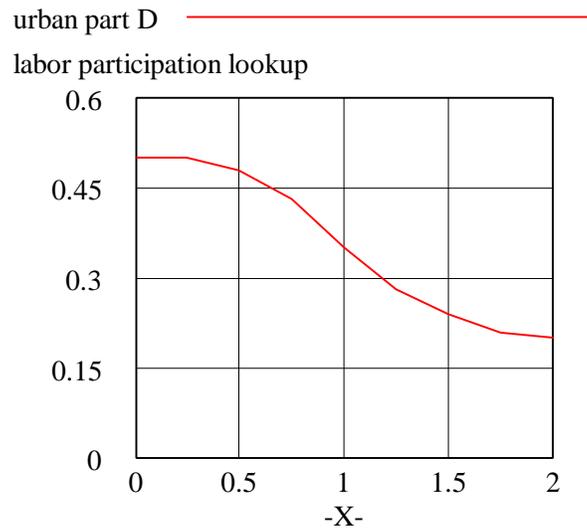labor participation fraction = labor participation lookup (labor availability * jobs per person)
        Units: dmnl
        The fraction of the total population that is willing and able to work.

labor participation lookup ([(0,0) - (2,0.6)], (0,0.5), (0.25,0.5), (0.5,0.48), (0.75,0.43), (1,0.35), (1.25,0.28), (1.5,0.24), (1.75,0.21), (2,0.2))
   Units: dmnl
        The function representing the relationship between labor availability and the labor participation fraction.

Notice that in order to have a variable labor participation fraction, you must model the labor force (or the labor participation fraction itself) as a stock. If you simply tried to connect labor availability to the labor participation fraction, which then connects back to labor availability through the labor force, your model would contain a feedback loop without a stock. Not only would this be incorrect, but your modeling software should give an error message and refuse to simulate the model.
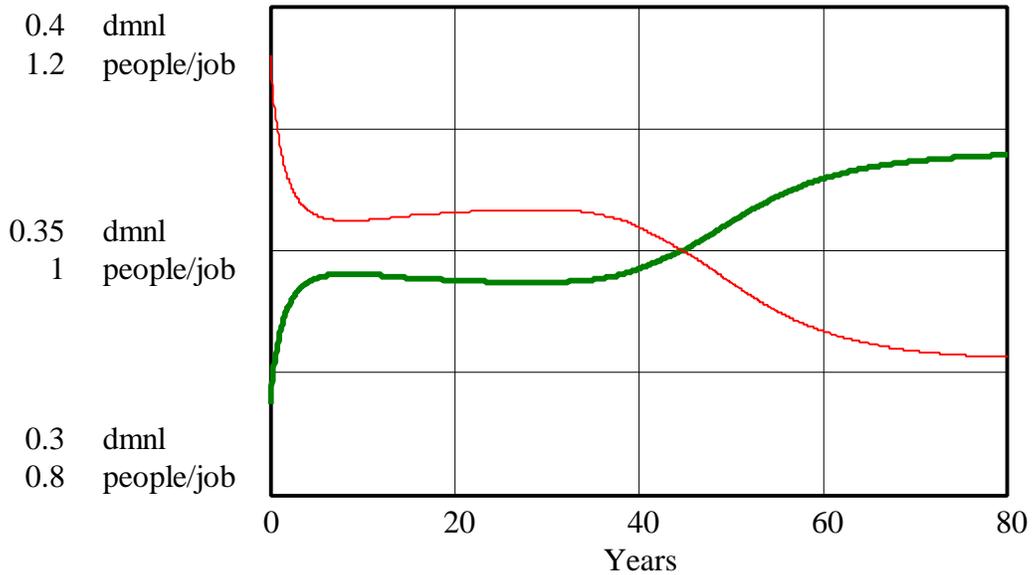
Model behavior:

Population and labor availability - part D

300,000	people	
1.2	people/job	
150,000	people	
1	people/job	
0	people	
0.8	people/job	

Years
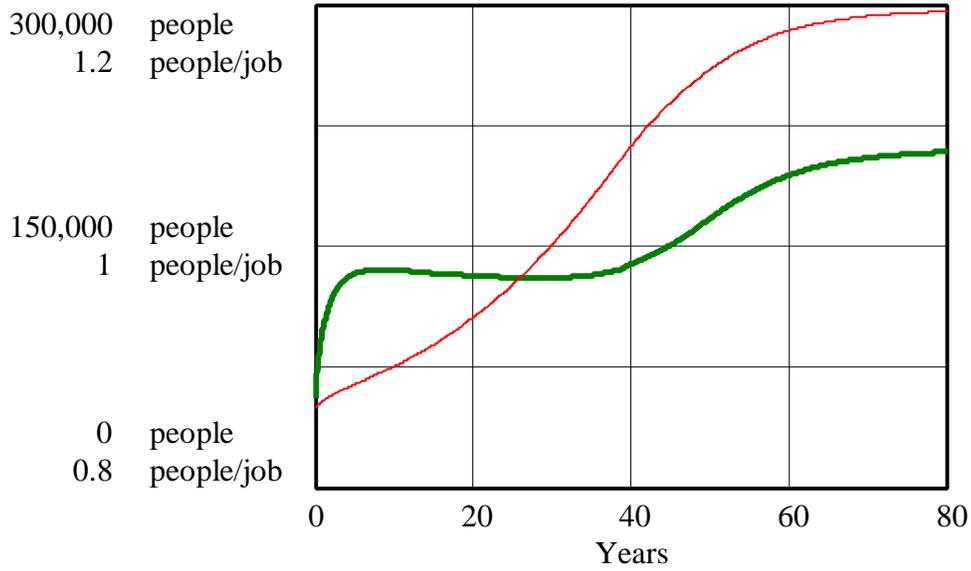
Population : urban part D ———————————— people
labor availability : urban part D ━━━━━━━━━ people/job

labor participation fraction and labor availability - part D

0.4	dmnl	
1.2	people/job	
0.35	dmnl	
1	people/job	
0.3	dmnl	
0.8	people/job	

Years

labor participation fraction : urban part D ———————— dmnl
labor availability : urban part D ━━━━━━━━━ people/job

The labor participation fraction is affected by labor availability. When labor availability is lower than 1, the labor participation fraction is higher than the normal value of 0.35. As soon as labor availability rises above 1, however, the labor participation fraction drops below 0.35.

The behavior of the system does not change drastically with the addition of a variable labor participation fraction. The equilibrium value of labor availability does not change significantly compared to the base run. The behavior of the stock of "Business Structures" does not change at all, while "Population" settles at only a slightly higher equilibrium value than in the base run:

Population - base run vs. part D

Population : urban — people
Population : urban part D — people
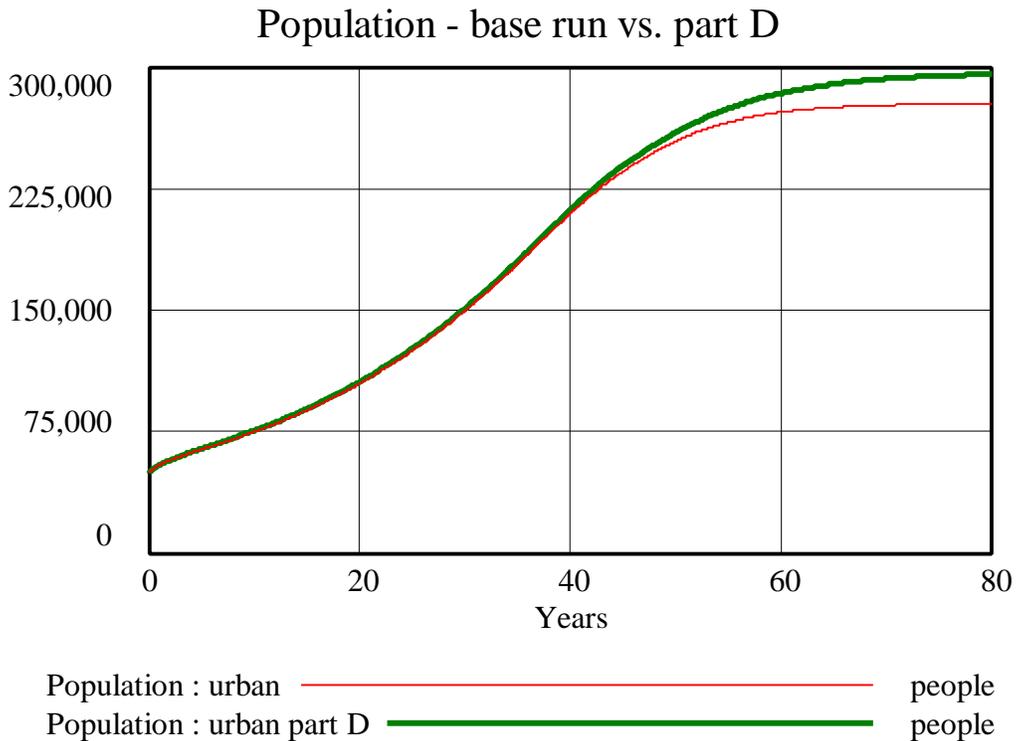
Economics Supply and Demand Corrections

In the Economics Supply and Demand paper (D-4506-3) referred to in this assignment there appears to be a mistake in the Time Step used in the model. The model apparently uses a Time Step of 1, which changes the behavior of the model slightly. The most prominent indication of this error is in questions 4 and 5 of the paper, which in the paper's solutions results in sustained oscillations. This is an error in the DT chosen. Had a smaller DT been used, such as .125, the model would exhibit damped oscillations.

This is an instructive mistake. First, the direction and degree of change in behavior from the changing "DESIRED INVENTORY COVERAGE" is important. The policy implications drawn from the model are correct, although the apparent behavior of sustained oscillations are false and slightly misleading. In handling this, there is an important distinction to be made between (a) exact model behavior, and (b) the effect on policy design. We should not depend on a model to replicate exactly an actual system. There are many things that the model does not include. However, we would like to expect that the model shows correctly the direction and approximate magnitude of change in behavior from a policy change.

Second, it is important to check for DT errors!!! This paper has gone through a number of revisions, but it is the information imbedded in the model that is the most difficult to discover! When opening a Vensim model it should be a habit to choose a small Time Step. It is also a good modeling habit to regularly halve your DT to check if any differences in behavior result (see the paper "Mistakes and Misunderstandings: DT error" (D-4695) on the GSP websight).

Third, it is a good practice to include your Time Step in the documentation! All of the solutions received so far have the Time Step documented, and this has made identifying errors easier.