

Guided Study Program in System Dynamics
System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management¹

Solutions to Assignment 23
Saturday, May 29, 1999

Reading Assignment:

Please read the following:

- Industrial Dynamics,² by Jay W. Forrester, Chapter 5
- Principles of Systems,³ by Jay W. Forrester, Chapter 5

1. Industrial Dynamics and Principles of Systems

Please read chapter 5 of Industrial Dynamics and chapter 5 of Principles of Systems carefully. The chapters cover some fundamental concepts to keep in mind when formulating system dynamics models.

The following two modeling exercises include a one-paragraph description of a characteristic behavior of a type of system. Each exercise then contains guidelines to help you conceptualize and formulate a model that demonstrates the dynamics of the system. You will be asked to simulate the model and analyze how the model responds to certain inputs. Finally, you will use some of the tests introduced in the paper by Ray Shreckengost, Dynamic Simulation Models: How Valid are They? (see assignment 15), to evaluate the validity of the model.

2. Modeling Exercise: An Ecological System

The following description of an ecological system is taken from “Concepts of Ecology”⁴ by E. Kormondy:

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² Forrester, Jay W., 1961. *Industrial Dynamics*. Waltham, MA: Pegasus Communications. 464 pp.

³ Forrester, Jay W., 1968. *Principles of Systems*, (2nd. ed.). Waltham, MA: Pegasus Communications. 391 pp.

⁴ Kormondy, E. J. (1969) *Concepts of Ecology*, Englewood Cliffs: Prentice-Hall, p. 110.

“...At a critical time in the life history of a given population, a physical factor such as light or a nutrient may be significant as a regulatory agent; at another time, parasitism, predation, or competition, or even some other physical factor may become the operative factor. As complex and as variable as the niche of any species is, it is unlikely that the regulation comes about by any single agency. However, there does appear to be considerable and mounting evidence, both empirical and theoretical, to suggest that populations are self-regulating through automatic feedback mechanisms. Various mechanisms and interactions appear to operate both in providing the information and in the manner of responding to it, and with the exceptional case of a catastrophe, the stimulus to do so appears to depend on the density of the population. The end effect is one of avoiding destruction of a population’s own environment and thereby avoiding its own extinction.”

You will be building, step by step, a first-order system dynamics model (that is, containing one stock) to study an ecological system in Alaska, initially inhabited by 100 otters. Otters do not reproduce as quickly as some animals lower on the food chain; they usually cannot reproduce until they are three years of age, and the young are born an entire year after conception. Although each pair of otters gives birth to one to three babies per year, few otters survive infancy. Assume that one baby otter survives infancy each year for every 4 adult otters in this particular ecological system. The 200-acre area of the ecosystem keeps the population in check due to limited natural resources. Under uncrowded conditions (less than 50 otters in the system), an otter lives on average about 15 years. When the ecosystem is crowded (more than 200 otters in the system), the habitat can no longer support all otters and about two out of five otters die per year. Furthermore, we know that initially (in 1984), the system is in equilibrium.

A. Start building the model by representing a stock of your choice. Formulate the stock’s flows. In your assignment solutions document, include the model diagram and documented equations.

The stock in this system is the number of otters. The number of otters is changed by births, which increase the number of otters, and by deaths, which reduce the number of otters.

First, consider the “births.” The rate of “births” depends on the “BIRTH FRACTION.” The article says that one baby otter survives infancy per four adult otters per year. Hence, the “BIRTH FRACTION” should equal 0.25 otters per otter per year. (To be more precise, note that only about 4/5, or 0.8 of the otter population are adults, and so the true birth fraction is $0.8 \cdot 0.25 = 0.2$ and not 0.25. For the rest of this exercise, however, we have used a “BIRTH FRACTION” of 0.25.)

The rate of “deaths” depends on the death fraction, which is the fraction of the population that dies each year. The death fraction is the inverse of the “life span.” We are told that the “life span” depends on the density of otters in the region. Thus, a lookup function,

“effect of density on life span,” is needed to show how average life span changes with density. Density, defined as the otter population divided by the area, must be non-dimensionalized before being used as an input to the lookup function. Therefore, a “NORMAL DENSITY” and “NORMAL LIFE SPAN” have to be chosen such that when the “density” equals the chosen “NORMAL DENSITY,” the “life span” equals the “NORMAL LIFE SPAN.”

Choose the otter density in 1984, 0.5 otters/acre to be the “NORMAL DENSITY.” Given that the system is in equilibrium in 1984, the death fraction will equal the “BIRTH FRACTION” = 0.25 per year. Hence, the “NORMAL LIFE SPAN” will be $1/0.25 = 4$ years.

Constructing the lookup function for the “life span multiplier” is the next step. Using the normal values selected above, and the information given in the system description, we can get the following points for the lookup function:

density ratio = density / NORMAL DENSITY

When “density ratio” is small, the output, “life span multiplier,” is large as otters live longer than the normal life span. We are told that as long as “density ratio” is less than 0.5 (that is, “density” is less than 0.25 otters per acre, which equals a population of 50 otters), “life span” is 15 years, equal to 3.75 times the “NORMAL LIFE SPAN.”

When “density ratio” is greater than 2, the “life span multiplier” will be 0.625. When “density” is greater than twice the normal density, death fraction equals 0.4 (two out of five otters die each year, as mentioned). Thus, average “life span” is $1/0.4 = 2.5$ years, which equals $2.5/4$, or 0.625 of the “NORMAL LIFE SPAN.”

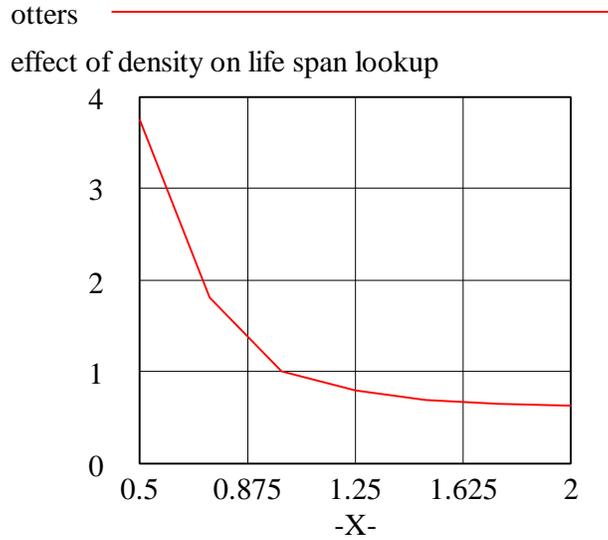
We now have the following points for our table function:

density ratio less than or equal to 0.5, output = 3.75

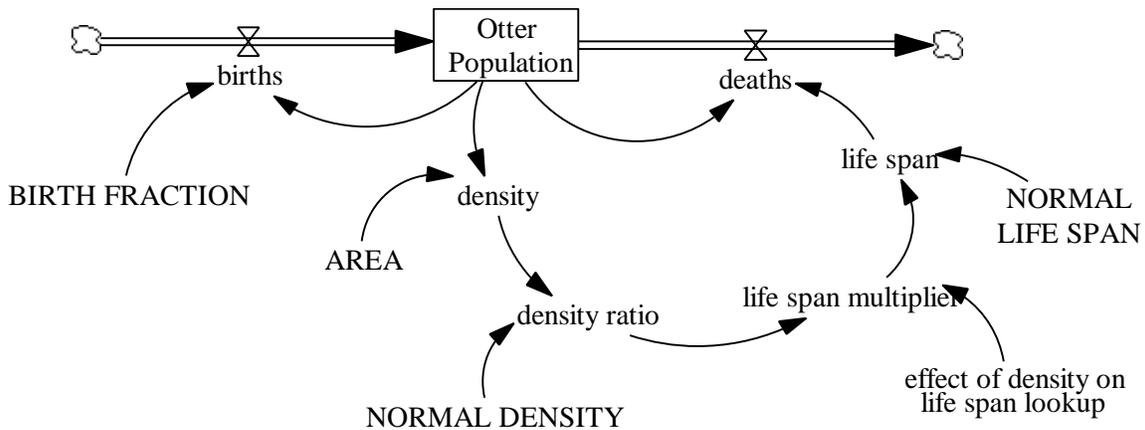
density ratio equal to 1, output = 1

density ratio greater than or equal to 2, output = 0.625

Using this data, we can construct a lookup function as shown below. Notice that in order to make the lookup function smooth and without sharp corners, we added a few more data points that were not specifically mentioned in the system description.



Model diagram:



Model equations:

AREA = 200
 Units: acre
 The total area of the ecosystem in consideration.

BIRTH FRACTION = 0.25
 Units: 1/year
 The birth fraction indicates how many young otters are born per existing otter each year. Some otters die when they are very young, and such otters are not accounted for.

births = Otter Population * BIRTH FRACTION
 Units: otter/year
 The number of otters born each year.

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deaths = Otter Population / life span

Units: otter/year

The number of otters that die each year.

density = Otter Population / AREA

Units: otter/acre

The number of otters per unit area of the ecosystem.

density ratio = density / NORMAL DENSITY

Units: dmnl

The ratio between current density and the normal density.

effect of density on life span lookup ((0,0) - (2,4)], (0.5,3.75), (0.75,1.8), (1,1),
(1.25,0.8), (1.5,0.7), (1.75,0.65), (2,0.625))

Units: dmnl

A lookup function showing how the life span of otters changes with density.

life span = NORMAL LIFE SPAN * life span multiplier

Units: year

The average life span of an otter.

life span multiplier = effect of density on life span lookup(density ratio)

Units: dmnl

The multiplier showing the effect of density on the life span of otters.

NORMAL DENSITY = 0.5

Units: otter/acre

The normal density is the density when the life span equals the NORMAL LIFE SPAN.

NORMAL LIFE SPAN = 4

Units: year

When the density equals the NORMAL DENSITY, life span equals the NORMAL LIFE SPAN.

Otter Population = INTEG (births – deaths, 100)

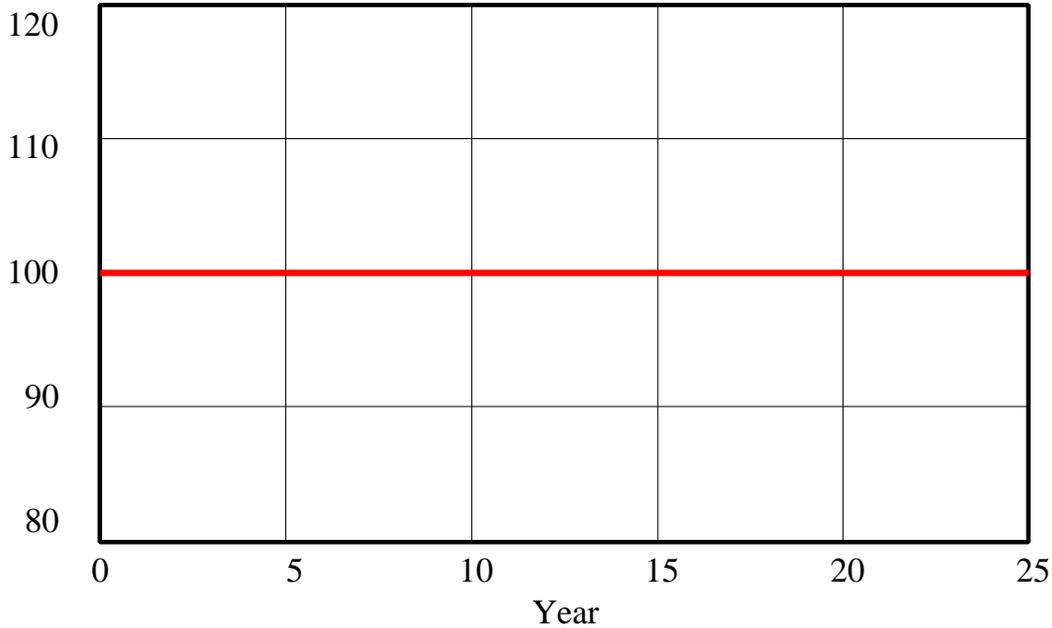
Units: otter

The total number of otters in the ecosystem at any time.

B. Draw a reference mode for the behavior of the stock over 25 years. Simulate the model for 25 years starting in 1984. In your assignment solutions document, include a graph of model behavior. Explain the behavior. Is the system in equilibrium? Why or why not?

Model behavior:

Behavior of Otter Population



Otter Population : otters ————— otter

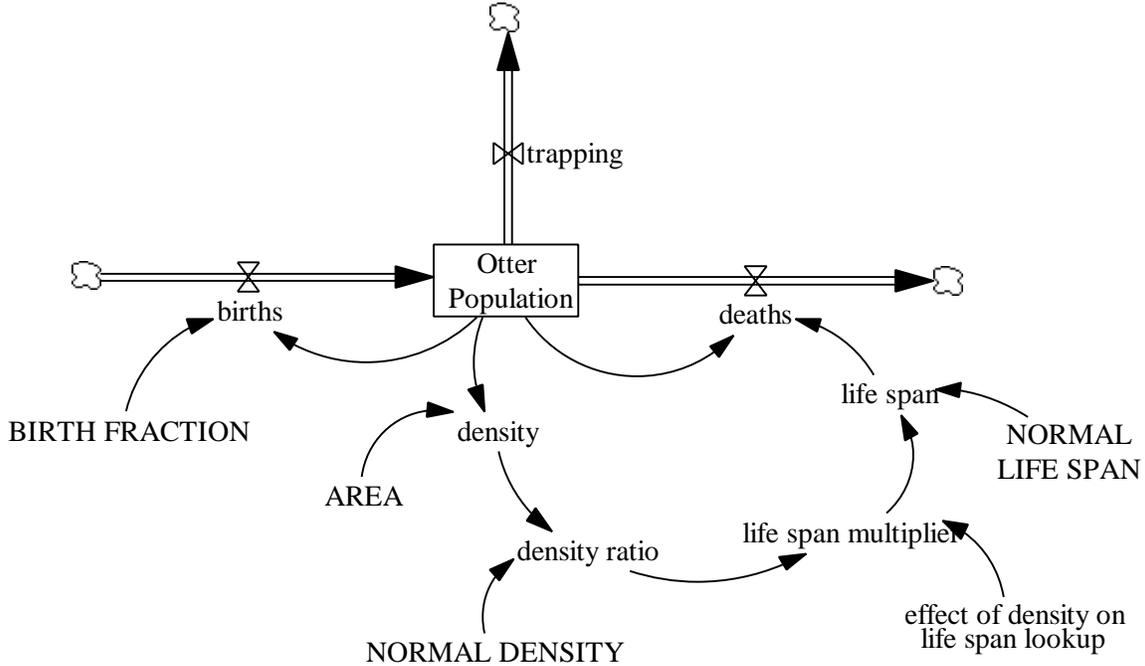
The system is in equilibrium. The initial value of “Otter Population” is such that the “density” equals the “NORMAL DENSITY,” and hence the “life span” equals the “NORMAL LIFE SPAN.” Therefore, the rate of “births” equals the rate of “deaths,” and the number of otters never changes.

Otters are valued by man for their extremely dense and lustrous fur. Many Alaskan fishermen supplement their income by trapping otters for their fur. The county preservation agency has asked fishermen to trap no more than 10 otters per year from this particular ecosystem.

C. Modify the model to represent the removal of ten otters from the system each year. In your assignment solutions document, include the modified model diagram and documented equations.

In this scenario, there is a new outflow from the “Otter Population” stock. The new outflow is called “trapping” and equals 10 otters per year.

Modified model diagram:



Modified model equations:

Otter Population = INTEG (births – deaths – trapping, 100)
 Units: otter
 The total number of otters in the ecosystem at any time.

trapping = 10
 Units: otter/year
 The number of otters trapped by fishermen each year.

D. Estimate the new equilibrium of the system. Simulate the model. In your assignment solutions document, include a graph of the model behavior. Explain the behavior. What is the new system equilibrium? Explain.

Calculating the new equilibrium value of “Otter Population” is difficult because of the nonlinear lookup function “effect of the density on life span lookup.” We can attempt, however, to find an approximate value.

For the system to be in equilibrium, set the inflow equal to the sum of the outflows:

$$\text{births} = \text{deaths} + \text{trapping}$$

$$0.25 * \text{Otter Population} = (\text{Otter Population} / \text{life span}) + 10$$

The “life span” is related to the “Otter Population” through the lookup function. A quick look at the above equation suggests that the life span will be greater than the life span in

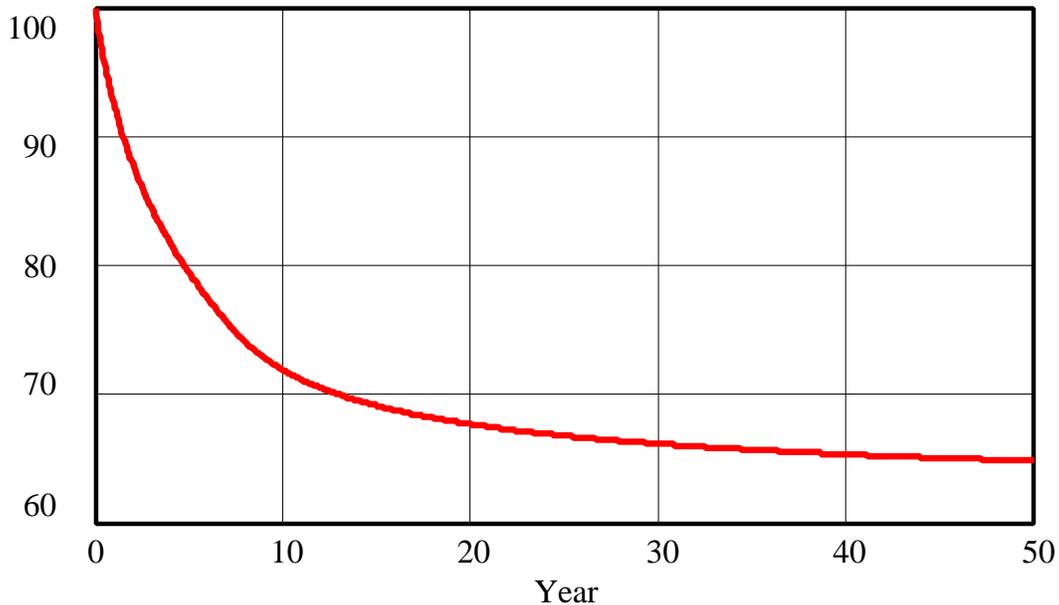
1984. From the equation, one can see that “Otter Population / life span” is less than “0.25 * Otter Population” because of the “+ 10” on the right hand side of the equation. Therefore, “life span” is greater than 4 years. Plug in various values for “life span” and find the corresponding “Otter Population” by checking the lookup table and solving the system equations. Then plug into the above equation and see if both sides of the equation are equal. You can also plug in various numbers for “Otter Population,” find the corresponding life span and do the calculations. The range for life span is more limited, hence it may be easier to guess the life span.

After some trial and error, one can find that a “life span” of 10 years gives an “Otter Population” approximately equal to 67 otters.

Simulating the model for a period of about 50 years shows that the equilibrium “Otter Population” is indeed 66.81 otters, and the equilibrium “life span” is 9.75 years.

The behavior of the system is asymptotic approach to the new equilibrium. As the total outflow from the stock is higher, the stock starts falling. A lower value of “Otter Population,” however, reduces the “density,” which in turn increases the average “life span” of otters. Hence, the rate of “deaths” becomes smaller, slowing down the decrease of “Otter Population” until the system reaches equilibrium. The new model behavior is shown below:

Behavior of Otter Population - part D



Otter Population : otters D ————— otter

In 1989, a large oil tanker known as the “Exxon Valdez” drifted onto a reef and spilled some 10.1 million barrels of oil into Alaska’s Prince William Sound. Because of the catastrophe, half the otter population of the ecosystem is killed.

E. Modify the model to represent this scenario (hint: use the PULSE function⁵). Set the outflow modeled in part C equal to zero for this scenario. That is, fishermen do not trap and kill any otters during this tragic time. In your assignment solutions document, include the modified model diagram and documented equations.

Make a new outflow from the stock of “Otter Population” called “sudden disaster.” Set the flow “trapping” equal to zero.

To kill half of the “Otter Population” instantaneously at time = 5, use the following equations:

FRACTION KILLED BY SUDDEN DISASTER = 0.5

Units: dmn1

The fraction of the total otter population that is killed in the accident.

Otter Population = INTEG (births – deaths – trapping – sudden disaster, 100)

Units: otter

The total number of otters in the ecosystem at any time.

sudden disaster = (1 / TIME STEP) * FRACTION KILLED BY SUDDEN DISASTER *
Otter Population * PULSE(TIME OF DISASTER, TIME STEP)

Units: otter/year

Otters killed by accidents such as oil spills.

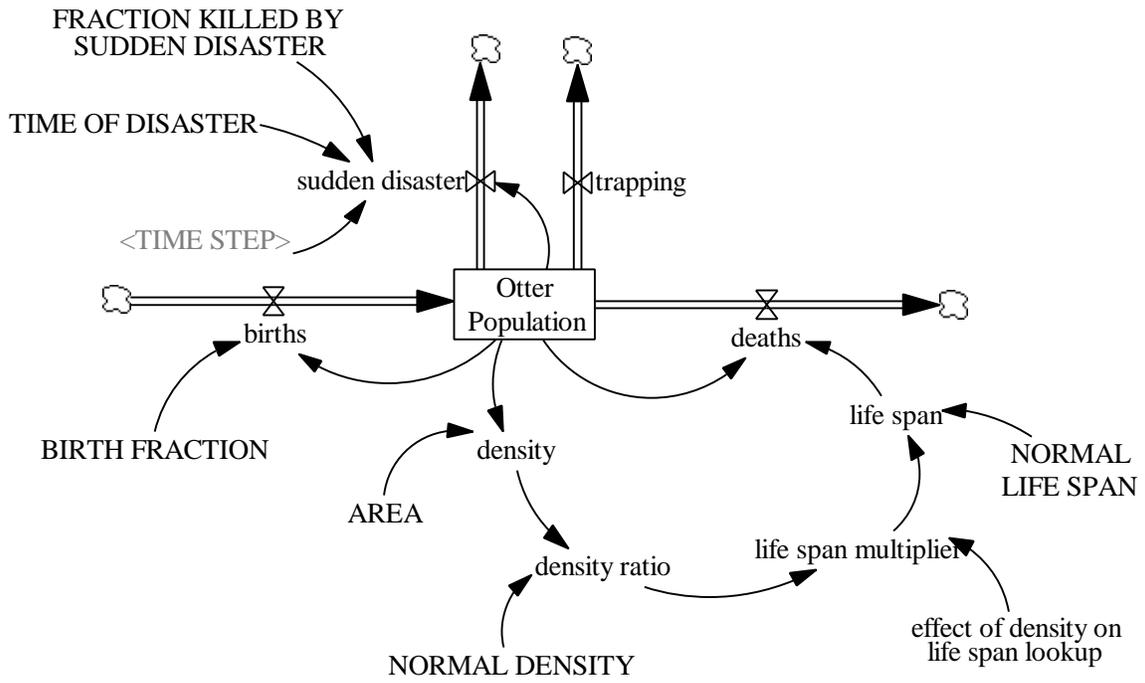
TIME OF DISASTER = 5

Units: year

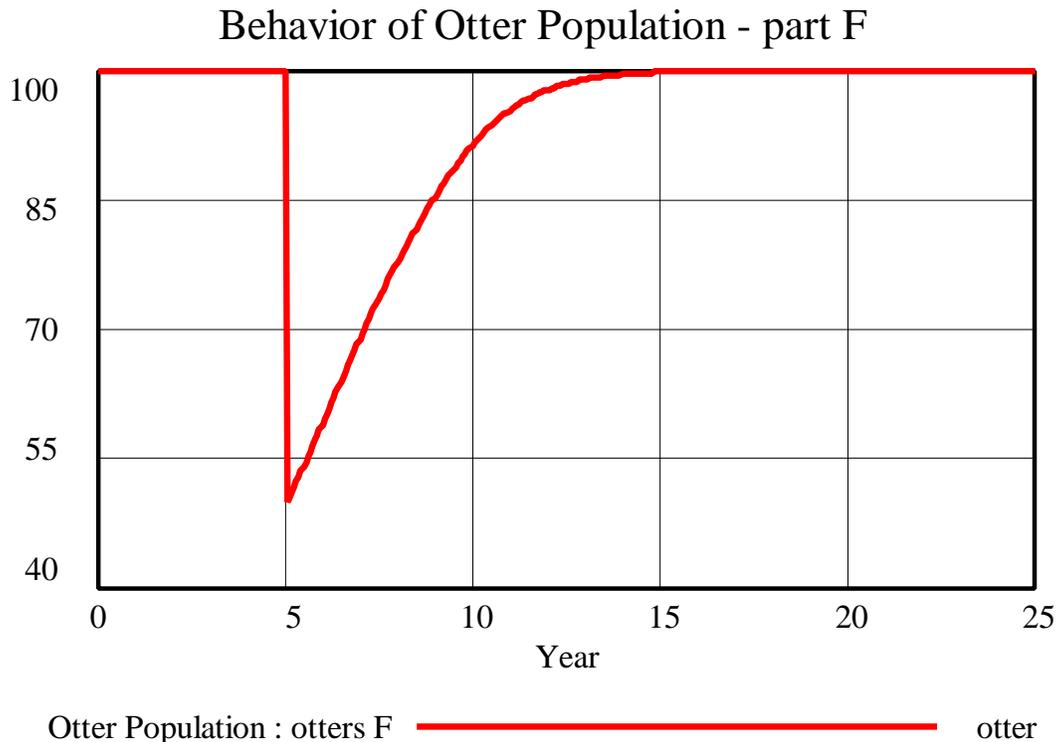
The time of the disaster.

⁵ Please refer to the general solutions to assignment 17, Exercise 1, part 1C, if you need help with formulating a PULSE function.

Modified model diagram:



F. Simulate the model. In your assignment solutions document, include a graph of the model behavior. Explain the behavior. Is the system “self-regulating through automatic feedback” as described in the passage?

Model behavior:

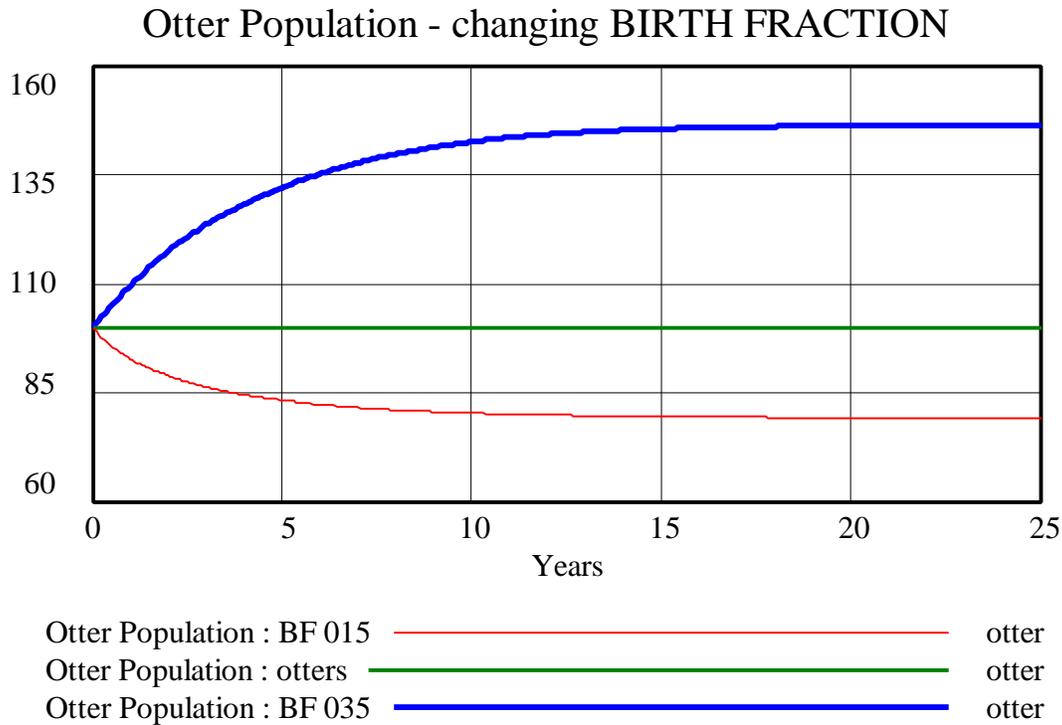
The system is indeed self-regulating as the passage describes. When the “Otter Population” is decreased dramatically, the low “density” leads to a very low rate of “deaths,” and the “Otter Population” begins to grow. As the “Otter Population” grows, “density” increases and therefore the rate of “deaths” also increases. When the “Otter Population” reaches 100, the rate of “births” is equal to the “deaths” and system is in equilibrium again.

G. Evaluate the validity of the model. You may want to use the behavior sensitivity test, among others. Describe the test(s) that would be most indicative of validity and the results of these tests.

Various tests can be used to validate this model. One approach is to test the sensitivity of the model to parameter and initial condition changes. Change the constants within a reasonable range and observe how the behavior of the system changes. This is the behavior sensitivity test. Systems usually do not react drastically to small changes in most parameters.

First, let us examine the effects of changing the “BIRTH FRACTION.” There is some uncertainty about the value of the “BIRTH FRACTION,” so one should make sure that the model’s behavior is not significantly affected if a different value of the “BIRTH FRACTION” is used. The graph below shows the behavior of “Otter Population” with

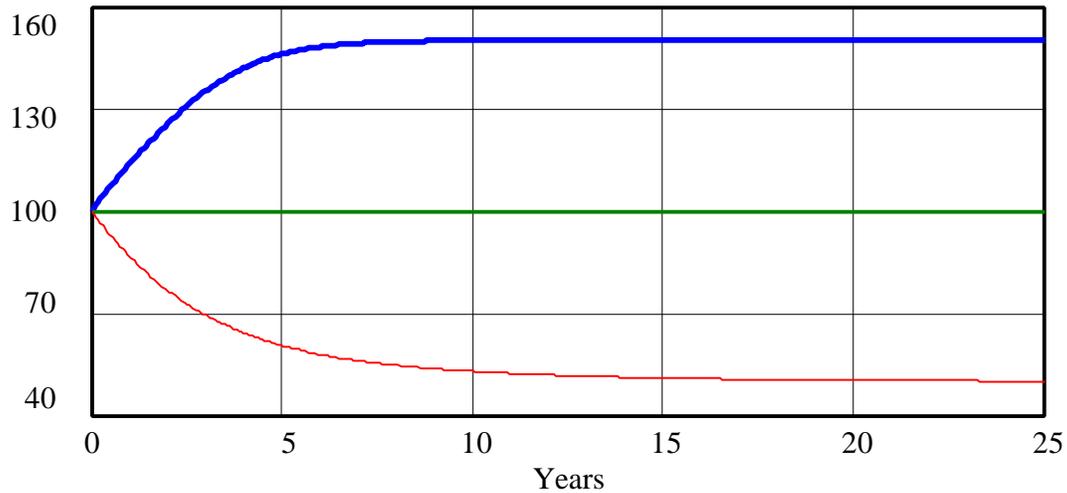
“BIRTH FRACTION” equal to 0.15 (“BF 015” simulation), 0.25 (the base run “otters” simulation), and 0.35 per year (“BF 035” simulation):



A lower value of “BIRTH FRACTION” decreases the inflow of “births” below the outflow of “deaths,” which causes the “Otter Population” to start decreasing immediately. As the “Otter Population” falls, however, the “density” also falls, resulting in a higher “life span.” Therefore, the outflow “deaths” decreases also, until the system reaches equilibrium again, with a lower equilibrium value of “Otter Population” than in the base run. Similarly, a higher value of “BIRTH FRACTION” results in a higher equilibrium value of “Otter Population.” Changing the “NORMAL LIFE SPAN” has similar effects on the behavior as changing the “BIRTH FRACTION.” A longer “NORMAL LIFE SPAN” decreases the rate of “deaths,” so the “Otter Population” starts growing, until the higher “density” reduces the actual “life span” and the system comes back to equilibrium with a higher equilibrium value of “Otter Population.” Similarly, a shorter “NORMAL LIFE SPAN” results in a lower equilibrium value of “Otter Population.”

Now, let us look at the effects of changing the “AREA.” Changes in the “NORMAL DENSITY” have similar effects as changes in the “AREA,” because both variables affect the actual “density.” The graph below shows the behavior of “Otter Population” with “AREA” equal to 100 (“area 100” simulation), 200 (the base run “otters” simulation), 300 acres (“area 300” simulation):

Otter Population - changing AREA

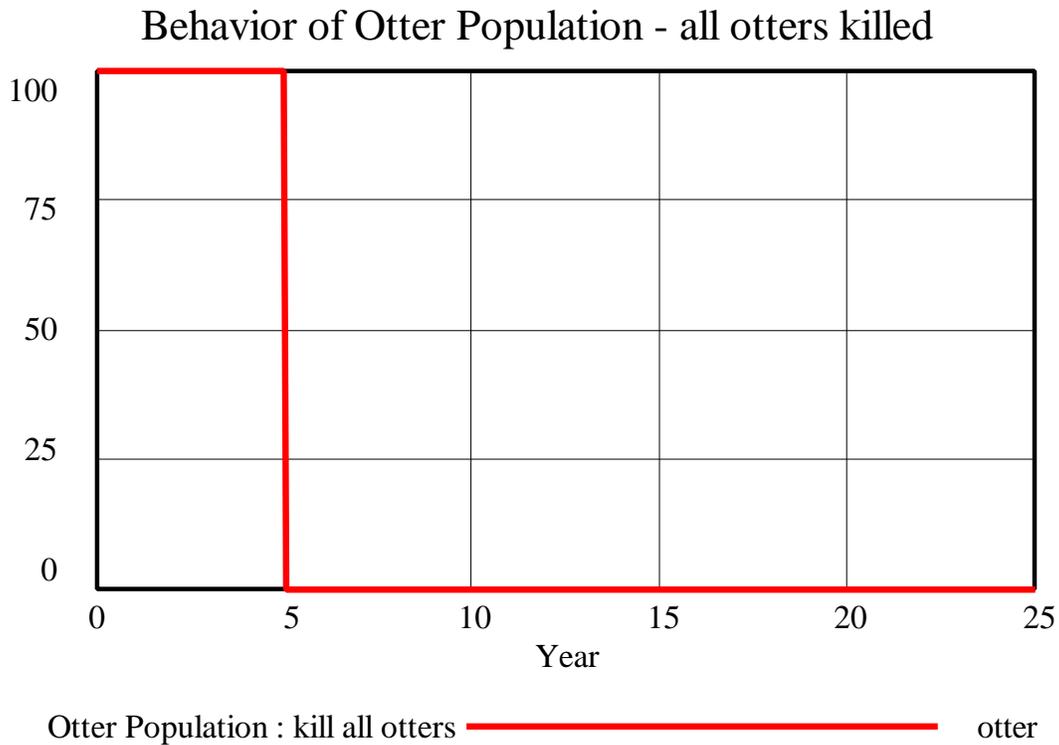


Otter Population : area 100 — otter
 Otter Population : area 300 — otter
 Otter Population : area 300 — otter

A smaller “AREA” increases the “density,” which results in a shorter “life span” and a higher rate of “deaths.” Hence, the “Otter Population” decreases until the “density” is low enough so that the rate of “deaths” equals the “births” and the system settles in equilibrium with a lower equilibrium value of “Otter Population.” Similarly, a larger “AREA” results in a higher equilibrium value of “Otter Population.”

Another test that can be used to examine the validity of the model is the extreme conditions test, such as killing all the otters. How does the model respond to such conditions? If the model shows that the population will start growing again, the model is not realistic. With no live otters, the population cannot grow. Similarly, what happens if a large number of otters is suddenly added to the existing population? Overcrowding should lead to a higher death rate until the population decreases to a sustainable level.

In the first test, try killing the entire “Otter Population” at the same time. Set the “trapping” outflow to 0 and set the “FRACTION KILLED BY SUDDEN DISASTER” equal to 1 in the “sudden disaster” outflow. The model then produces the following behavior:



As expected, when all otters are killed in a sudden disaster, the population never recovers.

Next, try adding a large number of otters to the system. Make an inflow to the stock of “Otter Population” called “adding otters”:

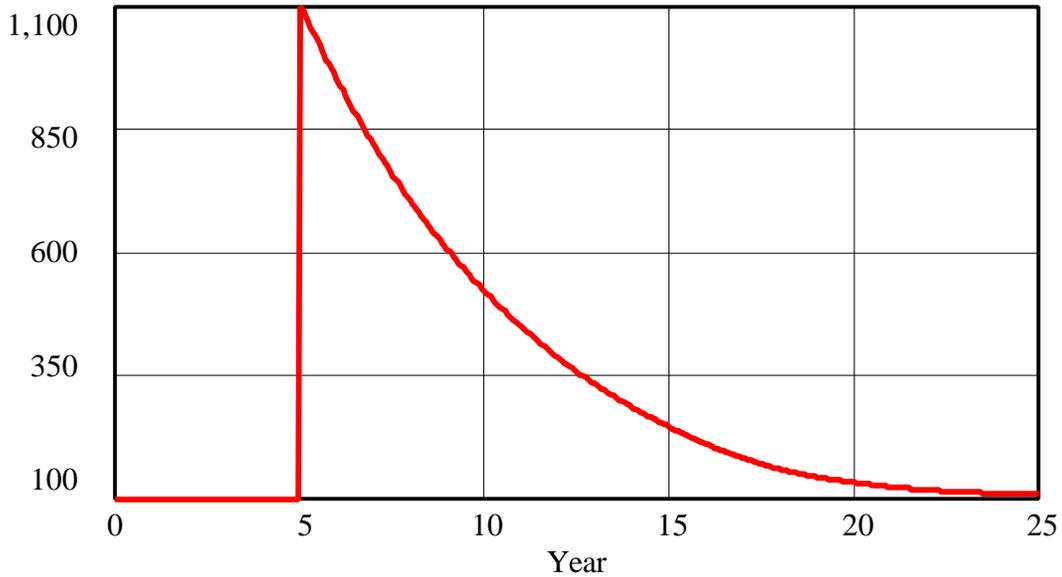
$$\text{adding otters} = (1/\text{TIME STEP}) * 1000 * \text{PULSE}(5, \text{TIME STEP})$$

UNITS: otters/Year

Otters brought from outside the ecosystem and added to the system.

This equation adds 1000 otters to the system over one time step. Make sure to set the “trapping” and “sudden disaster” outflows to 0. The model then produces the following behavior:

Behavior of Otter Population - adding otters

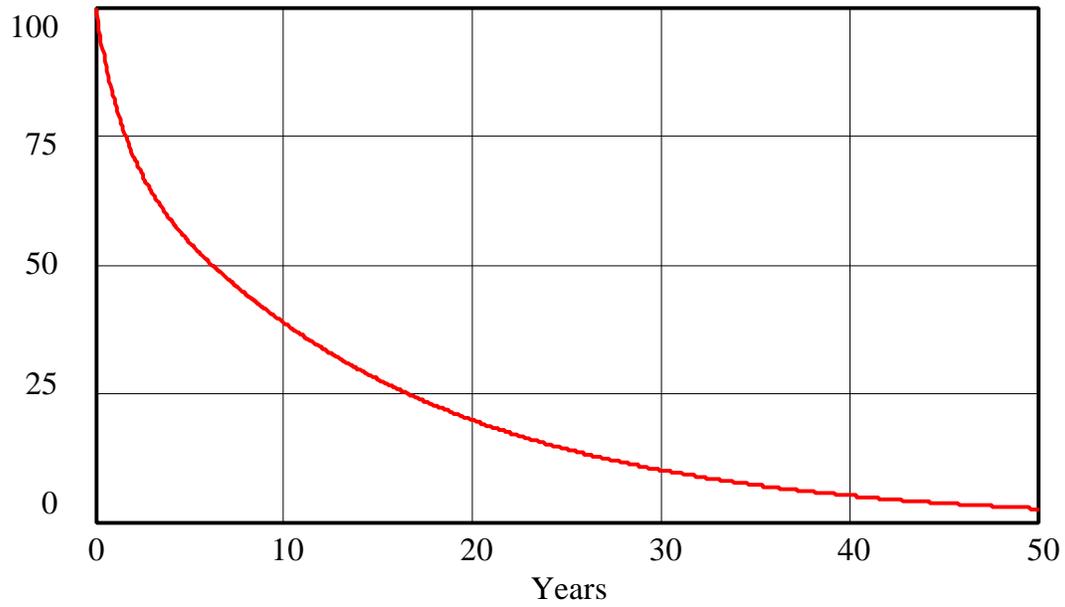


Otter Population : add otters ————— otter

Again, the result of the test is as expected. When a large number of otters are added to the system, the rate of “deaths” becomes higher than the rate of “births,” so the “Otter Population” decreases until equilibrium is reached again.

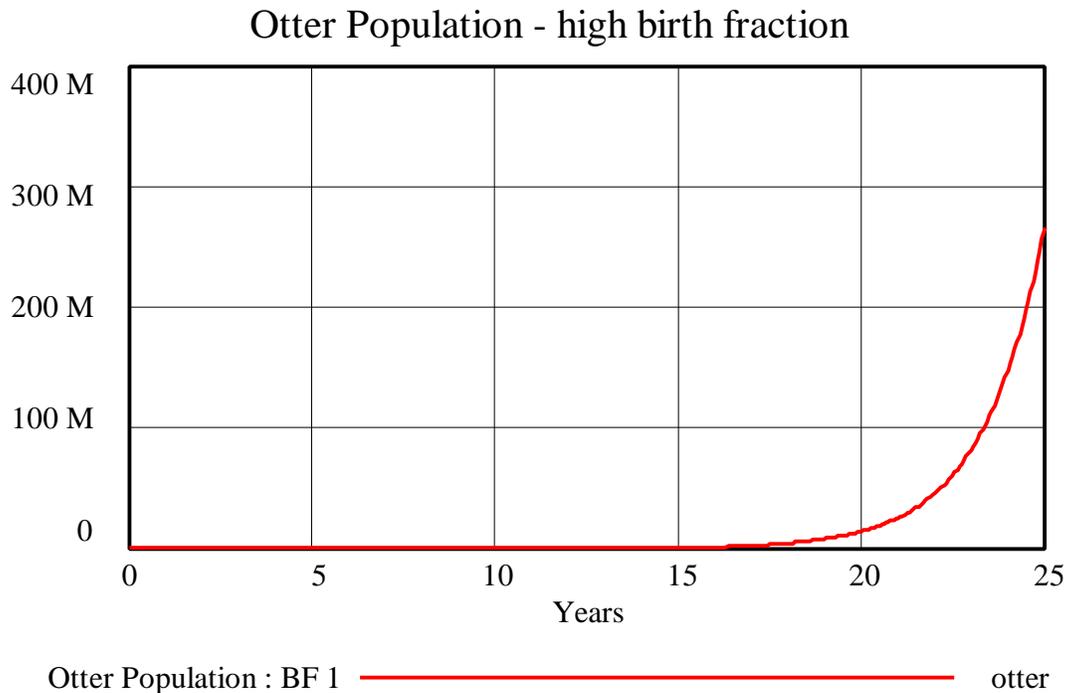
We can also try extreme conditions of the “BIRTH FRACTION.” When the “BIRTH FRACTION” is equal to 0 and hence no otters are born, the “Otter Population” should asymptotically decline to its new equilibrium value at 0. The graph below confirms this hypothesis:

Otter Population - no otters born



Otter Population : BF 0 ————— otter

If the “BIRTH FRACTION” is large, say 1 per year, the “Otter Population” should grow, but its growth should be limited by the effect of density on life span. The model generates the following behavior:



The model generates exponential growth, which is not realistic. This test shows that the model is not completely robust to large changes in the “BIRTH FRACTION.” Even though the “life span multiplier” reduces the “life span” well below the “NORMAL LIFE SPAN,” the “effect of density on life span lookup” function never allows the death fraction (the inverse of the “life span”) to become greater than the “BIRTH FRACTION.” Hence, the model produces exponential growth. If one wants to test the model with this particular extreme condition, the range of the lookup function would need to be extended to allow for an even shorter “life span.” Therefore, the current model is not robust to extreme changes in the “BIRTH FRACTION.” This also indicates that the model may be incomplete in some respect. The model does not account for shortages in food supply for otters when the population becomes much larger than the current population. Diminishing food supply will limit population growth even more rapidly than the crowding effect.

3. Modeling Exercise: A Predator/Prey System

The following description of a predator/prey system comes from “Fundamentals of Ecology”⁶ by Eugene Odum:

“...Interesting and only partly understood density variations are those which are not related to seasonal or obvious annual changes, but which involve regular oscillations or cycles of abundance with peaks and

⁶ Odum, Eugene P. (1971) *Fundamentals of Ecology*, Philadelphia: W. B. Saunders Co. pp. 190-192.

depressions every few years, often occurring with such regularity that population size may be predicted in advance. The best known cases concern mammals, birds, insects, fish and seed production in plants in northern environments. Among the mammals, the best-studied examples exhibit either a 9 to 10 year or a 3 to 4 year periodicity... Peaks of abundance often are followed by “crashes,” or rapid declines.”

Initially, a population of 1250 predators and 50000 prey coexist in an ecosystem encompassing 1000 acres of land. The prey reproduce much more rapidly than the predator (almost always the case with any predator/prey system), producing on average five offspring for every four prey each year. The predators, on the other hand, produce about one offspring for every four predators per year. Predators die either by natural causes (about ten per hundred predators each year), or by starvation, when prey is scarce. Prey die either by natural causes (about 50 per 100 prey each year) or are killed by predators.

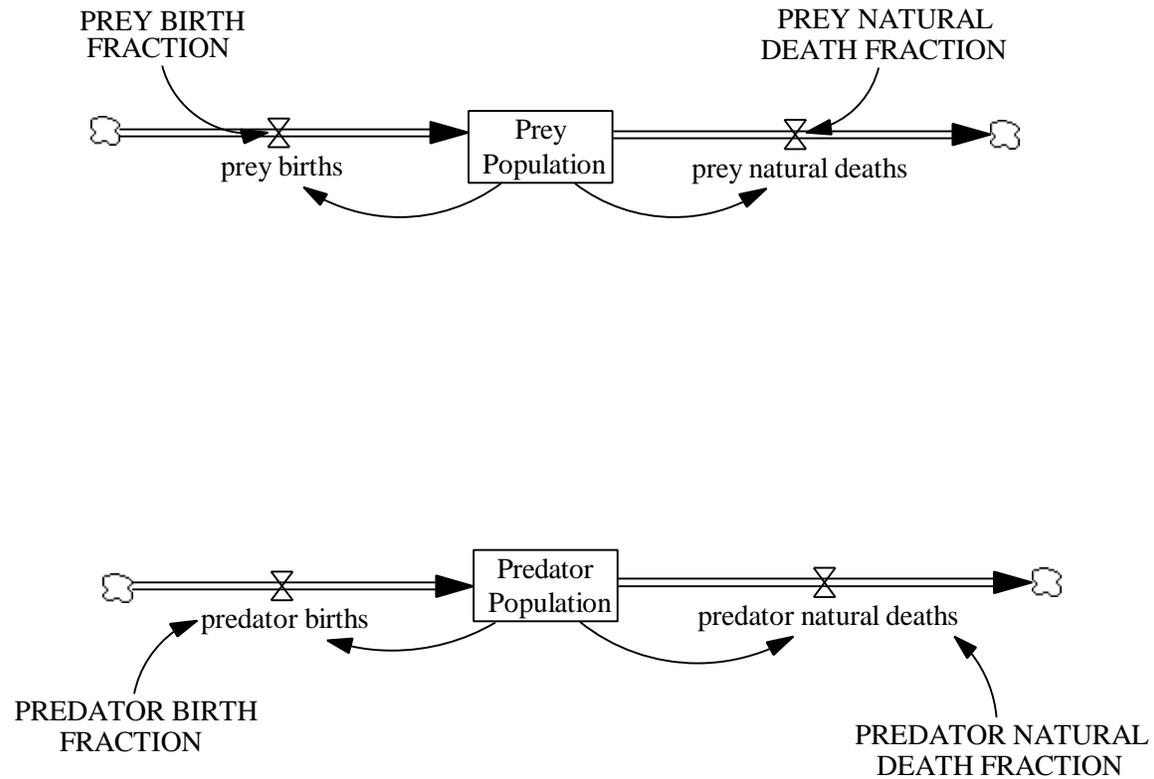
A. The ecology text hypothesizes that predator/prey models will exhibit oscillatory behavior. Do you agree? Why or why not?

From a systems modeling point of view, it is clear that the system has two stocks—the predator population and the prey population. As each stock depends on the other stock, it is quite likely that the system exhibits oscillatory behavior.

Initially, suppose that predator and prey populations are low. The prey density, and hence the prey killed rate will also be low, leading to an increase in prey population. Increasing prey population leads to longer predator life span as food for predators is more abundant, hence an increase in the predator population. As predator population increases, the number of prey killed increases and the prey population starts to decrease. Therefore, the predator population will also start to decrease due to scarcity of food. Once both prey and population are sufficiently low, the cycle will start all over again, resulting in a sustained oscillatory system.

B. Start building the model by creating stocks of “Predator Population” and “Prey Population.” Formulate the relevant inflows and outflows due to natural causes only. In your assignment solutions document, include the model diagram and documented equations.

Model diagram:



Model equations:

PREDATOR BIRTH FRACTION = 0.25

Units: 1/Year

The number of predators born per existing predator each year.

predator births = Predator Population * PREDATOR BIRTH FRACTION

Units: predator/Year

The number of predators born each year.

PREDATOR NATURAL DEATH FRACTION = 0.1

Units: 1/Year

The fraction of the predator population that dies due to natural causes each year.

predator natural deaths = Predator Population * PREDATOR NATURAL DEATH FRACTION

Units: predator/Year

The number of predators that die due to natural causes each year.

Predator Population = INTEG (predator births – predator natural deaths, 1250)

Units: predator

The total number of predators alive at any time.

PREY BIRTH FRACTION = 1.25

Units: 1/Year

The number of prey born per existing prey each year.

prey births = Prey Population * PREY BIRTH FRACTION

Units: prey/Year

The number of prey born each year.

PREY NATURAL DEATH FRACTION = 0.5

Units: 1/Year

The fraction of the prey population that dies each year due to natural causes.

prey natural deaths = Prey Population * PREY NATURAL DEATH FRACTION

Units: prey/Year

The number of prey that die due to natural causes each year.

Prey Population = INTEG (prey births – prey natural deaths, 50000)

Units: prey

The total number of prey alive at any time.

The nature of the closed predator/prey system is such that predators depend on the prey for subsistence. When there are few prey in the system, the predators are unable to catch much food, and many predators die. In fact, when there are no prey at all, the predators die twice as quickly as they are born. On the other hand, when there are as many as 100,000 prey within the ecosystem, only about 1 out of 16 predators die annually.

C. Formulate the outflow from “Predator Population” due to starvation. In your assignment solutions document, include the modified model diagram, documented equations, and graphs of any lookup functions.

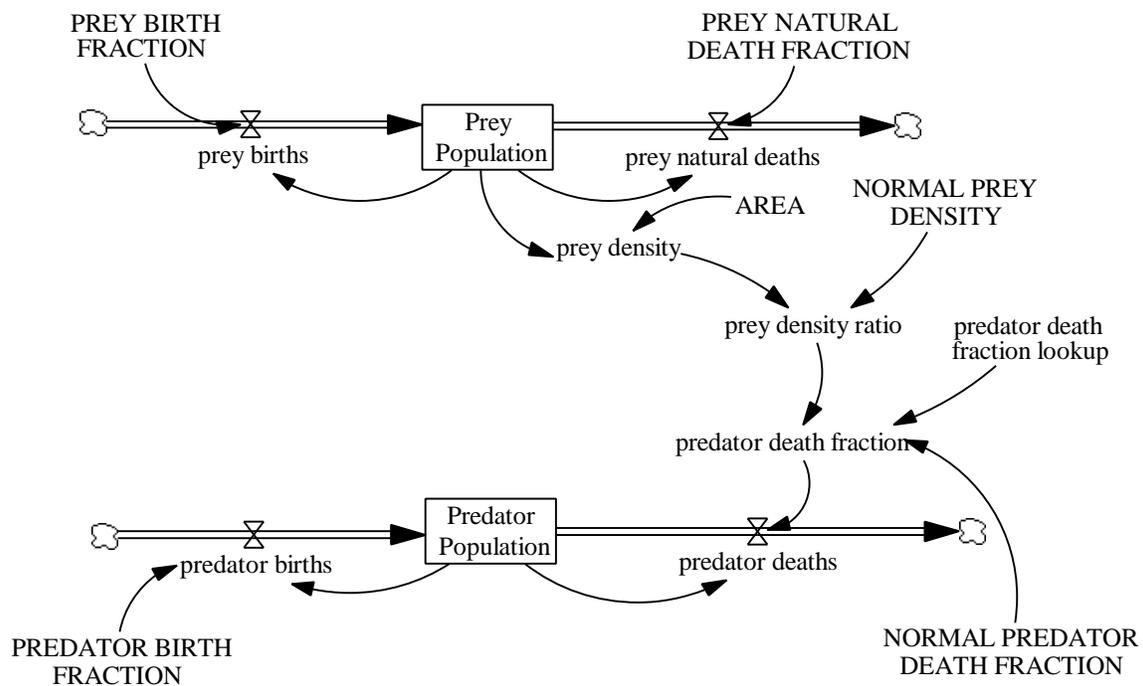
Predator starvation depends on the prey density. A table function is needed to relate the prey density to the death rate due to starvation. A normal death fraction due to starvation and a normal density must also be chosen to normalize the input and output from the table function.

We are told that when the prey density is zero, the predator death fraction is about 0.5 per year (twice the birth fraction). Therefore, the predator death fraction due to starving should be $(0.5 - \text{natural death fraction}) = 0.4$ per year. Similarly, when the “Prey Population” is high, the death fraction due to starvation is as low as $(1/16 - 0.1) = -0.0375$. This negative value is understandable when one considers both outflows together: the net death fraction is less than the natural death fraction because the high “Prey Population” leads to a very low death fraction. Even though the “natural predator deaths” and “predator deaths due to starving” can be modeled as two separate flows, they are essentially the same, so it is more correct to combine them (even though the question

suggests otherwise). The model now looks like the model below. The Appendix contains the model and table function with the two flows modeled separately.

Note: This exercise says that when prey density is zero, predator death fraction is about 0.5 per year. This means that average lifespan is $1/0.5 = 2$ years, which is not possible when there is no food. The average lifespan of a predator-like animal without any food is about three weeks to a month, hence death fraction when prey density is zero should equal about $1/(1/12) = 12$ per month. Sorry for our oversight and keep this in mind when building any population model. In the rest of the exercise, however, we use the information provided in the article.

Modified model diagram:



Choose the “NORMAL PREY DENSITY” to be 50 prey / acre (the density at the start of the simulation) and the “NORMAL PREDATOR DEATH FRACTION” to be 0.1 per year.

We now have the following points for our table function:

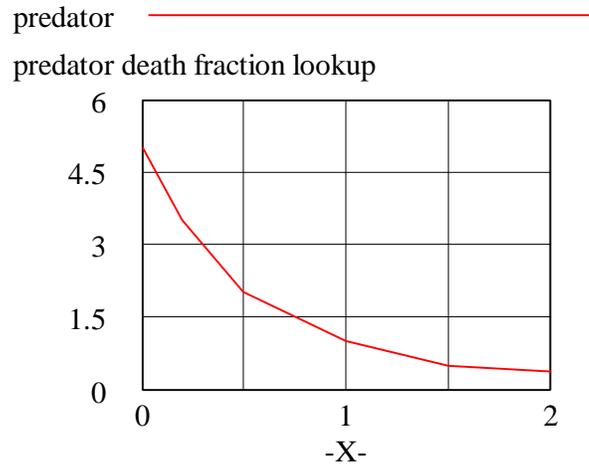
- when prey density = 0, input = 0, output = 5 (that is, “predator death fraction” = 0.5)
- when prey density = 50, input = 1, output = 1
- when prey density = 100, input = 2, output = 0.375

We can add more data points to make the lookup function more realistic and smooth.

predator death fraction lookup ([(0,0) - (2,5)], (0,5), (0.2,3.5), (0.5,2), (1,1), (1.5,0.5), (2,0.375))

Units: dmn1

The lookup function showing how the predator death fraction changes with prey density.



Modified model equations:

AREA = 1000

Units: acre

The total area of the system under study.

NORMAL PREDATOR DEATH FRACTION = 0.1

Units: 1/Year

The fraction of the predator population that die each year when the prey density equals the NORMAL PREY DENSITY.

NORMAL PREY DENSITY = 50

Units: prey/acre

The number of prey per unit area of the system for the predator death fraction to equal the NORMAL PREDATOR DEATH FRACTION.

predator death fraction = NORMAL PREDATOR DEATH FRACTION * predator death fraction lookup(preY density ratio)

Units: 1/Year

The fraction of the total number of predators that die each year.

predator death fraction lookup([(0,0) - (2,5)], (0,5), (0.2,3.5), (0.5,2), (1,1), (1.5,0.5), (2,0.375))

Units: dmn1

The lookup function showing how the predator death fraction changes with prey density.

predator deaths = Predator Population * predator death fraction

Units: predator/Year

The number of predators that die each year.

prey density = Prey Population / AREA

Units: prey/acre

The average number of prey per unit area of the system.

prey density ratio = prey density / NORMAL PREY DENSITY

Units: dmnl

The prey density normalized by the NORMAL PREY DENSITY.

The only piece of the puzzle missing now is the outflow from “Prey Population” due to killings by predators. When there are no prey in the ecosystem, predators cannot catch any prey. When there are 100,000 prey in the system, each predator catches about 125 prey per year. When there are 200,000 prey in the system, each predator kills 350 prey per year. Predators are satiated at 500 prey per predator each year, attainable when there are at least 400,000 prey in the ecosystem.

D. Formulate the outflow from “Prey Population” due to killings by predators. In your assignment solutions document, include the modified model diagram, documented equations, and graphs of any lookup functions.

The rate at which prey is killed by predators depends on the “prey density.” A lookup function is needed to show the relationship between “prey density” and prey killed per predator. The “NORMAL PREY DENSITY” will be the same as in part C, 50 prey per acre. As we are not told how many prey are killed per predator when the prey density is 50 prey per acre, let us estimate that it is about 30 prey per predator. The normal number of prey killed per predator can be therefore be 30.

Hence, the table function can be formulated as:

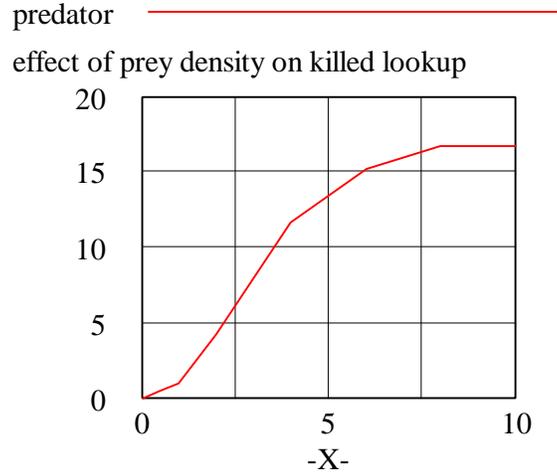
(Normal prey density = 50; Normal prey killed per predator = 30)

effect of prey density on killed lookup([(0,0)- (8,20)], (0,0), (0.5,0.5), (1,1), (2,4.17), (4,11.67), (8,16.67))

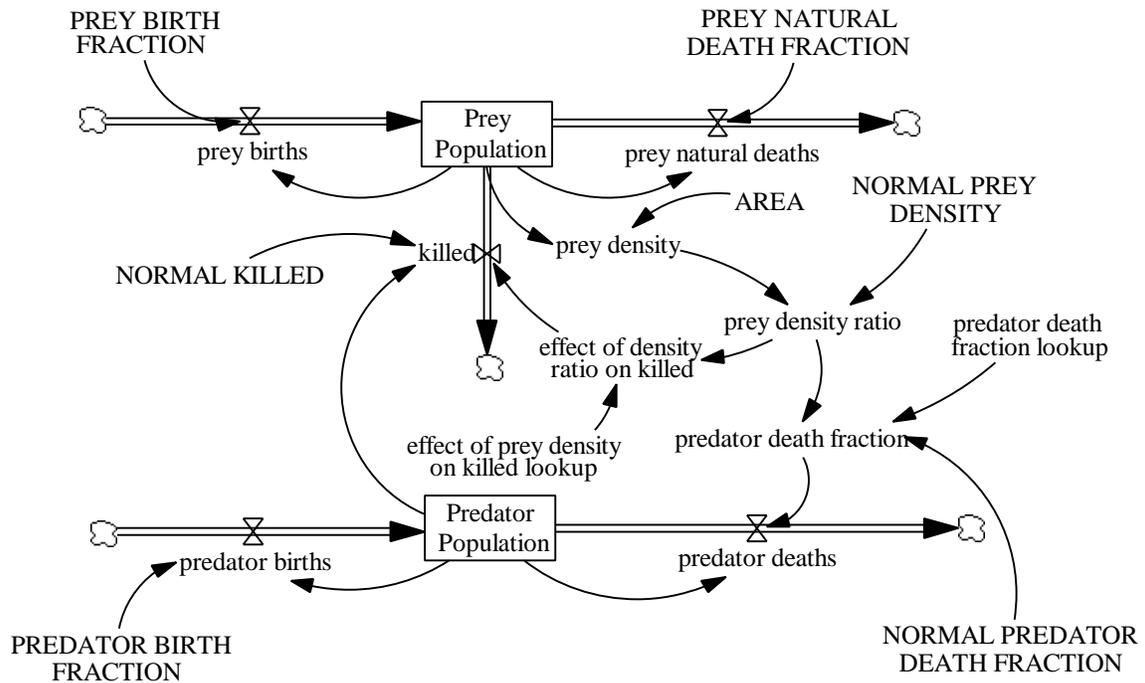
Units: dmnl

The lookup function showing how the number of prey killed by predators changes with prey density.

Thus, the lookup function looks like:



Modified model diagram:



Modified model equations:

effect of density ratio on killed = effect of prey density on killed lookup (prey density ratio)

Units: dmnl

The effect of prey density on the number of prey killed by predators each year.

effect of prey density on killed lookup $[(0,0) - (10,18)], (0,0), (0.5,0.5), (1,1), (2,4.17), (4,11.67), (6,15.2), (8,16.67), (10,16.67)$

Units: dmnl

The lookup function showing how the number of prey killed per predator changes with prey density.

$$\text{killed} = \text{Predator Population} * \text{NORMAL KILLED} * \text{effect of density ratio on killed}$$

Units: prey/Year

The number of prey killed by predators each year.

$$\text{NORMAL KILLED} = 30$$

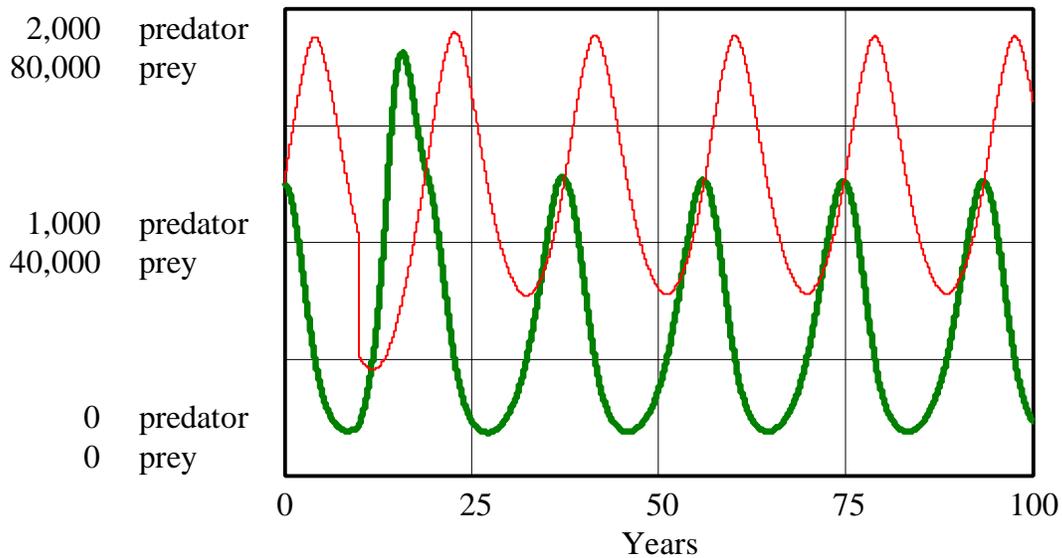
Units: prey / (predator * Year)

The number of prey killed per year when the prey density equals the NORMAL PREY DENSITY.

E. Simulate the model.⁷ In your assignment solutions document, include graphs of model behavior. What type of behavior do you observe? Explain the behavior.

The system demonstrates sustained oscillations as shown in the graph below. The behavior is in accordance with the observations in Eugene Odum's article. If the prey and predator populations indeed follow such a regular pattern, it is easy to predict their population at any time.

Behavior of predator and prey populations - pesticide



Assume for the rest of this exercise that the predator population represents insects and that the prey population represents crops (do not worry about changing the units here;

⁷ Make sure to use a Time Step that is small enough to have no significant effect on the model behavior.

just use the same model and observe the general behavior). Farmer Joe sees insects hovering over his crops and decides that they are keeping him from reaping a full harvest. Farmer Joe goes to town, buys several canisters of insecticide, and sprays it all over his crops. As a result, a year into a simulation, half of the insects are suddenly killed.

F. Simulate this policy change (hint: use the PULSE function). In your assignment solutions document, include graphs of model behavior. Explain the behavior. What are the short-term (during the first oscillations cycle following the policy change) and long-term (after several oscillations cycles) effects on the insect population and on the crops? Did the system react as Farmer Joe had expected?

To simulate the killing of the insects (predators), add an outflow called “pesticide deaths” from the stock of “Predator Population.” See Example 1 in part E of “An Ecological System” exercise earlier in this assignment. Using the formula in that example, one can formulate “pesticide deaths” as being an instantaneous process with the equations below:

FRACTION KILLED BY PESTICIDES = 0.5

Units: dmmnl

The fraction of predators killed by pesticides.

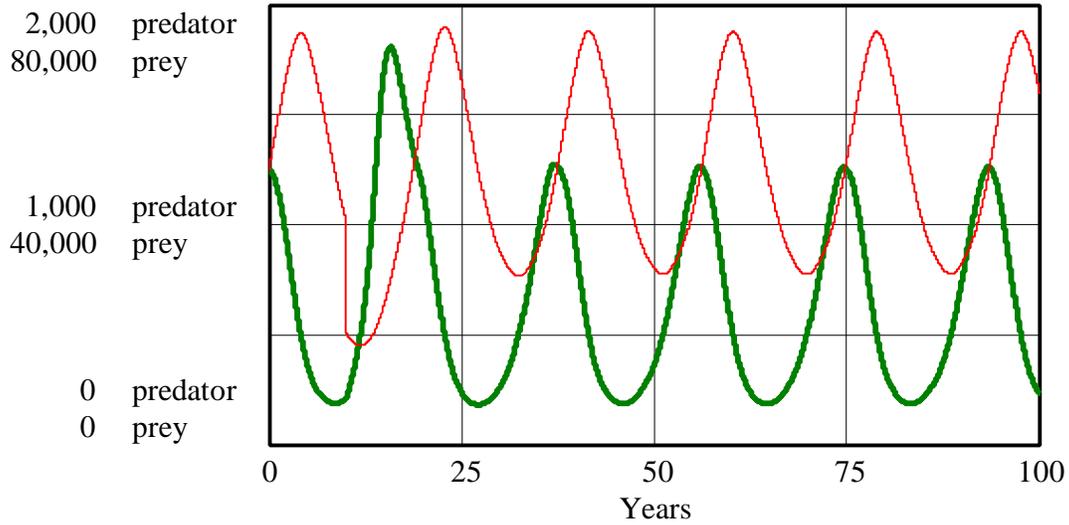
pesticide deaths = (1/TIME STEP) * Predator Population * FRACTION KILLED BY PESTICIDES * PULSE(10, TIME STEP)

Units: predator/year

The number of predators killed instantaneously by pesticides.

From the graph below one can see that in the short term, there is a decrease in the number of predators (insects). With time, however, the number of predators increases and the long term cyclical behavior is unaffected by the pesticide. This behavior is due to the self-regulating feature of this system.

Behavior of predator and prey populations - pesticide



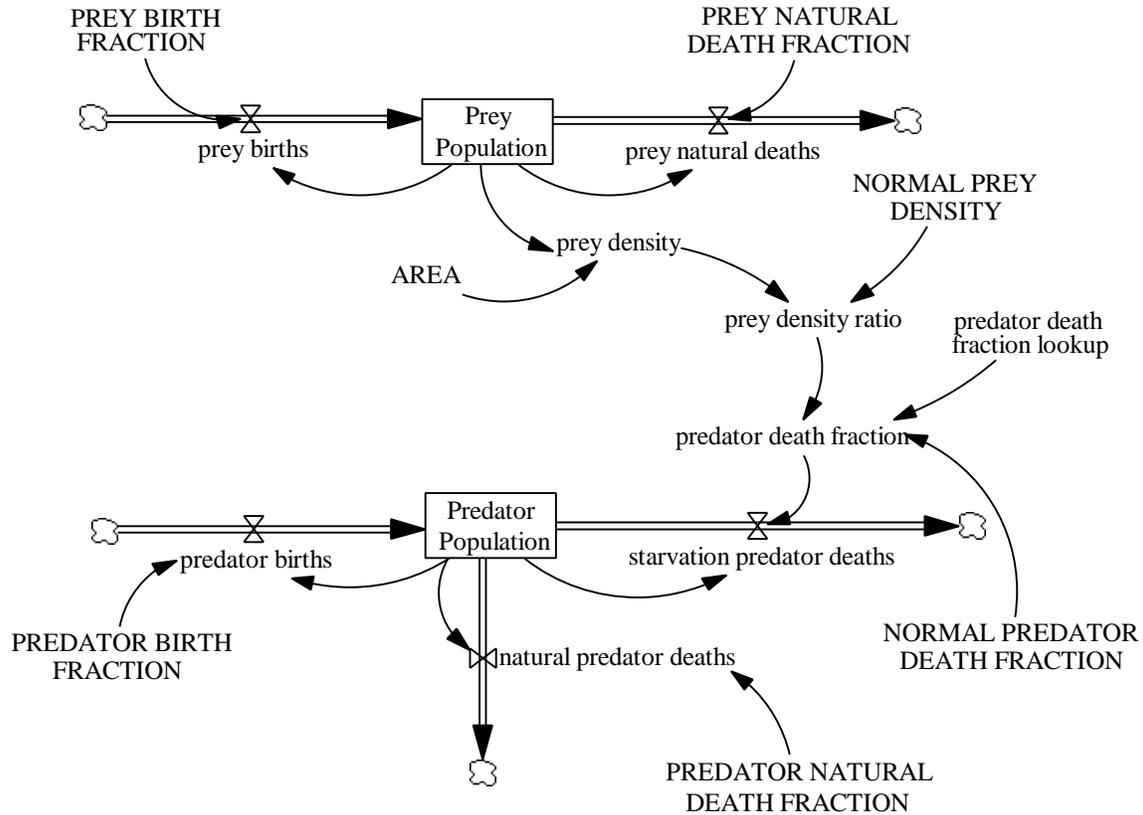
Predator Population : pesticide ————— predator
 Prey Population : pesticide ————— prey

G. Evaluate the validity of the model. In testing the model for validity and robustness, you may be interested in seeing whether the model is able to generate behavior that would not happen in real life (analogous behavior testing from the Shreckengost paper). Can you make one or both population stocks grow exponentially to infinity? Why would such a behavior happen? If such unrealistic behavior could result from the model, what does that say about the model? What is missing from the model that acts as a limiting factor in real life?

It is possible to make both populations grow exponentially to infinity by altering the table functions (specifically, the low-density output of the effect of density on killed lookup). In the real system, however, the effect of the vegetation, or the food supply of the prey should be considered. The vegetation will be unable to sustain the increasing prey population, leading to an increasing prey death rate, which will in turn control the predator population.

Appendix: Modeling “natural predator death” and “starvation predator death” as separate flows. See “Modeling Exercise: A Predator/Prey System” part C.

Model diagram:



Modified model equations:

natural predator deaths = Predator Population * PREDATOR NATURAL DEATH FRACTION
 Units: predator/Year
 Number of predators that die each year due to natural causes excluding starvations.

predator death fraction lookup $[(0,-0.7) - (2,4)]$, (0,4), (0.2,2.5), (0.5,1), (1,0), (1.5,-0.5), (2,-0.625))
 Units: dmnl
 The lookup function showing how the predator death fraction changes with prey density.

PREDATOR NATURAL DEATH FRACTION = 0.1
 Units: 1/Year
 The fraction of the predator population that dies due to natural causes each year.

predator prey two flows —————

predator death fraction lookup

