Guided Study Program in System Dynamics

System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management[1]

Solutions to Assignment #21
Thursday, April 15, 1999

Reading Assignment:

- Industrial Dynamics,[2] *by Jay W. Forrester, Chapter 2: An Industrial System*

Exercises:

1. Industrial Dynamics

Please read chapter 2 of Industrial Dynamics. *The chapter shows and explains the behavior of a simple model of a distribution system, similar to the system simulated in the "Beer Game." Please let us know if you have any questions. You do not need to submit anything for this reading assignment.*

2. Modeling Exercise

In this exercise, you are going to build a simple model of inventory ordering. We will continue to expand on this model in later assignments to eventually build a full model of the "Beer Game."

In New England, USA, a certain town is home to many prestigious colleges. The rowdy students in these centers for higher learning spend their evenings nestled in front of their computers with mugs of herbal tea. All of the students go to a local shop, The Tea Pot, to buy many varieties of tea, among them the well-loved Heavenly Seasonings brand, a caffeine-free, 100% natural herbal tea. Richard, the manager at The Tea Pot, displays some boxes of tea on his shelves, but keeps most of his tea in large crates in his supply room. These crates constitute his inventory of tea. Richard has hired you as a consultant to help him study the fluctuations of his inventory of Heavenly Seasonings tea.

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[2] Forrester, Jay W., 1961. *Industrial Dynamics.* Waltham, MA: Pegasus Communications. 464 pp.

Step 1: Inventory

Every week, The Tea Pot sells the contents of four crates of Heavenly Seasonings tea to the college students in the area. Today, Richard counted exactly twelve crates of Heavenly Seasonings tea in his supply room. In order to always have plenty of herbal tea on hand, Richard receives four crates of Heavenly Seasonings tea from his supplier every week.

A. Start building a model by representing the stock of the inventory of Heavenly Seasonings tea that Richard has in his supply room, along with the inflow and outflow to the stock. In your assignment solutions document, include the model diagram and documented equations.

Model diagram:

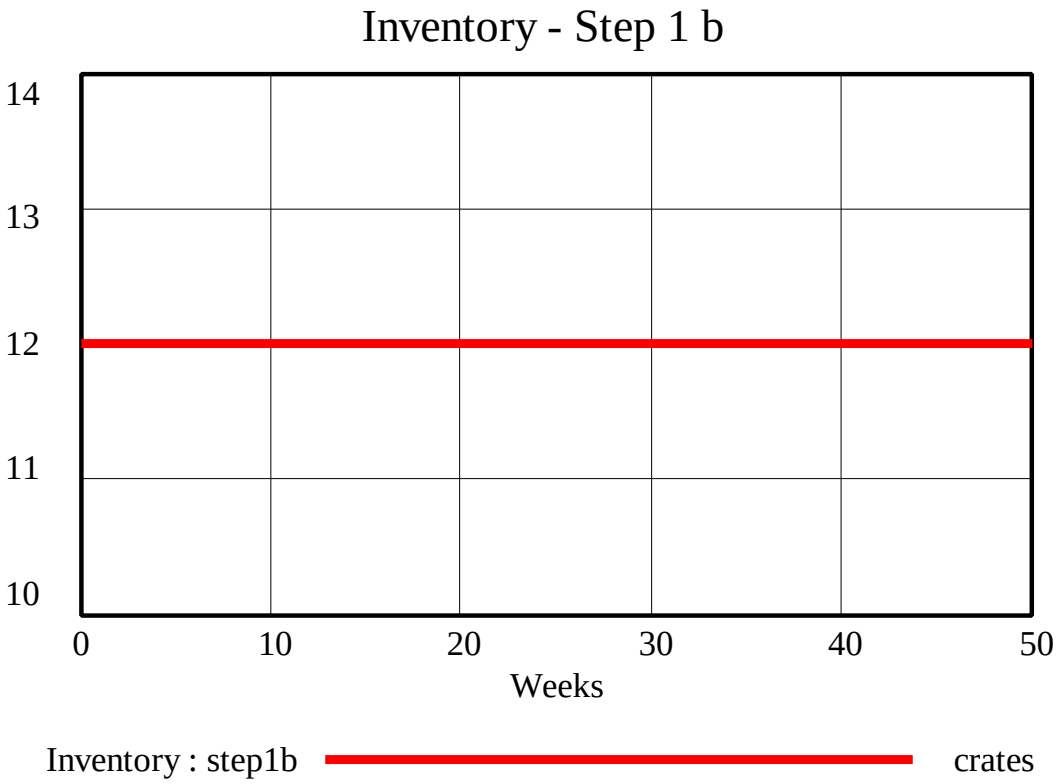
Model equations:

Inventory = INTEG (receiving - selling, 12)
Units: crates
The number of crates of Heavenly Seasonings tea in Richard's supply room.

receiving = 4
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard receives from his supplier every week.

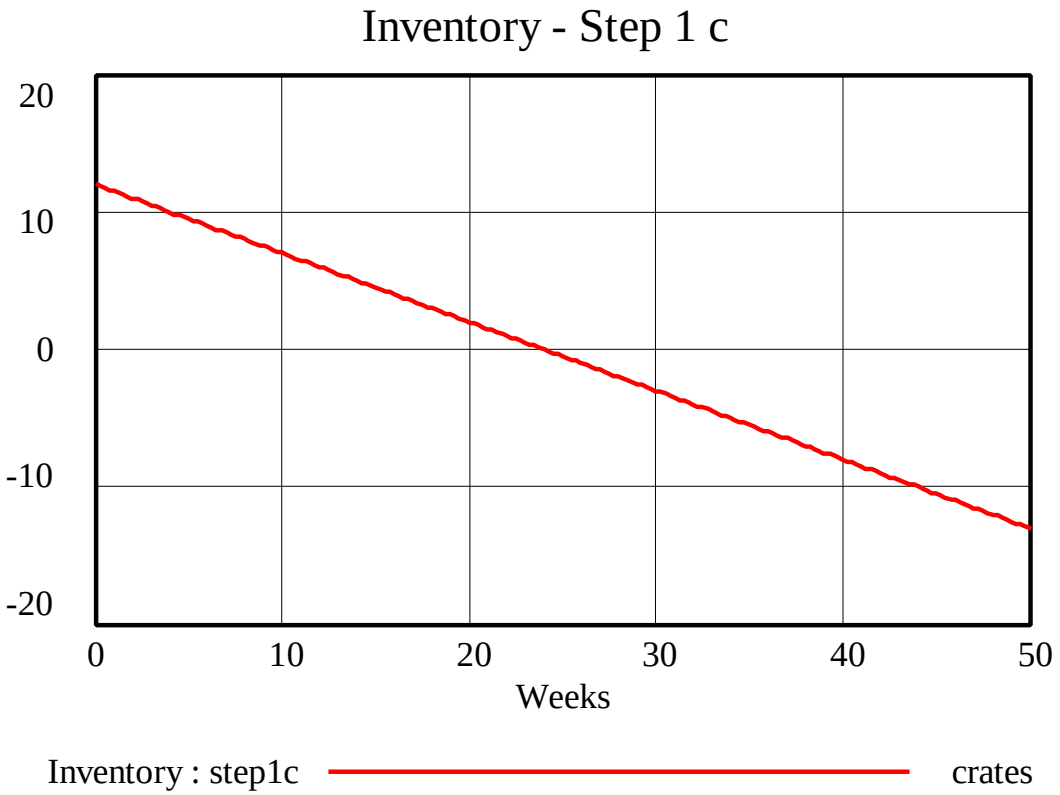
selling = 4
Units: crates/Week
The number of crates of Heavenly Seasonings tea that The Tea Pot sells to students every week.

B. Draw a reference mode for the behavior of the stock over a period of fifty weeks. Simulate the model. In your assignment solutions document, include a graph of the model behavior. What happens to the inventory of herbal tea? Why?



Because the flows of "receiving" and "selling" are equal, "Inventory" stays constant at the initial value.

C. Consider the scenario in which demand for Heavenly Seasonings tea is higher due to the introduction of a new flavor: Strawberry Kiwi Delight. Now college students are buying four and a half crates worth of Heavenly Seasonings tea a week. Richard is not sure that the new craze will last, so he continues receiving only four crates a week. Draw a reference mode for the behavior of The Tea Pot's inventory over the next fifty weeks. Then simulate the model. In your assignment solutions document, include a graph of model behavior in this scenario. What happens to the stock of inventory? Why?

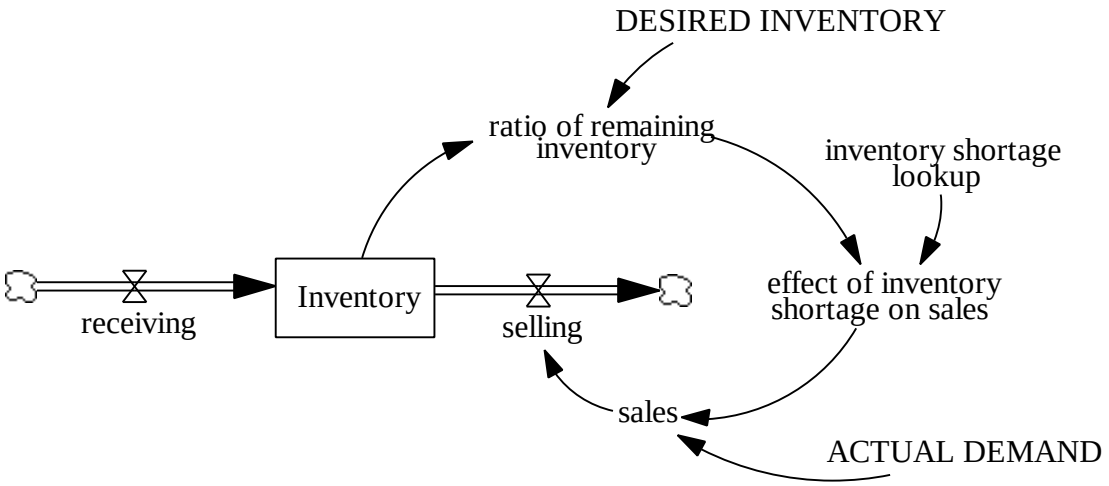


Because "selling" is now higher than "receiving," "Inventory" declines linearly by 0.5 crates every week. Note that, of course, in the real world inventories do not become negative. The next step improves the model.

We need to improve the model to take into account the fact that Richard may not always be able to satisfy the demand for Heavenly Seasonings tea. He can only sell tea if he has it on hand, in his supply room. As long as the inventory is above a certain fraction of the inventory that Richard desires to keep, sales will equal demand. As students deplete his inventory of tea, however, Richard starts to ration the tea. Richard would most likely not sell all of his Heavenly Seasonings tea to the first people who came into his store; he would probably keep a few boxes aside for employees or for his favorite customers. Hence, as The Tea Pot's inventory declines further, the inventory shortage will start having an effect on sales, and sales will fall below demand. When the ratio of the remaining to desired inventory drops to zero, the effect of inventory shortage will prevent any sales from taking place.

D. From the above description, improve the model to reflect Richard's inability to sell tea that he does not have. In your assignment solutions document, include the modified model diagram and documented equations.
Hint: You will need to use a lookup function. You will also want to create a variable, sales, and set the outflow of the stock of inventory equal to those sales.

Model diagram:



Model equations:

ACTUAL DEMAND = 4.5
Units: crates/Week
The actual number of crates of Heavenly Seasonings tea that students want to buy.

DESIRED INVENTORY = 12
Units: crates
The number of crates of Heavenly Seasonings tea that Richard would like to keep in his supply room.

effect of inventory shortage on sales = inventory shortage lookup (ratio of remaining inventory)
Units: dmnl
The effect of inventory shortage on sales is a function of the ratio of remaining inventory.

Inventory = INTEG (receiving – selling, 12)
Units: crates
The number of crates of Heavenly Seasonings tea in Richard's supply room.

inventory shortage lookup ([(0,0) - (1,2)], (0,0), (0.05,0.3), (0.1,0.55), (0.15,0.75), (0.2,0.9), (0.25,0.97), (0.3,1), (1,1))
Units: dmnl
The inventory shortage lookup function reflects the fact that as the ratio of remaining inventory decreases, Richard sells less tea.

ratio of remaining inventory = Inventory / DESIRED INVENTORY

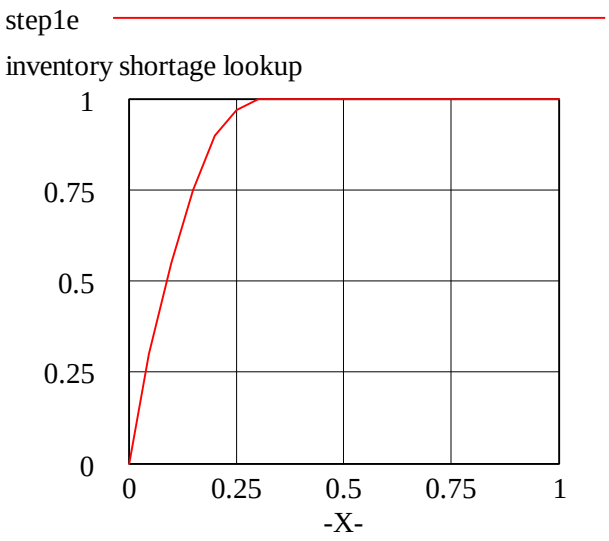
Units: dmnl
The ratio of the current inventory to the desired inventory.

receiving = 4
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard receives from his supplier every week.

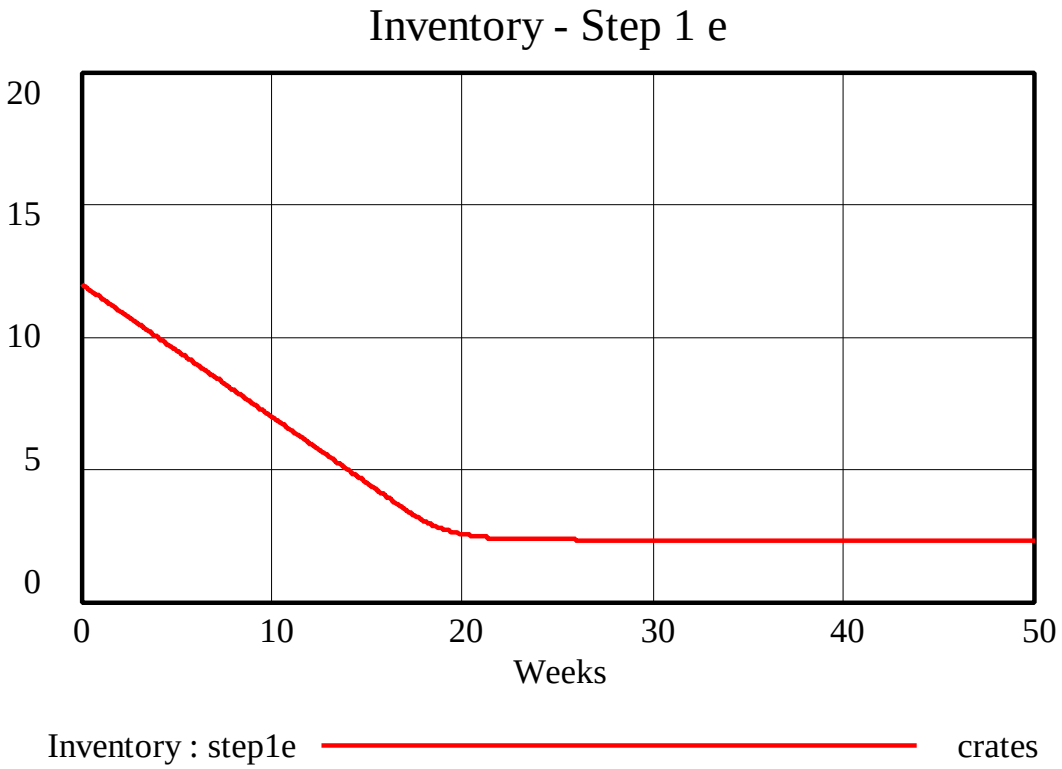
sales = ACTUAL DEMAND * effect of inventory shortage on sales
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard is able to sell.

selling = sales
Units: crates/Week
The number of crates of Heavenly Seasonings tea that The Tea Pot sells to students every week.

Graph of the lookup function:



E. Draw a new reference mode for the behavior of the stock of inventory in the Strawberry Kiwi Delight scenario. Simulate the model over a period of fifty weeks. In your assignment solutions document, include a graph of model behavior. What happens now to the stock of inventory? Why?



At first, "Inventory" declines linearly by 0.5 crates every week, as in part C. When the ratio of "Inventory" to "DESIRED INVENTORY" becomes low, however, the inventory shortage starts to have an effect on "sales." The outflow "selling" starts to decrease, and "Inventory" decreases more and more slowly, until "selling" is just equal to "receiving," and the stock settles at an equilibrium value of approximately two and a half crates of Heavenly Seasonings tea.

Step 2: Unfilled Orders

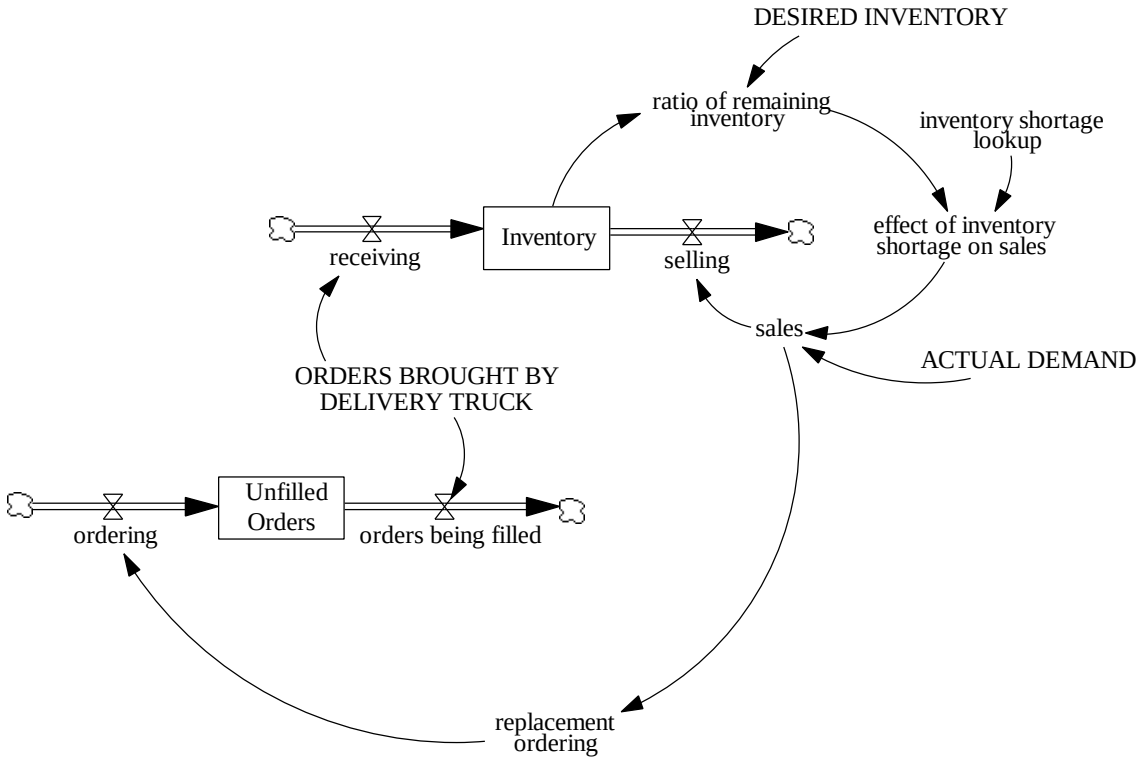
If Richard receives four crates of Heavenly Seasonings tea every week, it is because he placed orders for that tea with his supplier several weeks earlier. Now, Richard orders tea to replace the tea that he sells every week. In a notebook, Richard keeps a log of all of his transactions. He keeps track of how many orders he places every week and how many orders are filled every week when the delivery truck arrives. Today, Richard has unfilled orders for 16 cases of herbal tea.

A. Create a stock of "Unfilled Orders." What are the inflows and outflows to the stock? From the above description, formulate the equations driving the flows to the stock. In your assignment solutions document, include the new model diagram and documented equations.

Hint 1: The two stocks, "Inventory" and "Unfilled Orders" represent two different concepts and hence there cannot be a flow from one to the other.

Hint 2: Do not connect any flows to each other. Instead, formulate identical equations for the two flows if you wish them to have the same values.

Model diagram:



Modified model equations:

ordering = replacement ordering
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard orders from his supplier every week.

orders being filled = ORDERS BROUGHT BY DELIVERY TRUCK
Units: crates/Week
The number of orders that Richard has placed that are filled every week.

ORDERS BROUGHT BY DELIVERY TRUCK = 4
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard's supplier delivers to The Tea Pot every week.

receiving = ORDERS BROUGHT BY DELIVERY TRUCK
Units: crates/Week

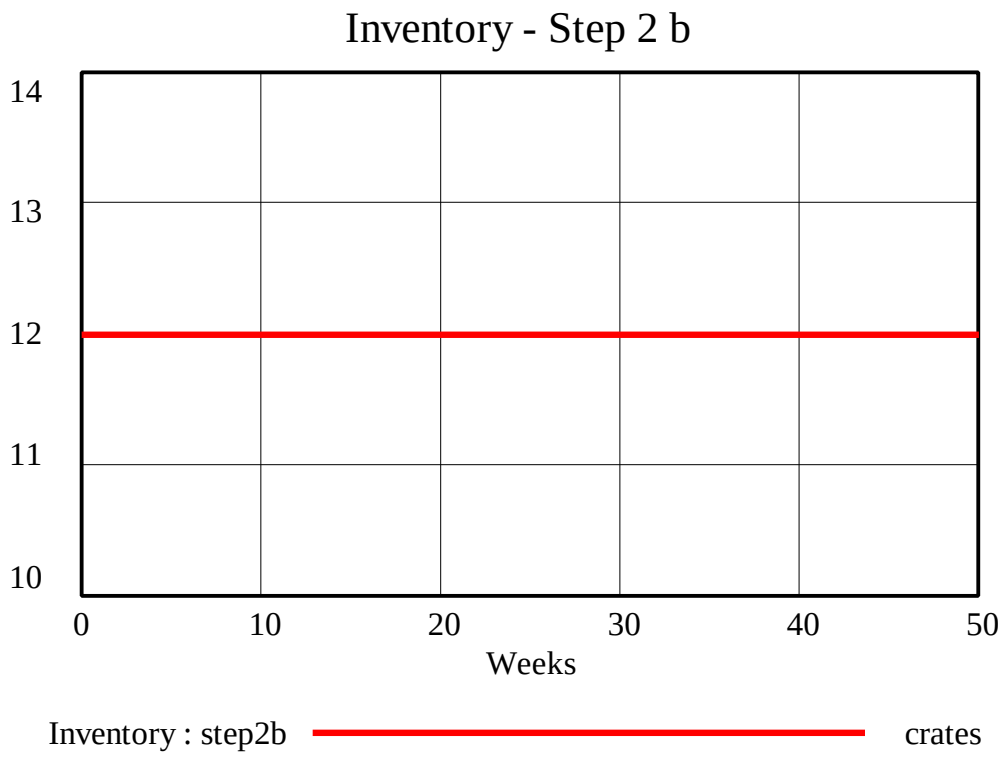
The number of crates of Heavenly Seasonings tea that Richard receives from his supplier every week.

replacement ordering = sales
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard has to order from his supplier every week to replace those that are sold to customers.

Unfilled Orders = INTEG (ordering - orders being filled, 16)
Units: crates
The number of orders that Richard has placed with his supplier but has not yet received.

Note that "Unfilled Orders" represents not the actual crates of tea but, rather, information about the crates, that is, the count of the number of crates that have been ordered. Because the two stocks of "Unfilled Orders" and "Inventory" represent two different concepts, it is incorrect to let the outflow from "Unfilled Orders" flow directly into the stock of "Inventory." Even though the flows "receiving" and "orders being filled" are always equal in measure, they are conceptually very different, and therefore must be modeled separately.

B. Draw reference modes for the two stocks under the original scenario (college students demand four crates worth of herbal tea a week) and then simulate the model over a period of fifty weeks. In your assignment solutions document, include graphs of the behavior of both stocks. What are the equilibrium values of the two stocks? Why?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Unfilled Orders - Step 2 b
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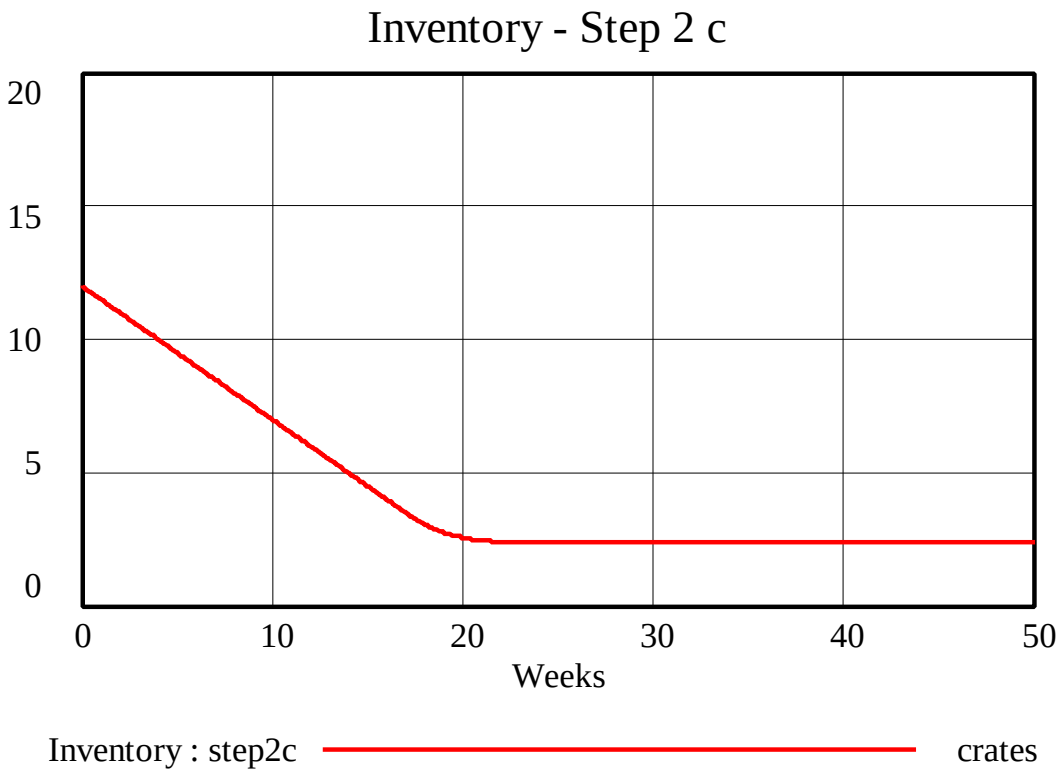
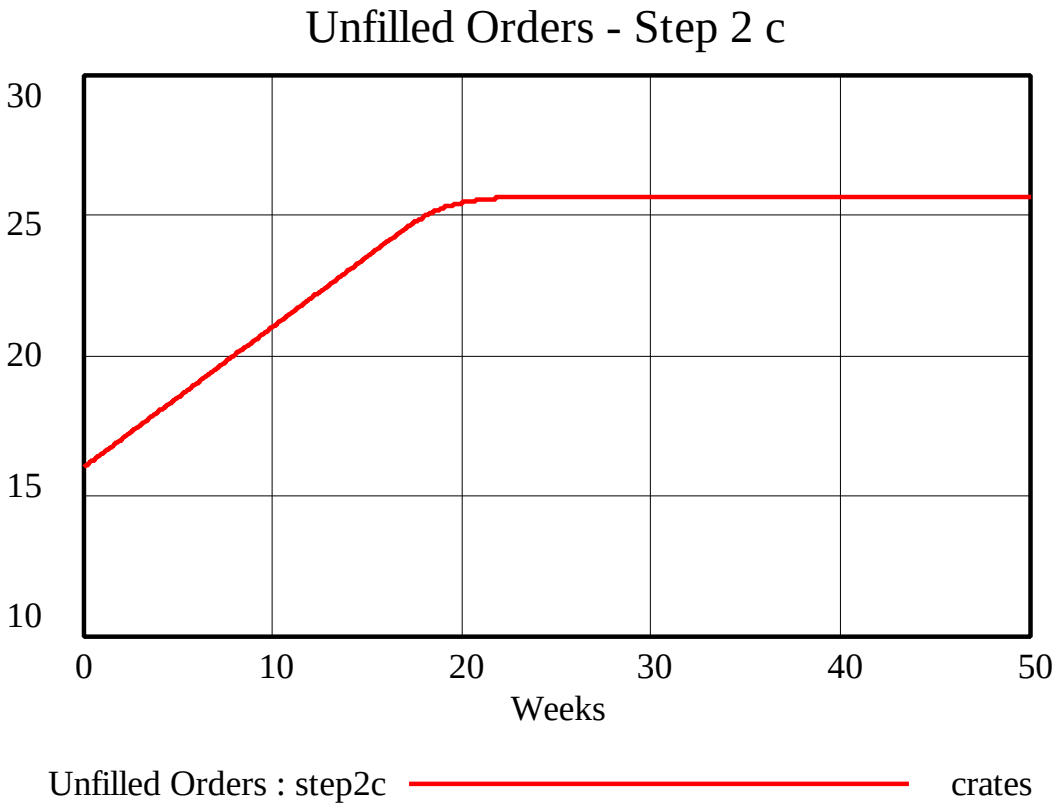
Weeks
Unfilled Orders : step2b
crates

The inflow into each stock equals its outflow, so each stock remains at its initial value throughout the simulation. The equilibrium value of "Inventory" is 12 crates, and the equilibrium value of "Unfilled Orders" is 16 crates of Heavenly Seasonings tea.

C. Draw reference modes for the two stocks under the Strawberry Kiwi Delight scenario. Simulate the model under this scenario over a period of fifty weeks. In your assignment solutions document, include graphs of the behavior of both stocks. What happens to the two stocks? Why?



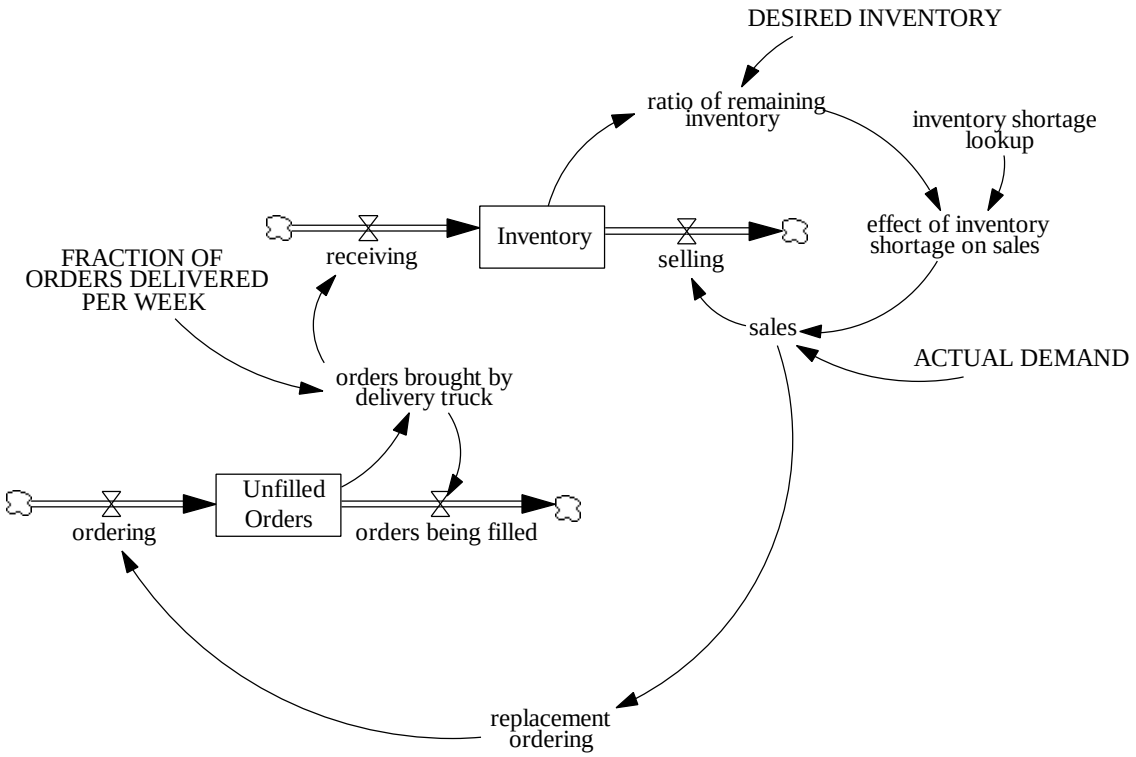


The rise in "ACTUAL DEMAND" leads to a rise in "sales," causing Richard to increase his "replacement ordering." The stock of "Unfilled Orders" therefore increases linearly at first. As "Inventory" decreases because of higher "sales," the ratio of "Inventory" to "DESIRED INVENTORY" falls, and the inventory shortage starts to decrease "sales." Richard then needs to order less Heavenly Seasonings tea to replace the sales, and the "Unfilled Orders" stock levels off at an equilibrium value of approximately 25 crates.

The model is still unrealistic. If Richard is selling more than 4 crates of Heavenly Seasonings tea, and ordering more than 4 crates of tea, he must eventually begin to receive the extra tea that he ordered. To improve the model, we need to improve the formulation of the rate at which Richard receives inventory. For now, let us assume that every week The Tea Pot receives one fourth of its unfilled orders, which implies an average delivery time of four weeks.

D. Improve the model by reformulating the inflow to the stock of inventory. Draw reference modes for the two stocks and then simulate the model for the Strawberry Kiwi Delight scenario over a period of 50 weeks. In your assignment solutions document, include graphs of the behavior of both stocks. What happens to the stocks? Why? What are the new equilibrium values of the two stocks?

Model diagram:

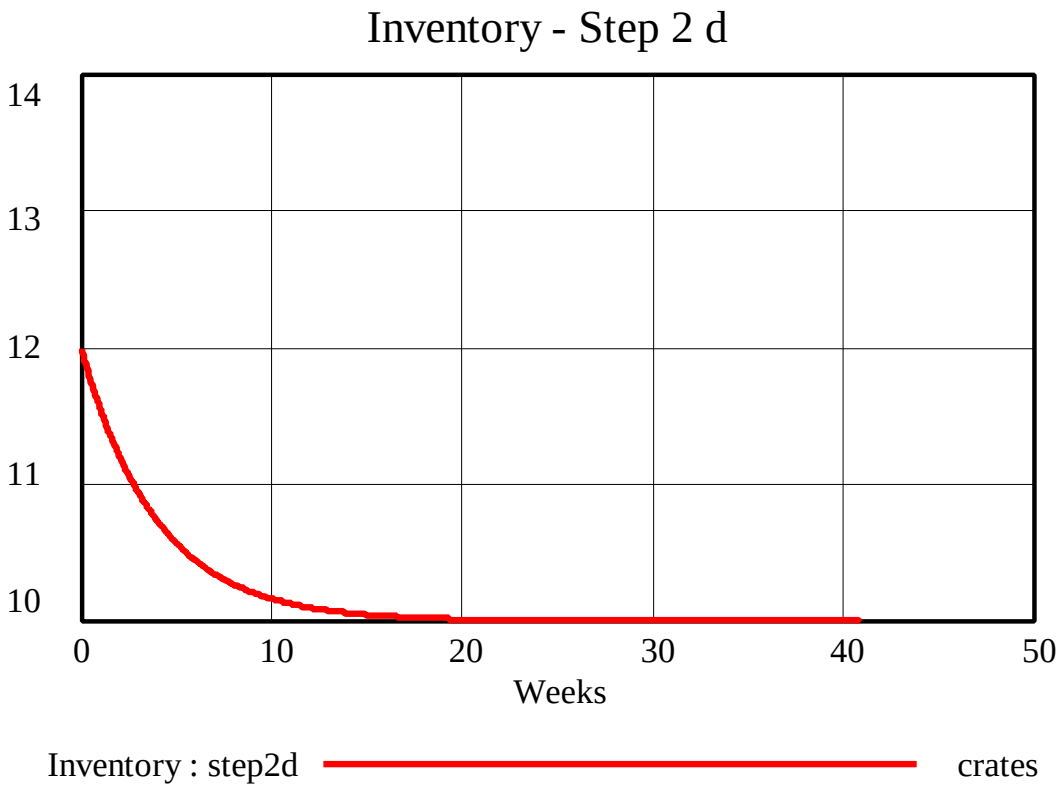


Modified model equations:

FRACTION OF ORDERS DELIVERED PER WEEK = 0.25
Units: 1/Week
The fraction of unfilled orders that The Tea Pot receives every week.

orders brought by delivery truck = Unfilled Orders * FRACTION OF ORDERS DELIVERED PER WEEK
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard's supplier delivers to The Tea Pot every week.

Model behavior: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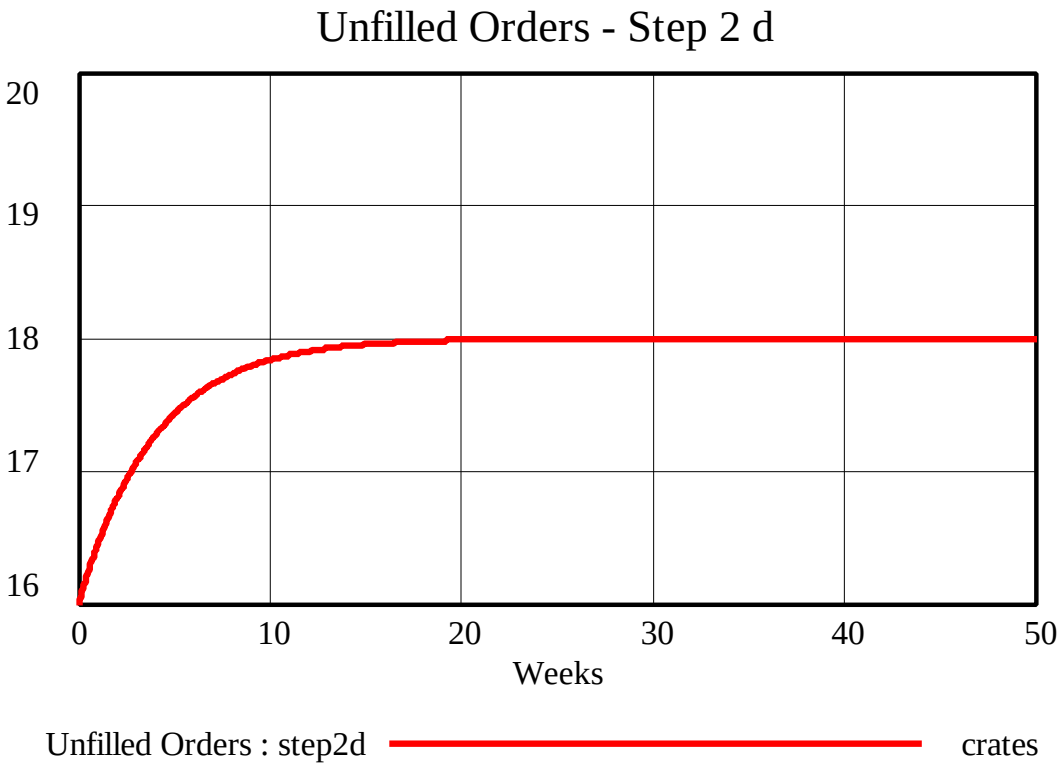
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crates

Let us first analyze the behavior of "Unfilled Orders." Initially, Richard has unfilled orders for 16 crates of tea. Because "ACTUAL DEMAND" suddenly increases to 4.5 crates per week, Richard's replacement ordering also increases, increasing the inflow into "Unfilled Orders" to a constant 4.5 crates per week. The number of "orders being filled," the outflow from the stock of "Unfilled Orders," equals only one-fourth of the current value of the stock, or 4 crates per week. Richard's rate of "ordering" therefore exceeds the rate at which his unfilled orders are being filled, and the stock of "Unfilled Orders" increases. The increase of the stock causes the outflow to increase until the outflow reaches the value of the inflow, 4.5 crates per week. When both "ordering" and "orders being filled" equal 4.5 crates per week, the stock "Unfilled Orders" settles at an equilibrium value of 18 crates.

The behavior of the stock of "Inventory" is determined by the behavior of the stock of "Unfilled Orders" because "receiving," the inflow to "Inventory," equals one fourth of the current value of "Unfilled Orders." The outflow from "Inventory," "selling," is a constant 4.5 crates per week. Because "receiving" only equals 4 crates per week at the beginning of the simulation, the stock of "Inventory" starts to decline and decreases until the inflow of "receiving" grows to equals the outflow of "selling." Receiving increases to 4.5 crates per week when the stock of "Unfilled Orders" finally equals 18 crates. Then the stock of "Inventory" settles at an equilibrium value of 10 crates.

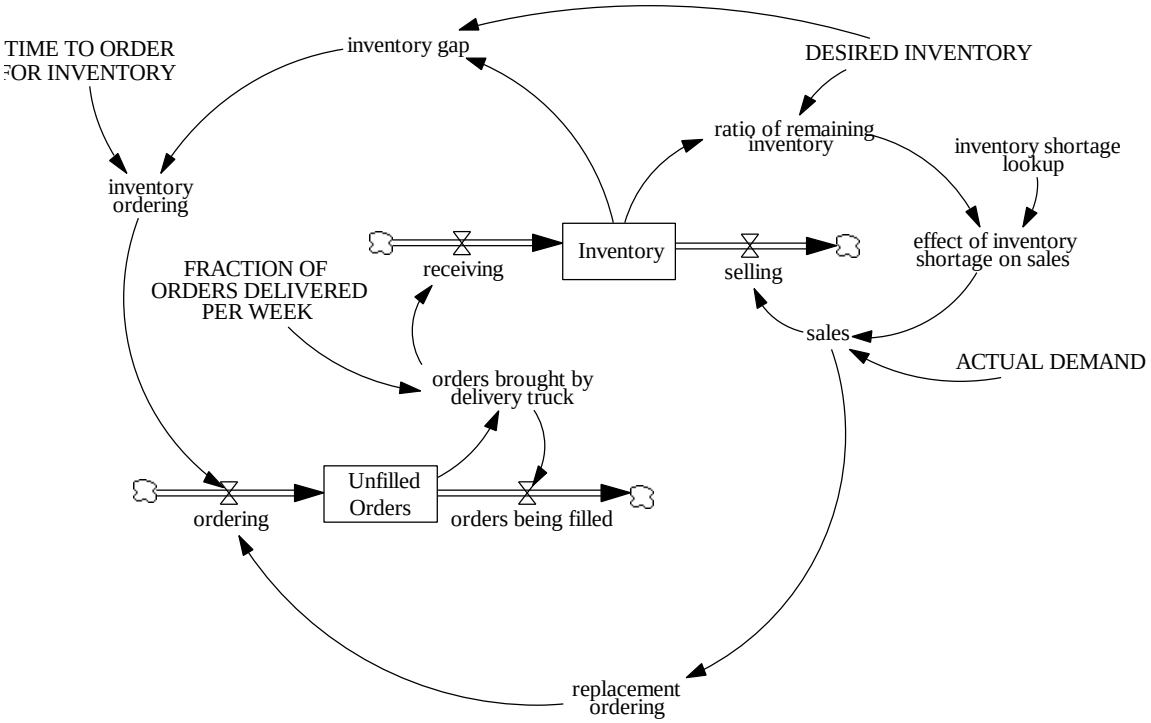
Notice that an increase in demand increases the equilibrium level of unfilled orders and decreases the equilibrium level of Richard's inventory. The next step allows Richard to maintain a stable inventory.

Step 3: Desired Inventory

When demand changes, the level of inventory in the supply room of The Tea Pot changes. If Richard wants to keep his inventory constant, he will need to order more tea, above and beyond his replacement order. Consider the Strawberry Kiwi Delight scenario. Richard has a desired inventory of twelve crates of Heavenly Seasonings tea. As his inventory is depleted, he needs to add extra crates of inventory correction ordering to his weekly replacement order. It takes two weeks to place an order to correct the inventory gap.

A. Add the new elements to the model. In your assignment solutions document, include the new model diagram and documented equations.

Model diagram:



Modified model equations:

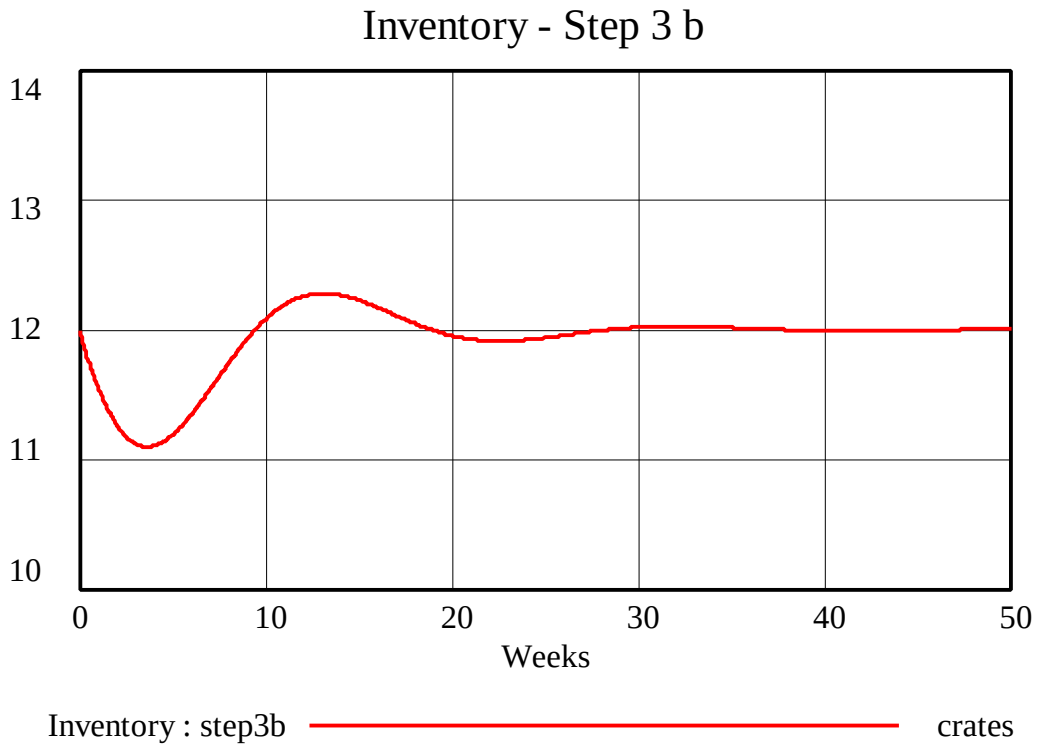
inventory gap = DESIRED INVENTORY – Inventory
Units: crates
The difference between Richard's desired inventory and the current inventory.

inventory ordering = inventory gap / TIME TO ORDER FOR INVENTORY
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard orders every week to correct for an inventory gap.

ordering = replacement ordering + inventory ordering
Units: crates/Week
The number of crates of Heavenly Seasonings tea that Richard orders from his supplier every week.

TIME TO ORDER FOR INVENTORY = 2
Units: Week
The time it takes Richard to order tea to correct for an inventory gap.

B. Draw reference modes for the behavior of the two stocks in the Strawberry Kiwi Delight scenario. Simulate the model for this scenario over a period of fifty weeks. In your assignment solutions document, include graphs of the behavior of both stocks and any other variables that might help you explain the behavior. Explain in detail the dynamics of the behavior that you observe.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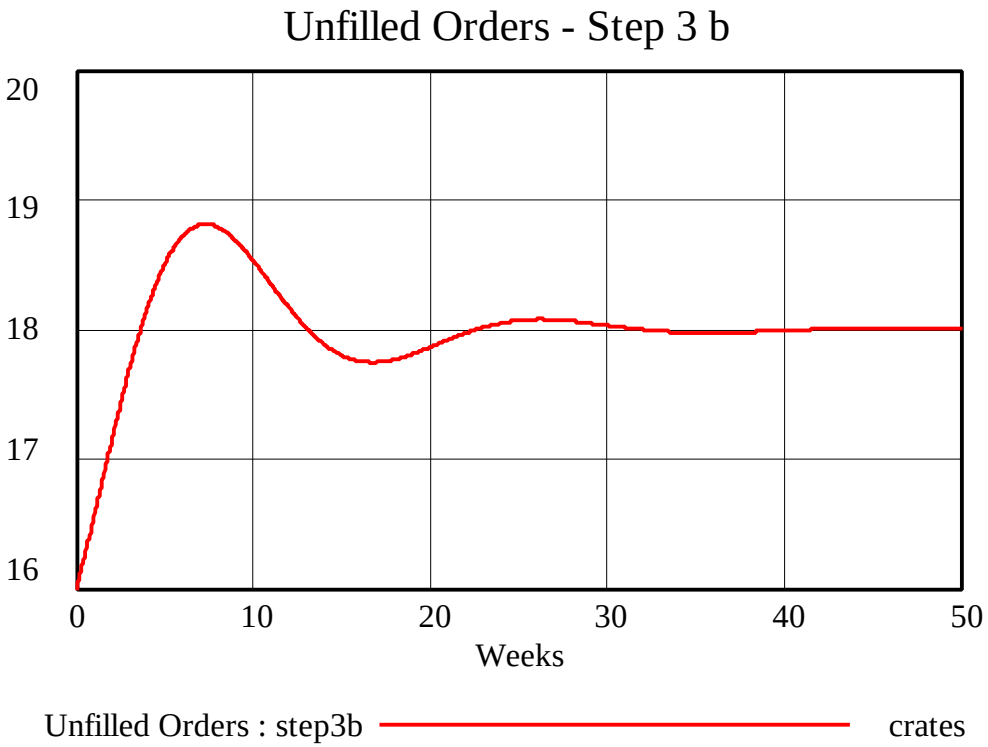
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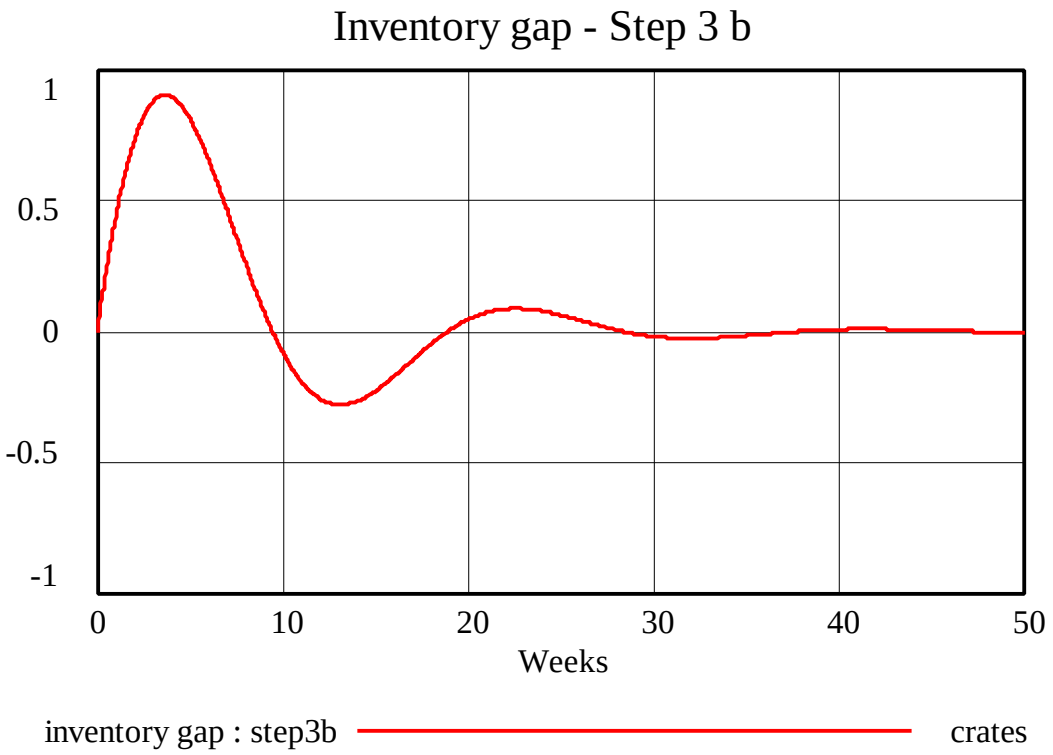
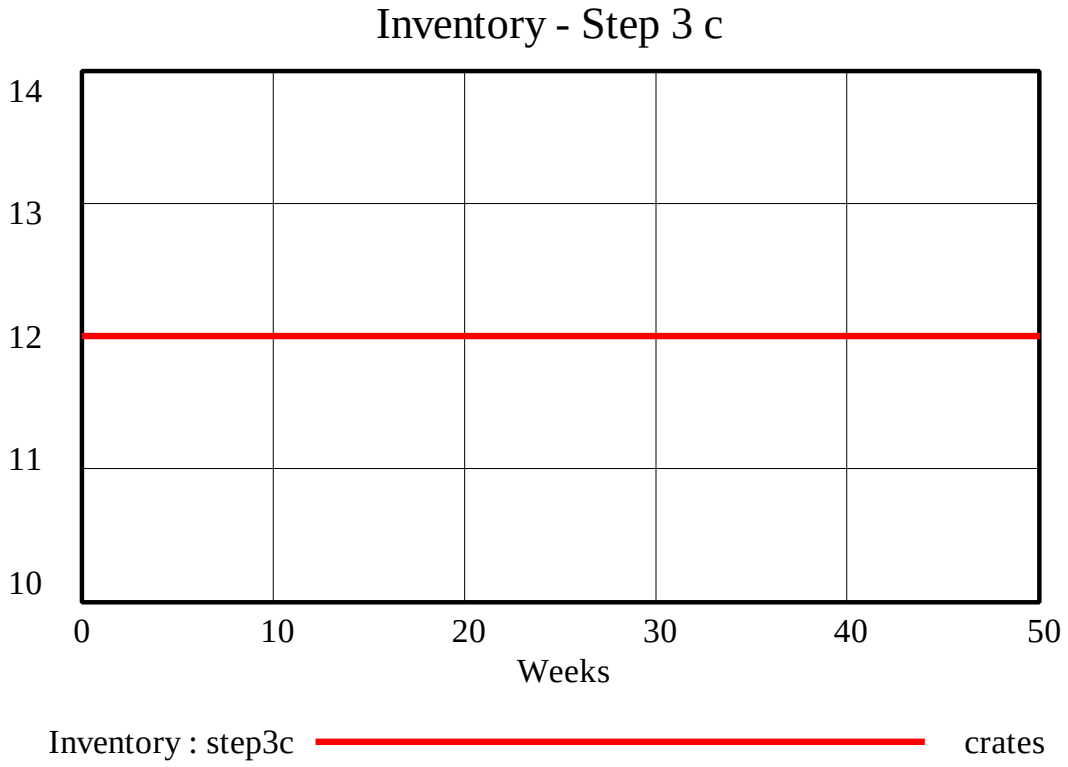
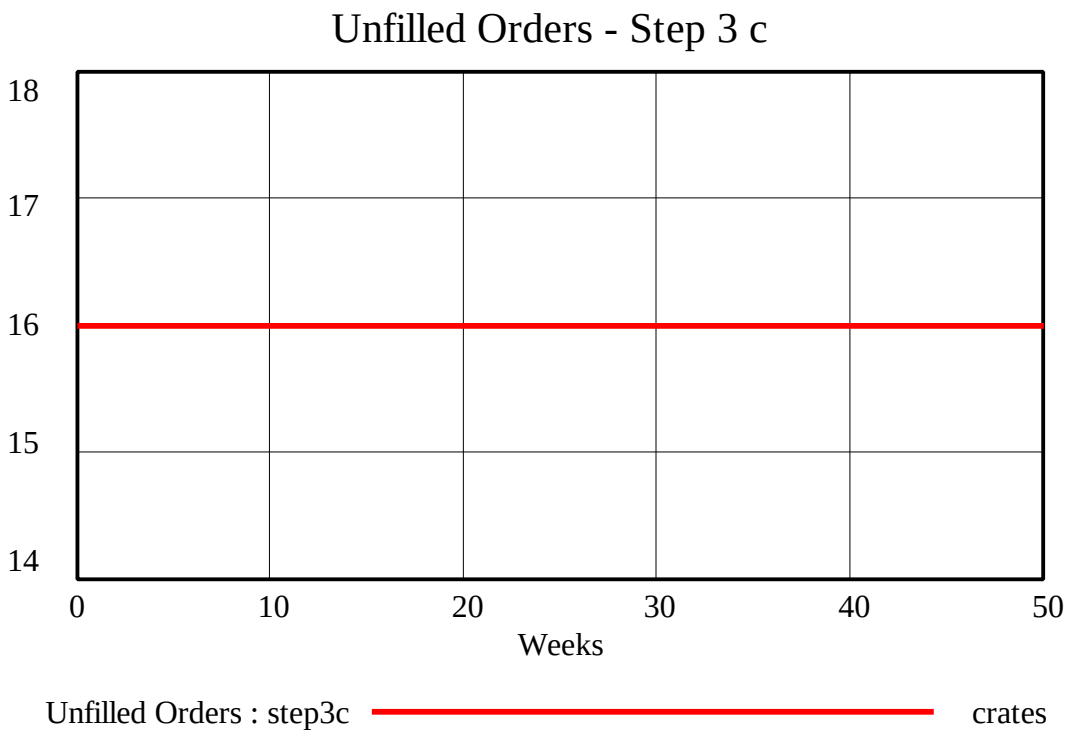


The system is now essentially a goal-gap system with a material delay. When Richard places inventory orders to close his inventory gap, he does not take into account the long delay in receiving the orders that he places, thereby causing the system to oscillate.
When actual demand increases, sales increase and the current inventory is somewhat depleted. Richard not only increases his replacement ordering in response to the additional sales, but he also begins to place inventory orders so that his inventory will not stabilize at a low equilibrium value, as in step 2C of this exercise. When actual demand first changes, Richard looks at his current inventory, considers his desired inventory, and then places an inventory order to close his inventory gap. The next time he goes to order, he looks again at his current inventory, considers his desired inventory, and then places another inventory order to close his inventory gap. The problem is that his current inventory does not yet contain the additional inventory orders that he placed in the previous time period. The inventory gap does not take into account the extra unfilled orders that he has placed but not yet received. Therefore, Richard places too many additional inventory-correction orders. When the additional orders finally begin to arrive, Richard realizes that his current inventory is becoming too large, so he places fewer orders. Unfortunately, he overcompensates again by not placing enough orders. Because of the delay in receiving orders, Richard places too many and then too few orders. Meanwhile, the stock of unfilled orders approaches its new equilibrium value, so Richard receives most of the orders that he should be receiving, his inventory stabilizes and he no longer needs to order additional crates of tea to correct his inventory.

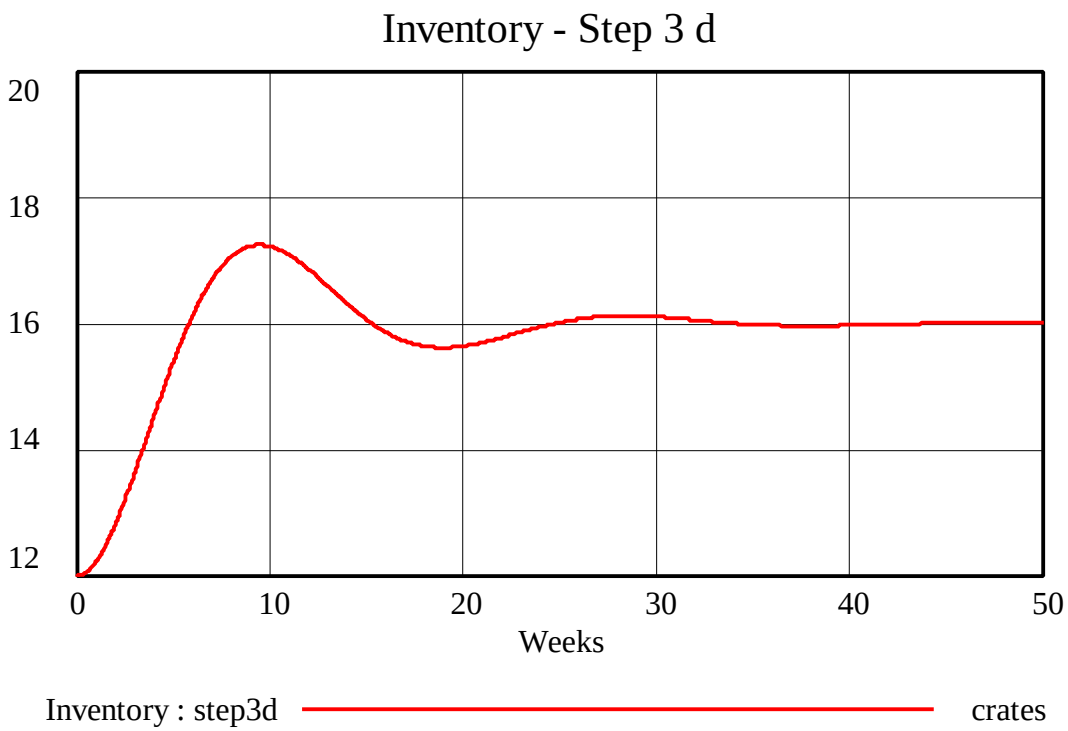
C. Consider again the original scenario in which college students buy only four crates worth of Heavenly Seasonings tea. Draw reference modes for the behavior of the two stocks. Simulate the model over a period of fifty weeks. In your assignment solutions document, include graphs of the behavior of both stocks. What behavior do you observe? Why?





When "ACTUAL DEMAND" is only four crates of tea per week, the system remains at equilibrium because the inflow and outflow into each stock are equal.

D. Consider again the original scenario. Richard decides to increase his inventory of Heavenly Seasonings tea from twelve crates to sixteen crates. Draw reference modes for the behavior of the two stocks over the next fifty weeks. Simulate the model. In your assignment solutions document, include graphs of the behavior of both stocks. What behavior do you observe? Why?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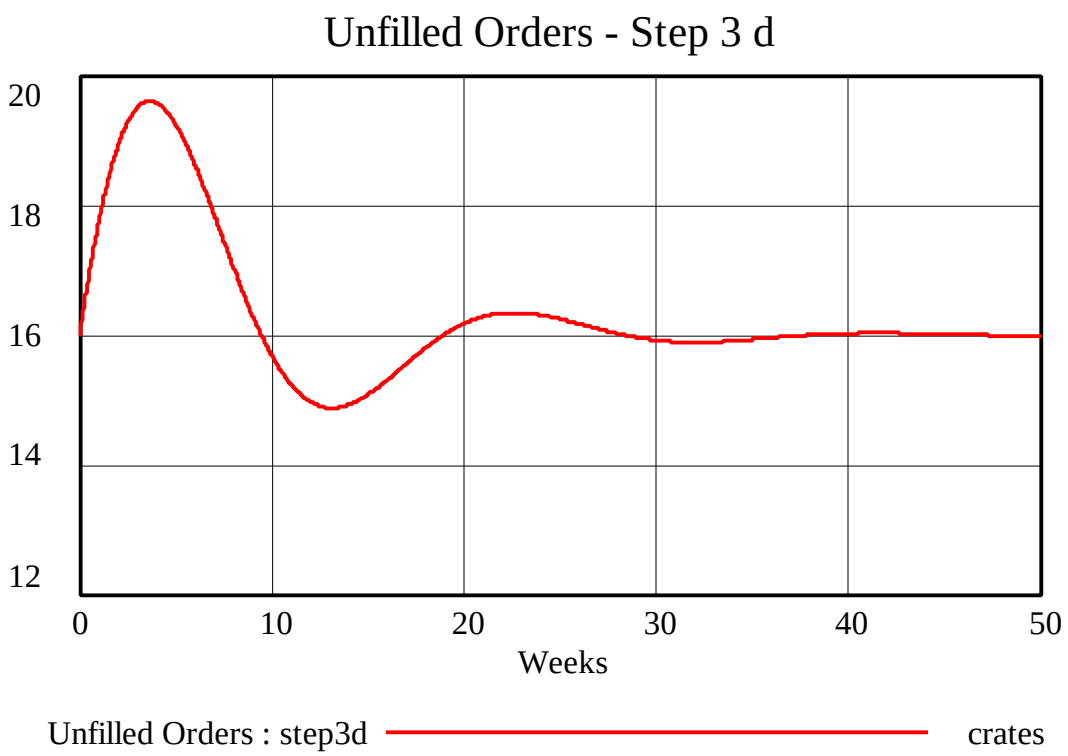
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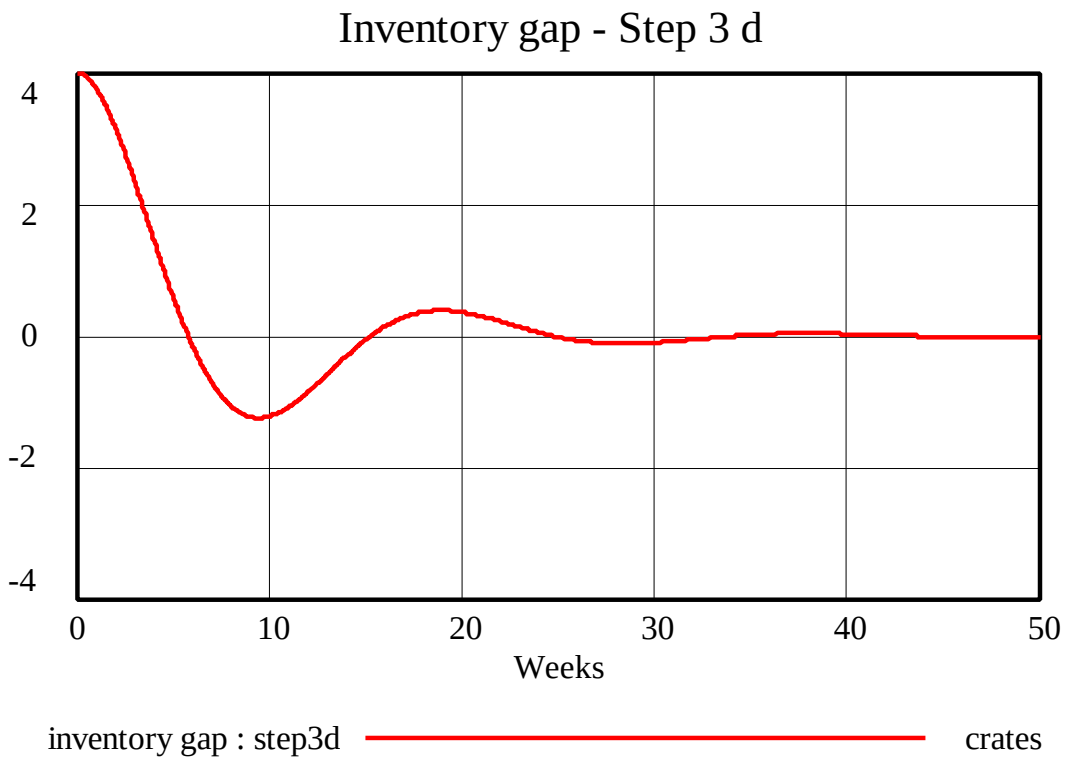


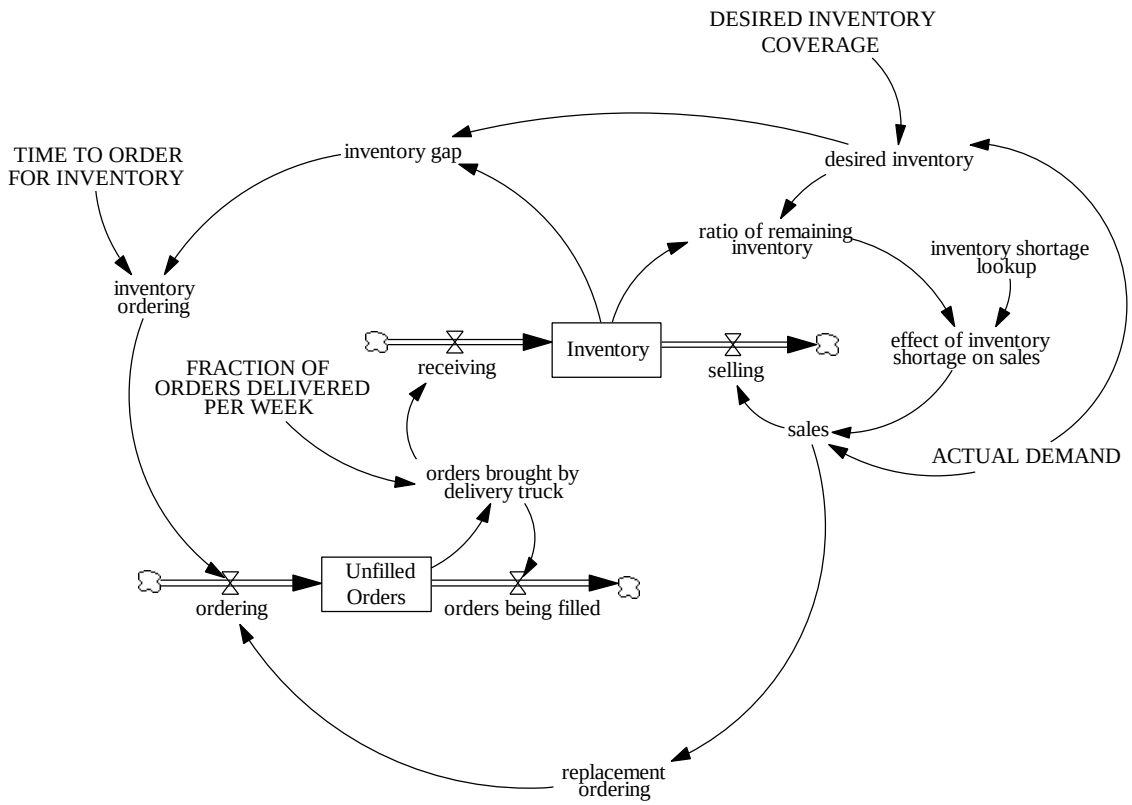
As in part A, the system generates damped oscillations that are triggered by a positive "inventory gap." In this case, however, the gap is the result of a higher "DESIRED INVENTORY," rather than of a decreasing "Inventory."

Step 4: Desired Inventory Coverage

The model is still not very realistic. In the real world, managers do not desire a certain level of inventory but a certain level of inventory coverage. Desired inventory coverage, measured in days, weeks, or months, is simply the ratio of desired inventory to demand per unit time. If actual demand is four crates of tea a week, then a desired inventory of 12 crates represents an inventory coverage equal to three weeks. If Richard wants an inventory coverage of four weeks, then his desired inventory would be 16 crates.

A. Add the new elements to the model. In your assignment solutions document, include the new model diagram and documented equations.

Model diagram:



Modified model equations:

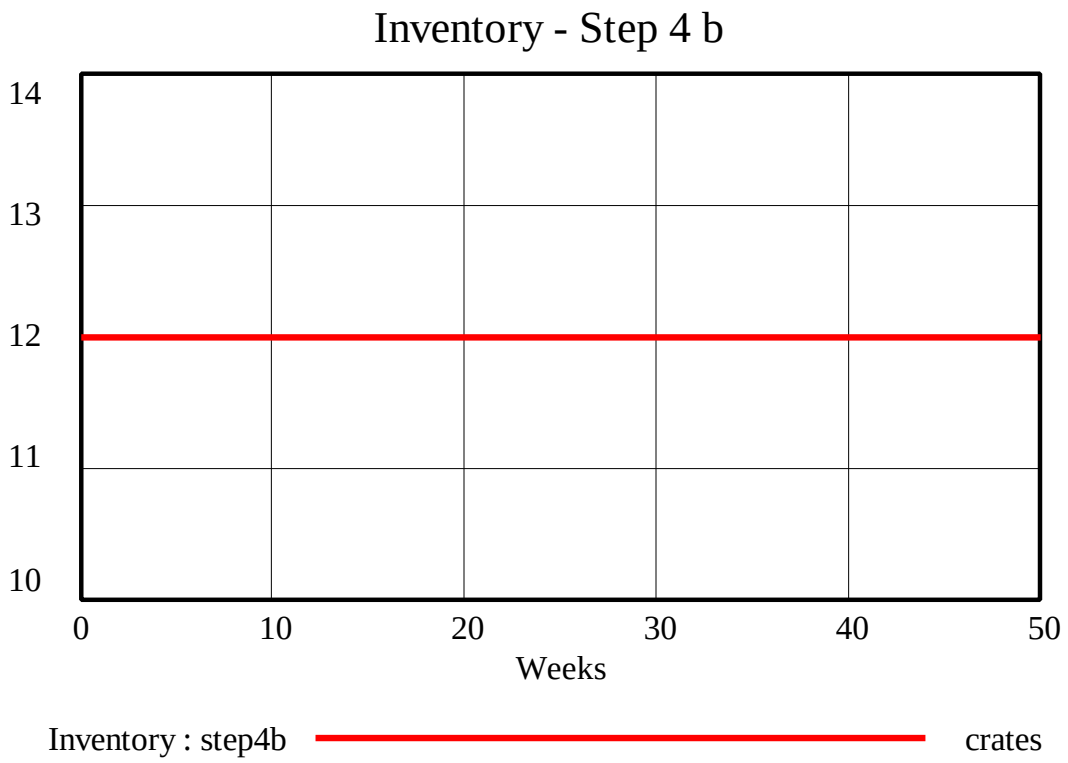
desired inventory = ACTUAL DEMAND * DESIRED INVENTORY COVERAGE
Units: crates
The number of crates of Heavenly Seasonings tea that Richard would like to keep in his supply room.

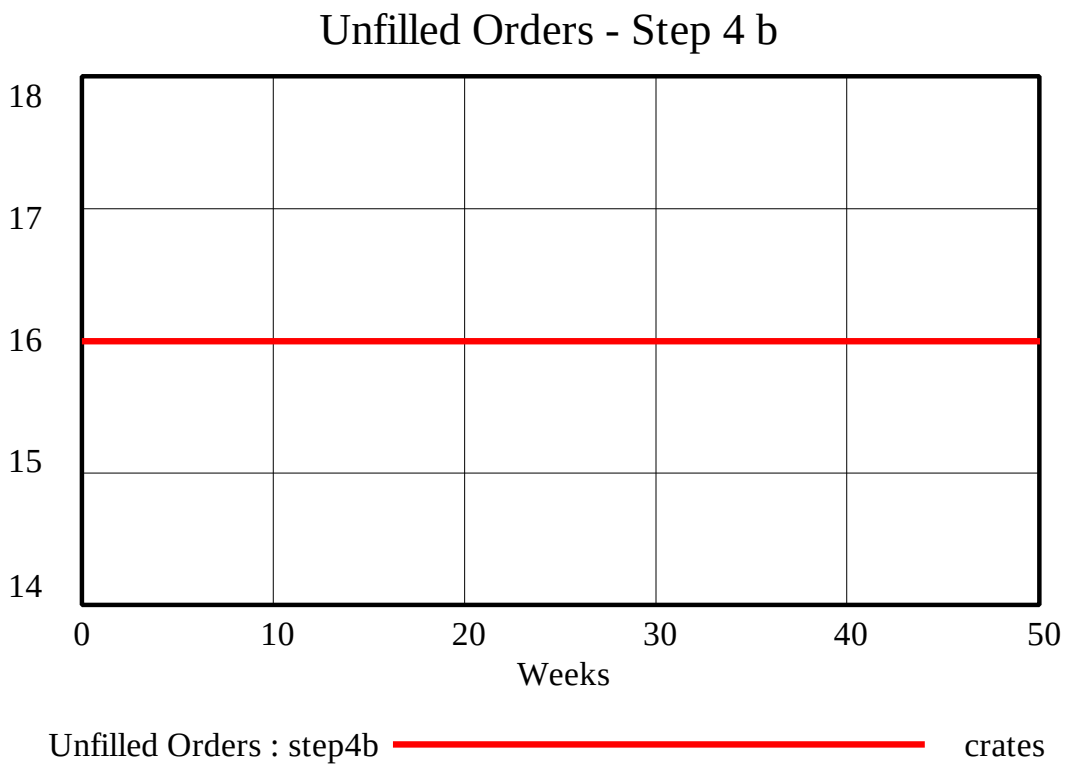
DESIRED INVENTORY COVERAGE = 3
Units: Week
The number of weeks worth of demand for Heavenly Seasonings tea that Richard wants to keep in his supply room.

Notice that the desired inventory is based on actual demand. In the real world, a company has no way of knowing what actual demand might be. They have available the sales figure after actual demand has been influenced by many factors (such as the effect of inventory shortage). In a more comprehensive setting, product quality, marketing effectiveness, ease of placing orders, or reputation would also affect sales. Situations arise where sales are as little as 10% of actual demand and the company has no idea that sales could be higher.

B. First consider the original scenario with demand of four crates a week, in which Richard's desired inventory coverage is three weeks. Draw reference modes for the behavior of the two stocks over the next fifty weeks. Simulate the model. In your

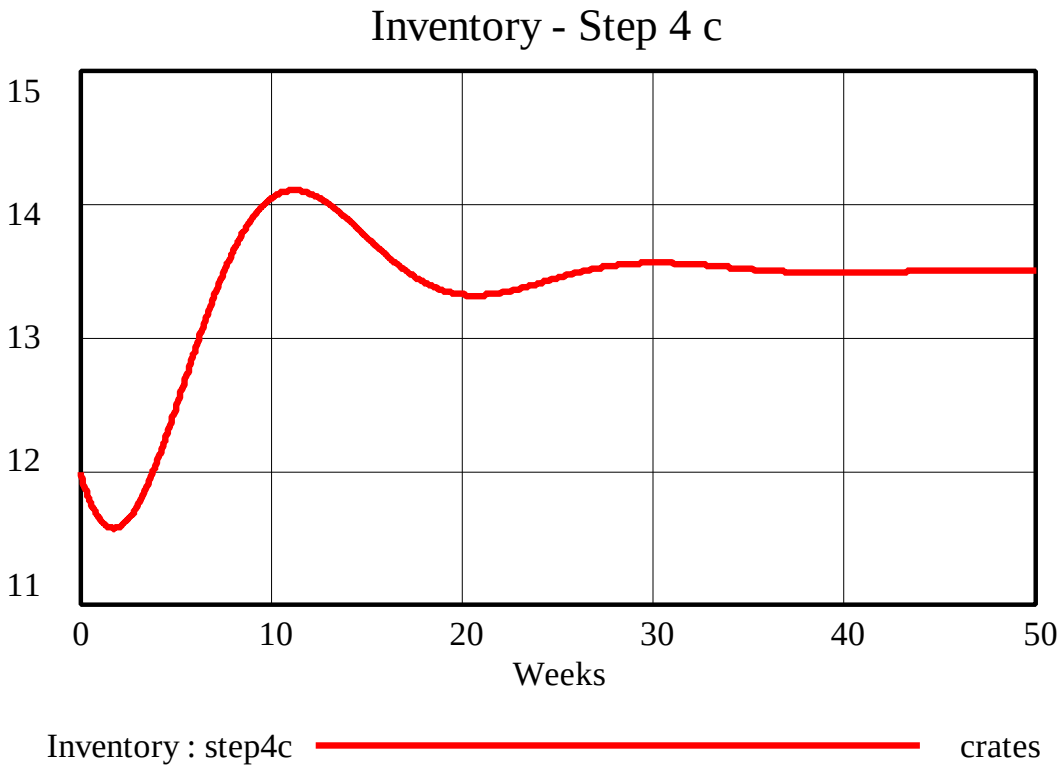
assignment solutions document, include graphs of the behavior of both stocks. What behavior do you observe? Why?

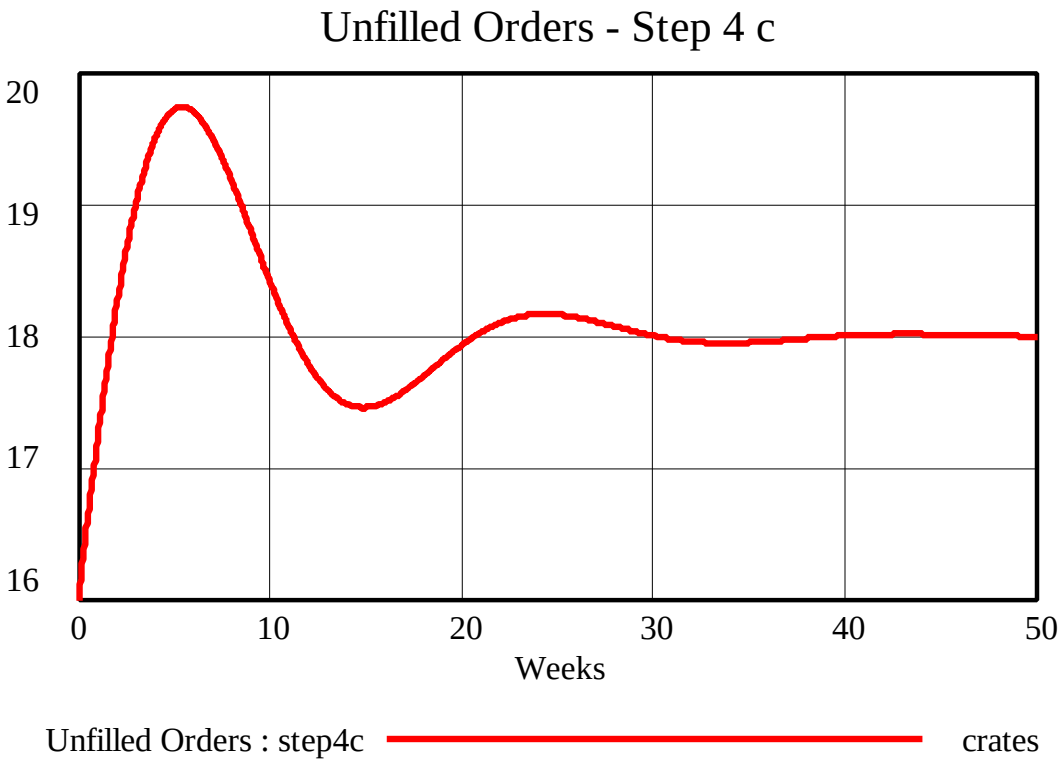




Because the actual demand is 4 crates a week and the inventory coverage is 3 weeks, the desired inventory is 12 crates. The desired inventory equals the current inventory, so there is no inventory gap. The net flows to both stocks are zero, so the system is in equilibrium.

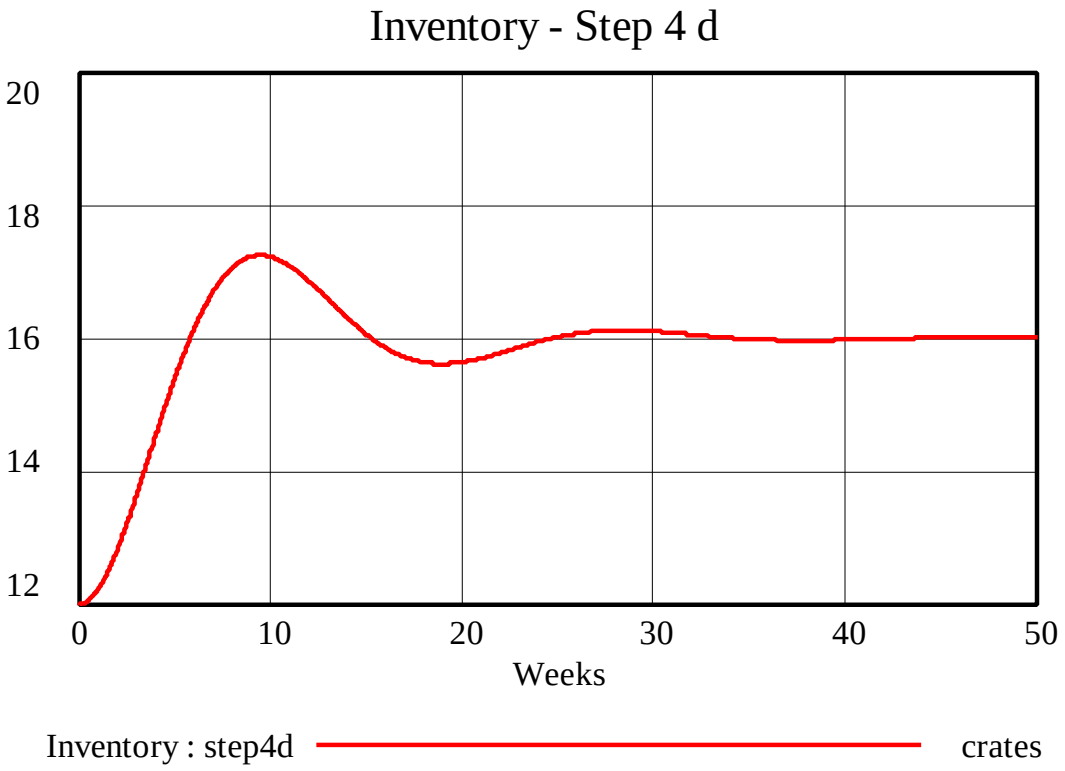
C. Consider now the Strawberry Kiwi Delight scenario with demand of four and a half crates a week. Assume that Richard's desired inventory coverage is still three weeks. Draw reference modes for the behavior of the two stocks over the next fifty weeks. Simulate the model. In your assignment solutions document, include graphs of the behavior of both stocks. What behavior do you observe? Why?

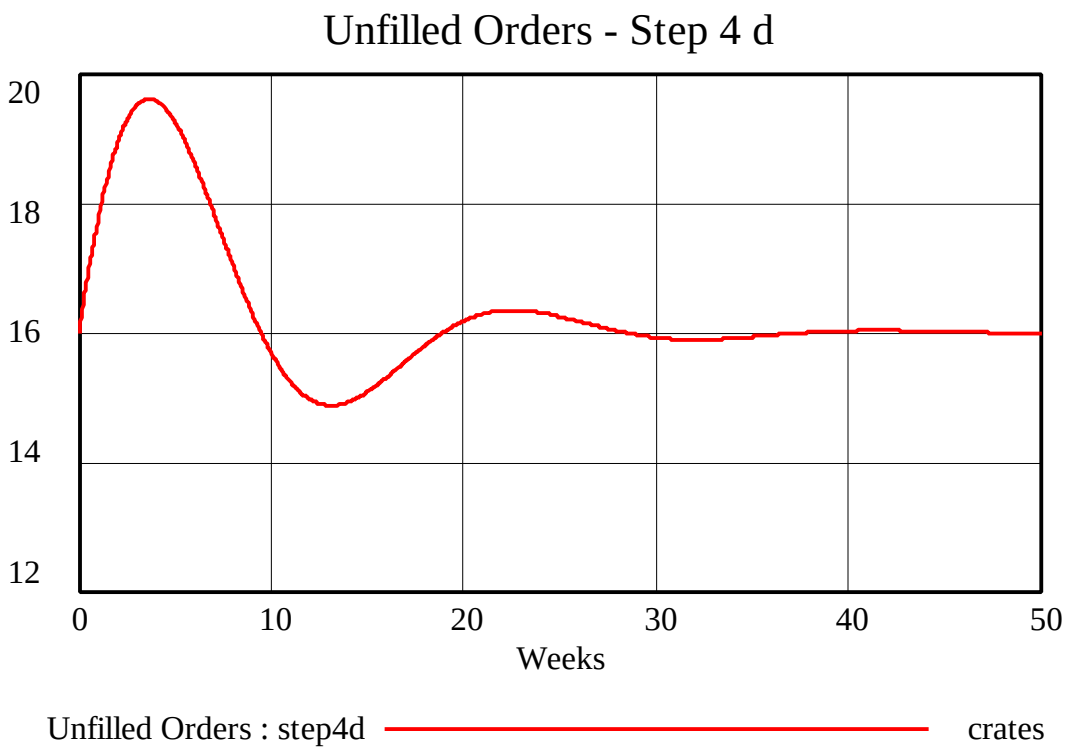

Inventory - Step 4 c
15
14
13
12
11
0
10
20
30
40
50
Weeks
Inventory : step4c
crates


Unfilled Orders - Step 4 c
20
19
18
17
16
0
10
20
30
40
50
Weeks
Unfilled Orders : step4c
crates

The model produces the same dynamic behavior as in step 3b, but with a higher equilibrium value of "Inventory."

D. Finally consider the scenario in which demand is back to four crates a week, but Richard now desires an inventory coverage of four weeks. Draw reference modes for the behavior of the two stocks over the next fifty weeks. Simulate the model. In your assignment solutions document, include graphs of the behavior of both stocks. What behavior do you observe? Why?





Again, the model produces the same behavior as in step 3d.