

Guided Study Program in System Dynamics

System Dynamics in Education Project

System Dynamics Group

MIT Sloan School of Management¹

Solutions to Assignment #17

Friday, March 12, 1999

Reading Assignment:

There is no reading assignment.

Exercises:

1. Fishbanks Modeling Exercise

Fish Banks, Ltd. is a role-playing game developed by Dennis L. Meadows at the University of New Hampshire. Dr. Meadows developed the game to inform people about using natural resources effectively and prudently. Although the game originally targeted corporate managers and public officials, anyone can benefit from the insights gained by playing the game.

In the Fish Banks game, teams of players manage their own fishing companies. At the beginning of the game, each fishing company has equal amounts of money and fishing ships. Each company has the same operating costs and technology. At the beginning of every simulated year, the teams make decisions about buying or selling ships, whether to fish or not, and where to fish. The object of the game for each company is to maximize total profits.

In this assignment you will conceptualize, formulate, and simulate the system dynamics model that underlies the Fish Banks game. The fishing system model can be segmented into two subsystems: the fish sector and the ship sector. You will build the two subsystems separately and observe the behavior of each subsystem. Then, you will combine the two models to form the final fishing system model.

Part 1 Fish Population

Forest Lake is a small lake nestled in the Pocono Mountains in northern Pennsylvania. Today, approximately 120,000 fish, mainly wall-eyed pike and bass, swim in the deep and shallow waters of Forest Lake. Forest Lake is a mid-sized lake, with a

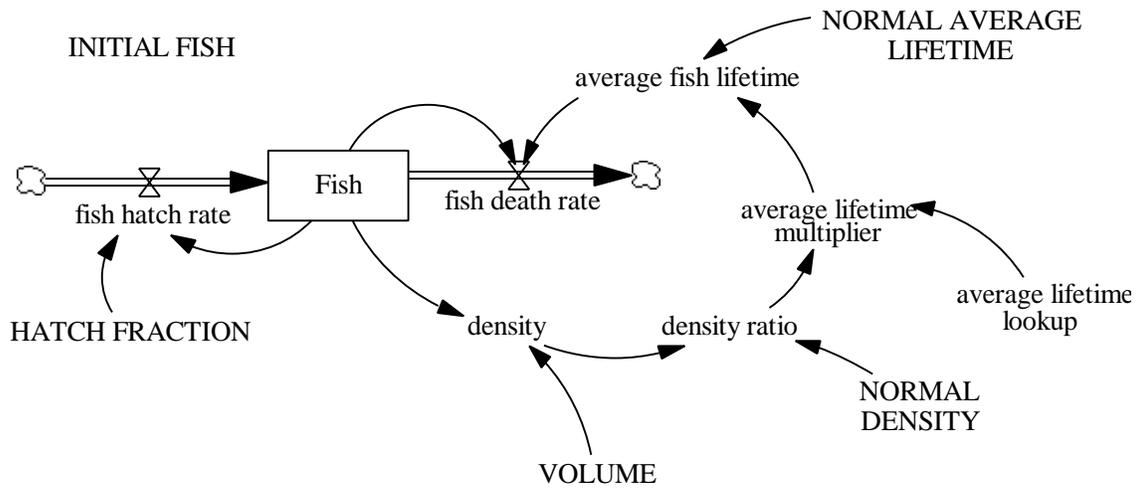
¹ Copyright © 1999 by the Massachusetts Institute of Technology. Permission granted to distribute for non-commercial educational purposes.

surface area of 20,000 square meters and an average depth of 10 meters. Although female fish lay several thousand eggs a year, on average only three of each female's eggs will hatch successfully each year. When the density of fish is at an optimum level, as it is now, the fish live eight months on average. The beautiful bass that give proud fishermen a reason to boast are typically three to four years old, but those fish are few and far between. The average lifetime of the fish is short because most fish die young; big fish devour the little ones. The average lifetime of the fish, however, varies with the crowding in the lake because of the competition for food and other nutrients. For example, if the lake were twice as dense in fish, the average lifetime of the fish would drop down to approximately four and a half months. If the lake were half as dense in fish, the average lifetime of the fish would rise to nine months.

A. From the above description, formulate a stock and flow model of the fish population. In your assignment solutions document, please include the model diagram, documented equations, and a graph of any lookup functions that you use in the model.

Hint: the model should include a constant that represents the carrying capacity of the lake.

Model diagram:



Model equations:

$$\text{average fish lifetime} = \text{NORMAL AVERAGE LIFETIME} * \text{average lifetime multiplier}$$

Units: year

The actual fish lifetime.

$$\text{average lifetime lookup} ([(0,0) - (2,2)], (0,1.2), (0.25,1.18), (0.5,1.125), (0.75,1.07), (1,1), (1.25,0.91), (1.5,0.77), (1.75,0.625), (2,0.5625))$$

Units: dimensionless

The lookup for average lifetime.

D-4752-1

average lifetime multiplier = average lifetime lookup(density ratio)

Units: dimensionless

The relationship between death fraction and density ratio.

density = Fish / VOLUME

Units: fish/cubic meter

Density is the number of fish found per cubic meter in Forest Lake.

density ratio = density / NORMAL DENSITY

Units: dimensionless

The ratio of actual density over maximum density.

Fish = INTEG (fish hatch rate – fish death rate, INITIAL FISH)

Units: fish

The number of fish in Forest Lake.

fish death rate = Fish / average fish lifetime

Units: fish/year

The number of fish that die in a year.

fish hatch rate = Fish * HATCH FRACTION

Units: fish/year

The fish hatch rate is the number of fish hatched per year. It is computed by multiplying the total fish population by a hatch fraction.

HATCH FRACTION = 1.5

Units: 1/year

Hatch fraction is the average number of offspring per year per fish. It is the fractional increase in the population per year. A hatch fraction of 1.5 per year means that every female fish (half the fish population) will have an average of 3 offspring per year that hatch successfully.

INITIAL FISH = 120000

Units: fish

The initial number of fish in Forest Lake.

NORMAL AVERAGE LIFETIME = 2/3

Units: year

Fish live an average of 8 months (or 2/3 of a year) under conditions of normal density.

NORMAL DENSITY = 0.6

Units: fish/cubic meter

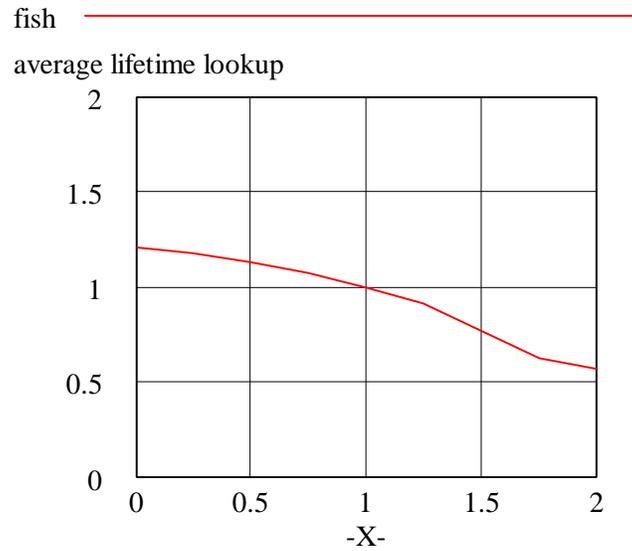
The normal density of fish in Forest Lake.

VOLUME = 200000

D-4752-1

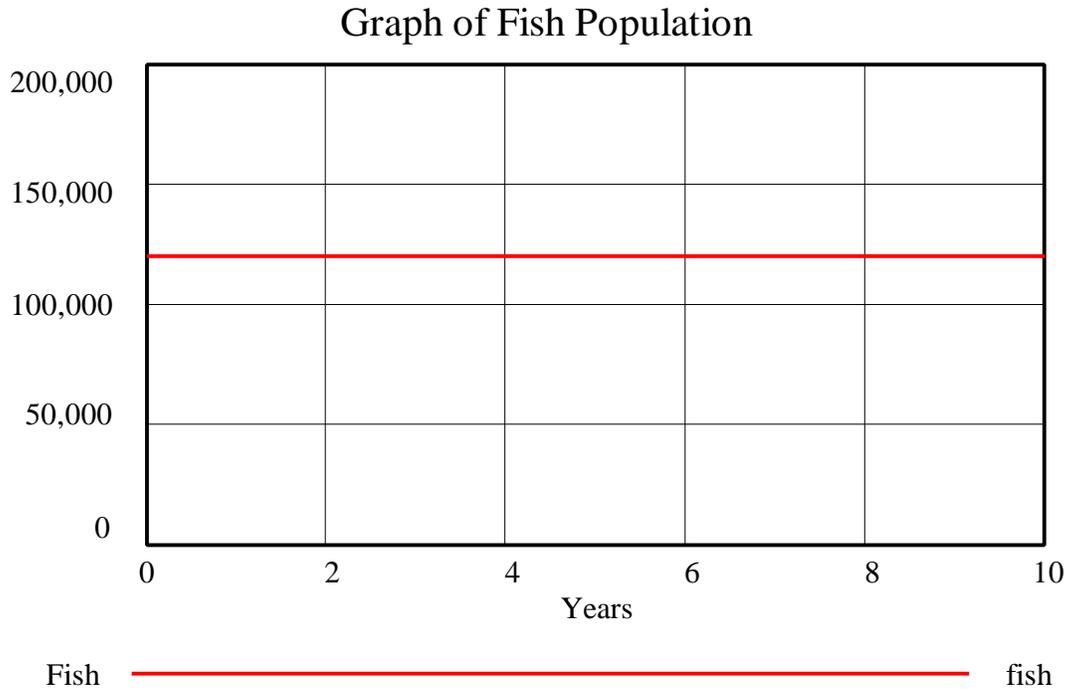
Units: cubic meter
The size of Forest Lake in cubic meters.

Graph of the lookup function:



B. Draw a reference mode for the behavior of the fish population over a period of 10 years. Then simulate the model. In your assignment solutions document, please include a graph of the behavior of the fish population. Explain.

Model behavior:



The number of fish stays constant throughout the simulation. The initial number of fish is such that the “density” equals the “NORMAL DENSITY,” so the “average fish lifetime” equals “NORMAL AVERAGE LIFETIME.” Hence, “fish death rate” equals “fish hatch rate,” and the system remains in equilibrium.

C. Imagine that some local ruffians decided to go fishing with dynamite last night. They kill half of the fish in the lake and skim a few off the surface of the water to take home. Draw a reference mode describing the behavior of the fish population for the next 10 years. Then simulate the model in this scenario. In your assignment solutions document, include a graph of the behavior of the fish population. Explain.

To represent this event, add an outflow from “Fish” called “dynamite death.” Introduce the constants “TIME OF DYNAMITE DEATH,” “FRACTION KILLED BY DYNAMITE” and use the ghost variable function to add “<TIME STEP>” to the model. The ghost variable button is next to “flow” button on the horizontal toolbar in Vensim PLE. The equation for “dynamite death” will be:

$$\text{dynamite death} = (\text{FISH} * \text{FRACTION KILLED BY DYNAMITE}) * (1 / \text{TIME STEP}) * \text{PULSE}(\text{TIME OF DYNAMITE DEATH}, \text{TIME STEP})$$

$$\text{FRACTION KILLED BY DYNAMITE} = 0.5 \text{ (dmnl)}$$

$$\text{TIME OF DYNAMITE DEATH} = 1 \text{ year}$$

This formulation is a little confusing, but important. It shows how to model instantaneous events. The equation is explained below:

(FISH * FRACTION KILLED BY DYNAMITE) shows that a certain fraction of the fish population is killed by dynamite.

When you use the PULSE function, it creates a pulse of height 1. Therefore, to create a pulse of different heights, you have to multiply the PULSE function by the desired height. The above part of the equation gives the PULSE function the desired height.

The syntax of the PULSE FUNCTION is:

PULSE (start time, duration)

(Start time and duration are expressed in the time units of the model, in this case years.)

To show an instantaneous process (that is, process of duration one time step), we use "TIME STEP" as the duration.

Suppose we set up the equation as:

dynamite death = (FISH * FRACTION KILLED BY DYNAMITE) * PULSE(TIME OF DYNAMITE DEATH, TIME STEP)

This formulation would kill 50% of the fish (depending on the fraction killed) at an annualized rate. The number of fish killed in one time step would be very small. Hence, the height of the pulse, (FISH * FRACTION KILLED BY DYNAMITE) has to be changed accordingly. This is achieved by multiplying (FISH * FRACTION KILLED BY DYNAMITE) by (1/TIME STEP). (1/TIME STEP) is generally a large number because time step is small. What is effectively done is the annualized number of fish killed is increased, so that actual number killed during one time step is exactly half of the Fish population at the time of the pulse.

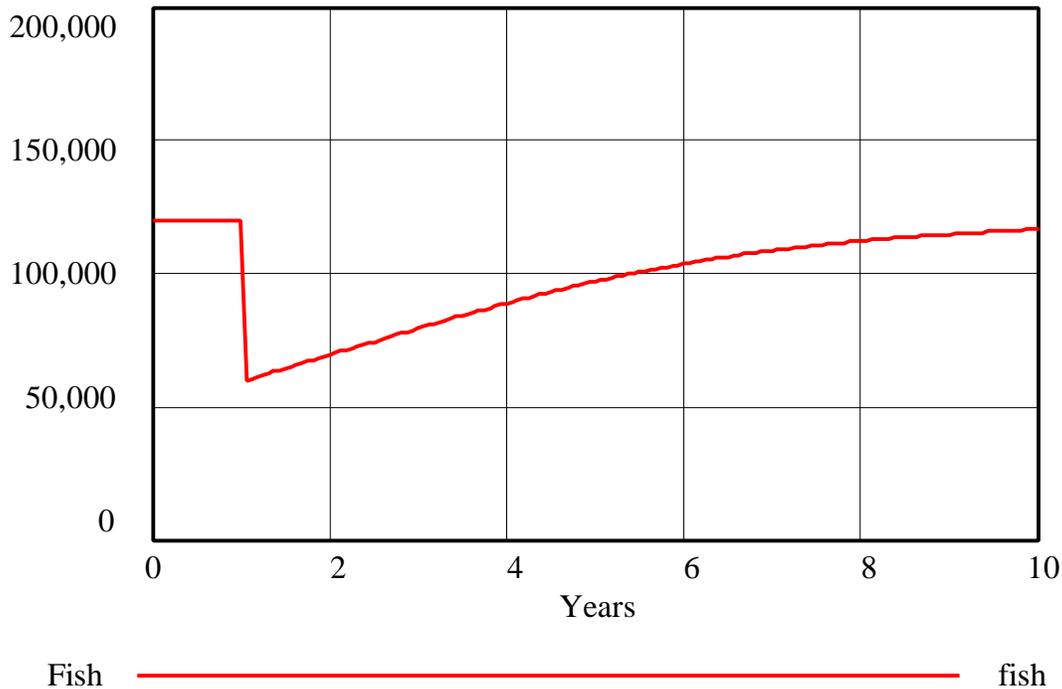
You could also simplify the equation by setting the pulse to have duration of one year. Then your equation could simply be:

dynamite death = (FISH * FRACTION KILLED BY DYNAMITE) * PULSE(TIME OF DYNAMITE DEATH, 1)

This formulation would kill the desired fraction of fish over a period of one year. When you run the simulation, however, you would notice that the number of fish does not drop to exactly 60,000 but to about 78,000. This is because during the one-year duration of the pulse, fish are still reproducing and dying through the usual mechanisms because the blast is not instantaneous. Simply setting the pulse duration to one time unit often is a good approximation, though you may sometimes want more exact simulations.

Model behavior:

Graph of Fish Population - Part 1 C

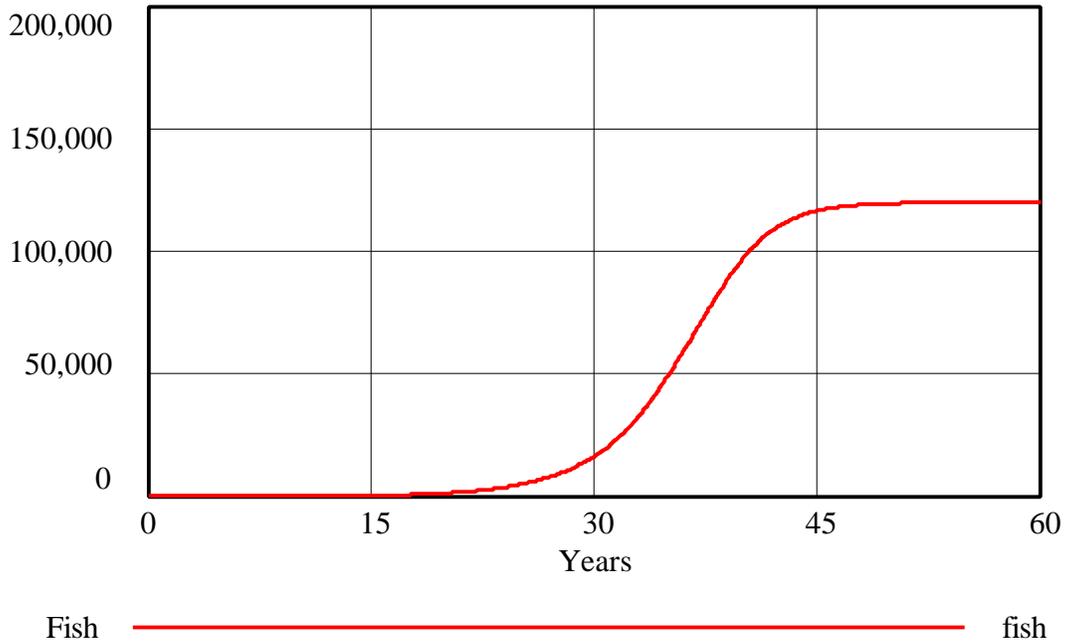


With a lower number of “Fish” after the explosion at time = 1 year, the “Fish” population grows back asymptotically to its equilibrium value. As the population of “Fish” increases, the “density” of fish in Forest Lake increases. Hence, crowding increases and the “average fish lifetime” decreases, so the population stabilizes at the same equilibrium value as before.

D. Imagine Woodlock Pond, a body of water almost exactly identical to Forest Lake, just a few miles away. The fish in Woodlock Pond, however, died out many years ago. A child and his father catch ten small fish in a plastic bag filled with water, drive over to Woodlock Pond, and drop the fish into the pond. Draw a reference mode describing the behavior of the fish population in Woodlock Pond. Simulate the model. In your assignment solutions document, include a graph of the behavior of the fish population. Explain how and why the behavior that you observe differs from the behavior that you observed in the previous scenario in part C. Extend the time horizon as long as necessary to observe the entire significant behavior.

Model behavior:

Graph of Fish Population - Part 1 D



In the scenario in part C, even though the fish population is drastically depleted, the fish population is still dense enough that it feels the effect of crowding as a limit to growth, so growth is asymptotic throughout the simulation. In the current scenario, however, the fish population is initially so sparse that crowding has no effect and the population begins to grow exponentially. As the population grows, the higher density of fish in the lake begins to act as a limit to growth by reducing the average lifetime. The population then begins to grow asymptotically, generating an overall S-shaped growth pattern. Note that the growth of the fish population after approximately year 35 (when the population reaches 60,000 fish) is identical to the growth of the fish population in the scenario from part C.

Part 2 Catching fish

Four families live in the woods surrounding Forest Lake. Currently, each family has one fishing boat that they take out onto the water every day in search of fish. The families are able fishermen, and the waters are densely populated with fish, so every day each boat brings back twenty fish. The families realize, however, that if the waters were less dense in fish their daily catch would suffer. In fact, if the waters were three quarters as dense in fish, each boat would only be able to catch nineteen fish a day. If the waters were half as dense in fish, each boat would only catch ten fish a day. If the waters were only one fourth as dense in fish as they are today, the families would only catch one fish a day.

The relationship between fish density ratio and catch per boat.

Fish = INTEG (fish hatch rate – fish death rate – total catch per year, INITIAL FISH)

Units: fish

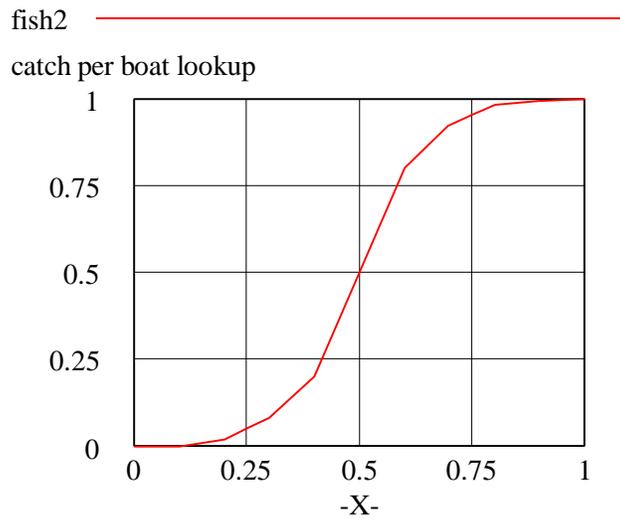
The number of fish in Forest Lake.

total catch per year = catch per boat * BOATS

Units: fish/year

The number of fish caught by the families over the course of a year.

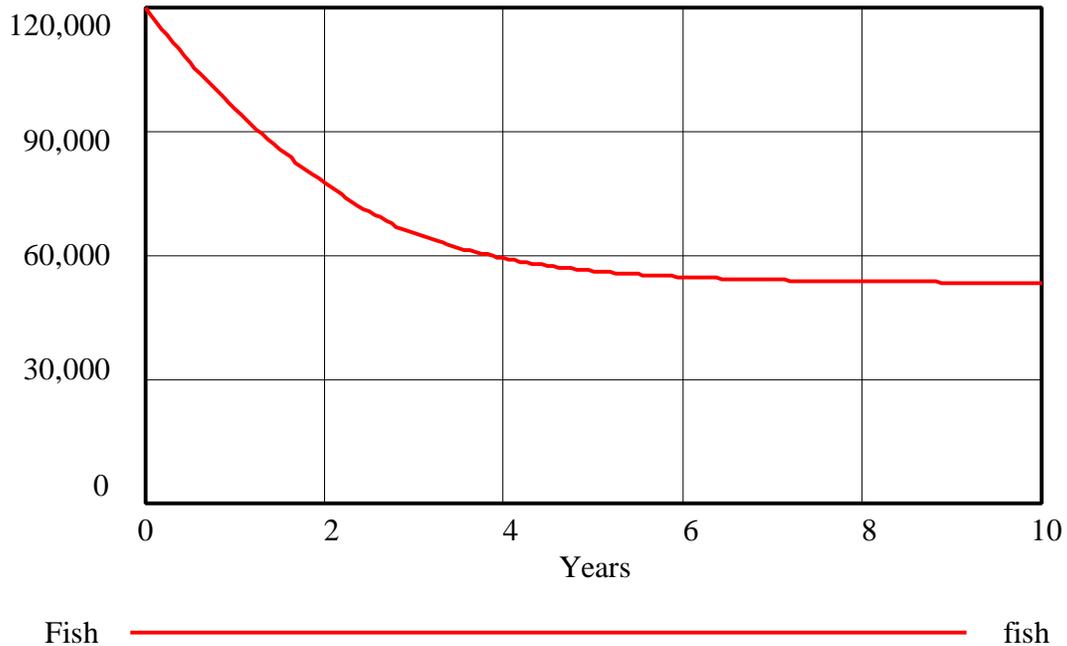
Graph of the lookup function:



B. Draw a reference mode for the behavior of the fish population over a period of 10 years. Simulate the model. In your assignment solutions document, include a graph of the behavior of the fish population. What is the new equilibrium level of the fish population? Why?

Model behavior:

Graph of Fish Population - Part 2 B

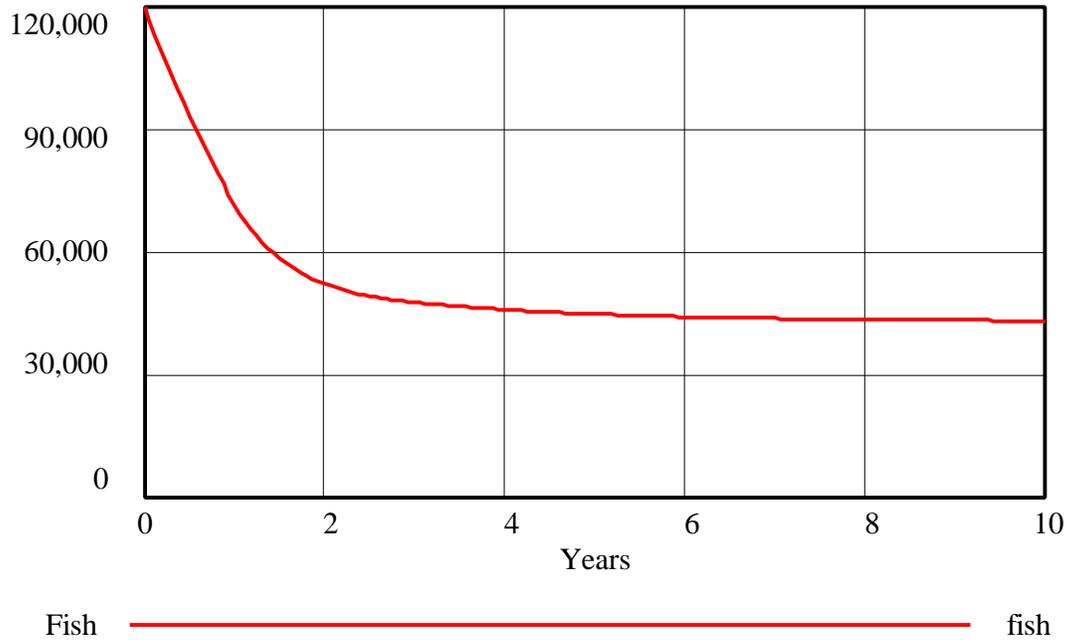


The new equilibrium of the fish population is around 53,000 fish. As long as there are more than 12,000 fish in the lake (that is, if the fish density is higher than one-tenth of the normal density), fish will be harvested from the lake. The additional outflow of fish harvesting lowers the final equilibrium value of the fish population.

C. Imagine that the families hear about a new net that can double the daily catch of any boat. The families immediately buy these new nets. Draw a reference mode for the behavior of the fish population over a period of 10 years. Simulate the model. In your assignment solutions document, include a graph of the behavior of the fish population. What is the new equilibrium level of the fish population? Why?

Model behavior:

Graph of Fish Population - Part 2 C

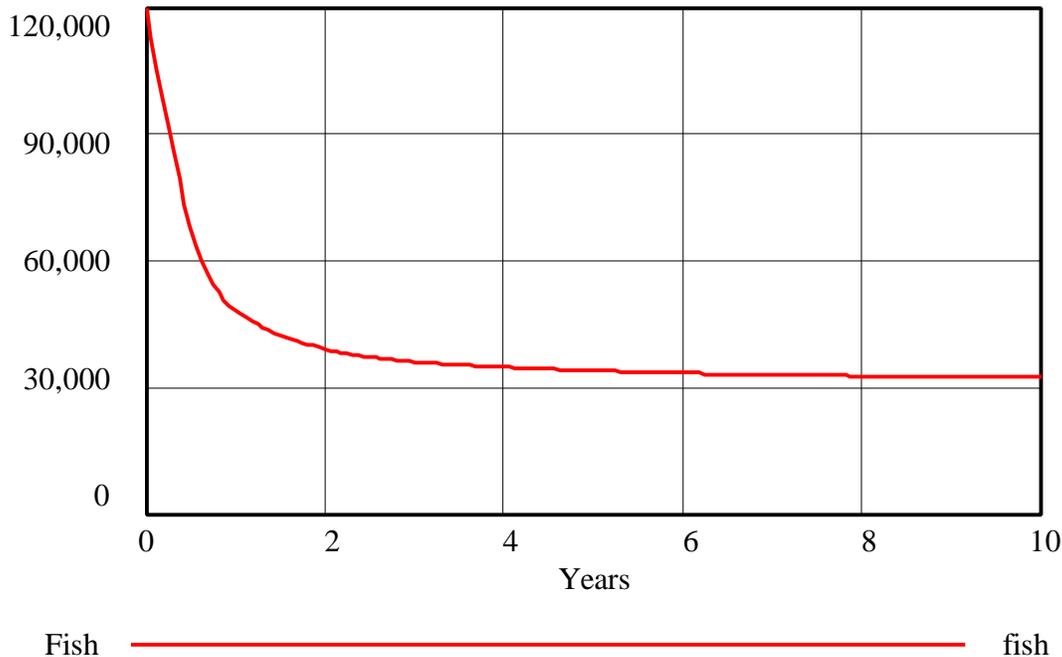


The new equilibrium of the fish population is around 43,000 fish. The new nets enable the fishermen to twice as much fish at each fish density. Strengthening the fish harvesting outflow decreases the equilibrium value of the fish population.

D. For some unexplainable reason, fish steaks become the latest craze in Harrisburg, PA. The price of fish rises. The families begin to make quite a profit selling their fish at the local market. They invest their profits in the acquisition of extra fishing boats. Now each family has new nets and two boats. Draw a reference mode for the behavior of the fish population over a period of 10 years. Simulate the model. In your assignment solutions document, include a graph of the behavior of the fish population. What is the new equilibrium level of the fish population? Why?

Model behavior:

Graph of Fish Population - Part 2 D



The new equilibrium of the fish population is around 32,000 fish. The additional ships and new nets further double the total catch per year at each fish density. Strengthening the outflow decreases the equilibrium value of the fish population.

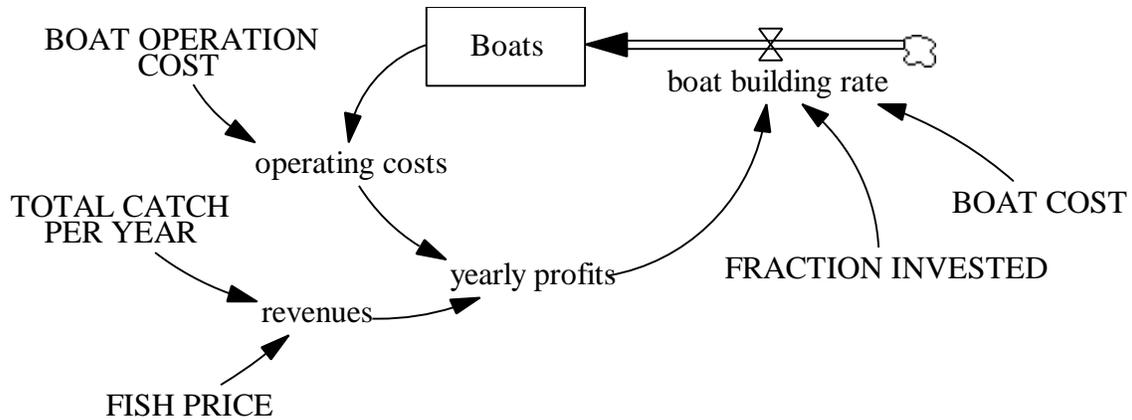
Part 3 Building boats

What determines how many fishing boats each family has? Each family starts out with one boat. The boats still use the old nets. Each family has decided to invest 50% of their yearly profits from fishing into building new boats. The family determines its yearly profits by subtracting operating costs from revenues. Each boat costs \$10,000 a year to operate. A fish sells on the local markets for about \$5. A new boat costs \$20,000.

A. From the above description, formulate a stock and flow model of the total number of boats around Forest Lake. In your assignment solutions document, include the model diagram and documented equations.

Hint: Assume that the total catch per year is a constant $20 \text{ fish/boat/day} * 4 \text{ boats} * 365 \text{ days/year} = 29,200 \text{ fish/year}$. We recommend that you build this model in a separate Vensim PLE document from the previous sector.

Model diagram:



Model equations:

$$\text{boat building rate} = \text{FRACTION INVESTED} * \text{yearly profits} / \text{BOAT COST}$$

Units: boat/year

The boat building rate is the number of new boats that are built each year. It is determined by the amount of money invested each year in new boats divided by the boat cost.

$$\text{BOAT COST} = 20000$$

Units: dollars/boat

The boat cost is the amount of money it costs to build a new boat.

$$\text{BOAT OPERATION COST} = 10000$$

Units: dollars/boat/year

The amount of money it costs to operate a boat each year.

$$\text{Boats} = \text{INTEG}(\text{boat building rate}, 4)$$

Units: boat

The number of boats around Forest Lake.

$$\text{FISH PRICE} = 5$$

Units: dollars/fish

Fish price is the amount of money a fisherman can sell a fish for as soon as it is caught.

$$\text{FRACTION INVESTED} = 0.5$$

Units: dimensionless

The fraction invested is the fraction of the profits each year that are reinvested in building new boats.

$$\text{operating costs} = \text{Boats} * \text{BOAT OPERATION COST}$$

Units: dollars/year

The total operation cost is the number of boats times the operating cost per boat each year.

revenues = FISH PRICE * TOTAL CATCH PER YEAR

Units: dollars/year

Revenues are the product of fish caught per year times the price of fish.

TOTAL CATCH PER YEAR = 29200

Units: fish/year

The total number of fish caught per year.

yearly profits = revenues – operating costs

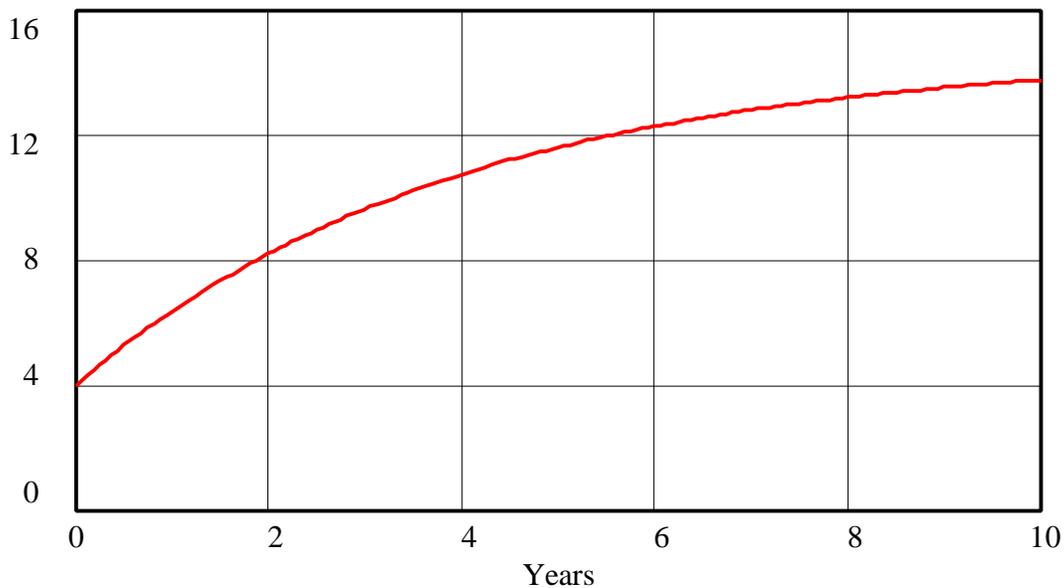
Units: dollars/year

Profits each year are the difference between revenues and costs.

B. Draw a reference mode for the behavior of the total number of boats on Forest Lake over a period of 10 years. Simulate the model. In your assignment solutions document, include a graph of the behavior of the total number of boats. At the given conditions, how many boats will fish in Forest Lake in ten years?

Model behavior:

Total Number of Boats on Forest Lake - Part 3 B



Boats : fish3  boat

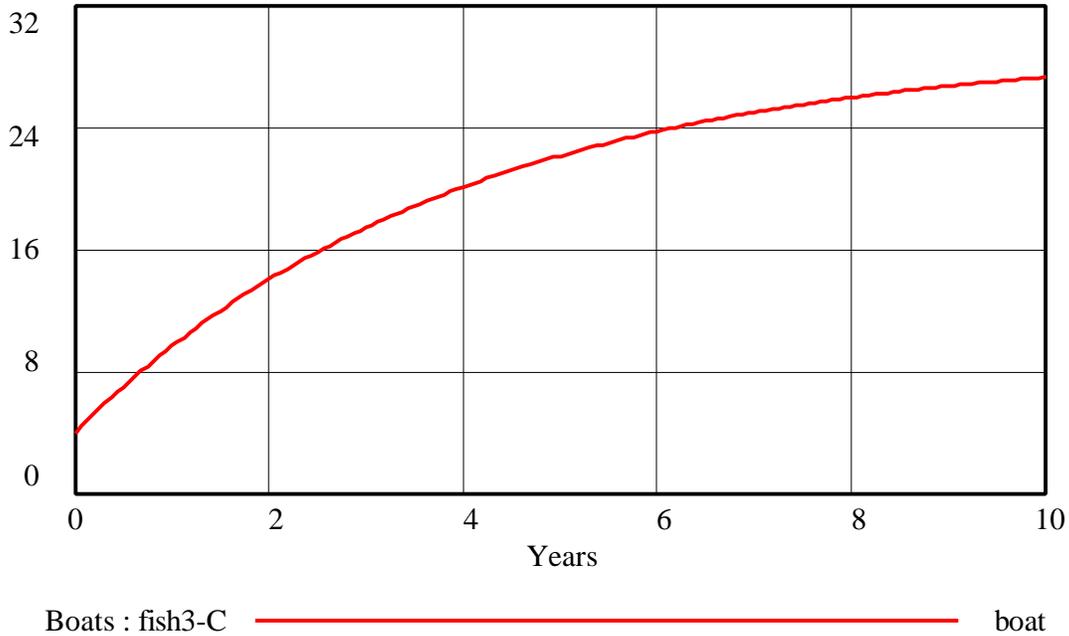
In ten years, approximately 14 boats will fish in Forest Lake.

C. With the special nets, the total catch per year doubles. Draw a reference mode for the behavior of the total number of boats on Forest Lake after each family purchases the

new nets. Simulate the model. In your assignment solutions document, include a graph of the behavior of the total number of boats. At the given conditions, how many boats will fish in Forest Lake in ten years?

Model behavior:

Total Number of Boats on Forest Lake - Part 3 C



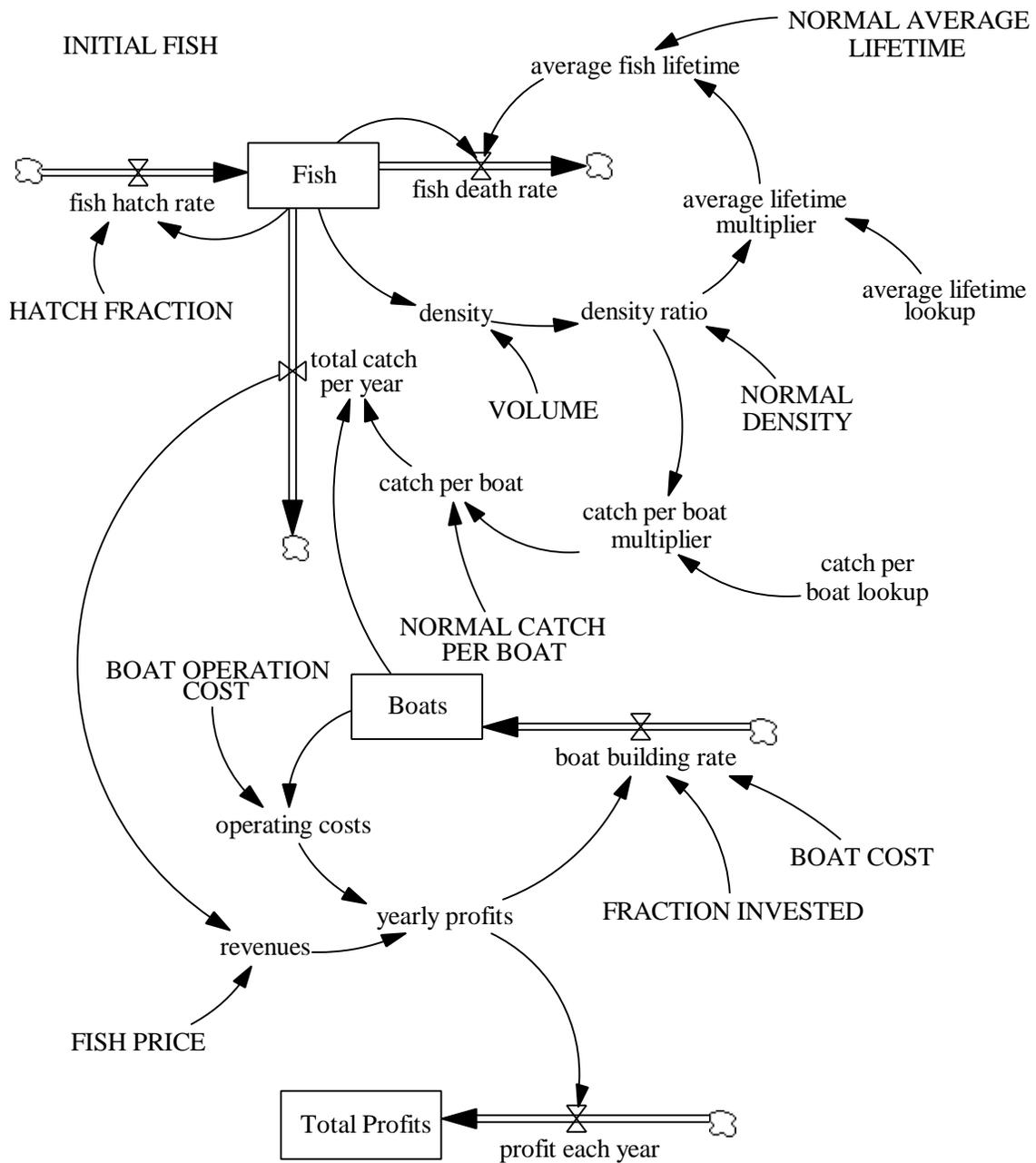
With the new nets the families catch more fish, sell more fish, and therefore make more profits that they can, in turn, reinvest in building more boats. In this scenario, approximately 28 boats fish in Forest Lake in ten years.

Part 4 Bringing it all together

Now we are ready to bring together both sectors of the model.

A. Copy the model from part C and paste it into the model from part B. Use connectors to link the variables that represent the total number of boats on the lake and the total catch per year of fish between the two models. In addition, create a stock that keeps track of the total profits of the families by summing up their yearly profits. In your assignment solutions document, include the complete model diagram and documented equations. Identify all feedback loops in the model.

Model diagram:



Model equations:

average fish lifetime = NORMAL AVERAGE LIFETIME * average lifetime multiplier
 Units: year
 The actual fish lifetime.

average lifetime lookup ((0,0) - (2,2)], (0,1.2), (0.25,1.18), (0.5,1.125), (0.75,1.07), (1,1), (1.25,0.91), (1.5,0.77), (1.75,0.625), (2,0.5625))
 Units: dimensionless

The lookup for average lifetime.

average lifetime multiplier = average lifetime lookup(density ratio)

Units: dimensionless

This is the nonlinear dimensionless relationship between death fraction and density ratio.

boat building rate = FRACTION INVESTED * yearly profits / BOAT COST

Units: boat/year

The boat building rate is the number of new boats that are built each year. It is determined by the amount of money invested each year in new boats divided by the boat cost.

BOAT COST = 20000

Units: dollars/boat

The boat cost is the amount of money it costs to build a new boat.

BOAT OPERATION COST = 10000

Units: dollars/boat/year

The amount of money it costs to operate a boat each year.

Boats = INTEG (boat building rate, 4)

Units: boat

The number of boats around Forest Lake.

catch per boat = NORMAL CATCH PER BOAT * catch per boat multiplier

Units: fish/boat/year

The number of fish a boat catches per year.

catch per boat lookup([(0,0) - (1,1)], (0,0), (0.1,0), (0.2,0.02), (0.25,0.05), (0.3,0.08), (0.4,0.2), (0.5,0.5), (0.6,0.8), (0.7,0.92), (0.75,0.95), (0.8,0.98), (0.9,0.99), (1,1))

Units: dimensionless

The lookup function for catch per boat.

catch per boat multiplier = catch per boat lookup(density ratio)

Units: dimensionless

The relationship between fish density ratio and catch per boat.

density = Fish / VOLUME

Units: fish/cubic meter

Density is the number of fish found per cubic meter in Forest Lake.

density ratio = density / NORMAL DENSITY

Units: dimensionless

The ratio of actual density over maximum density.

D-4752-1

Fish = INTEG (fish hatch rate – fish death rate – total catch per year, INITIAL FISH)

Units: fish

The number of fish in Forest Lake.

fish death rate = Fish / average fish lifetime

Units: fish/year

The number of fish that die in a year.

fish hatch rate = Fish * HATCH FRACTION

Units: fish/year

The fish hatch rate is the number of fish hatched per year. It is computed by multiplying the total fish population by a hatch fraction.

FISH PRICE = 5

Units: dollars/fish

Fish price is the amount of money a fisherman can sell a fish for as soon as it is caught.

FRACTION INVESTED = 0.5

Units: dimensionless

The fraction invested is the fraction of the profits each year that are reinvested in building new boats.

HATCH FRACTION = 1.5

Units: 1/year

Hatch fraction is the average number of offspring per year per fish. It is the fractional increase in the population per year. A hatch fraction of 1.5 per year means that every female fish (half the fish population) will have an average of 3 offspring per year that hatch successfully.

INITIAL FISH = 120000

Units: fish

The initial number of fish in Forest Lake.

NORMAL AVERAGE LIFETIME = 2/3

Units: year

Fish live an average of 8 months (or 2/3 of a year) under conditions of normal density.

NORMAL CATCH PER BOAT = 7300

Units: fish/(year*boat)

The normal number of fish that a boat can catch and carry home in a year.

NORMAL DENSITY = 0.6

Units: fish/cubic meter

The normal density of fish in Forest Lake.

operating costs = Boats * BOAT OPERATION COST

Units: dollars/year

The total operation cost is the number of boats times the operating cost per boat each year.

profit each year = yearly profits

Units: dollars/year

The profit made each year is equal to the yearly profits.

revenues = FISH PRICE * total catch per year

Units: dollars/year

Revenues are the product of fish caught per year times dollars per fish.

total catch per year = catch per boat * Boats

Units: fish/year

The number of fish caught by the families over the course of a year.

Total Profits = INTEG (profit each year, 0)

Units: dollars

The total economic profits are the accumulation over time of the yearly profits.

VOLUME = 200000

Units: cubic meter

The volume is the size of Forest Lake in cubic meters.

yearly profits = revenues – operating costs

Units: dollars/year

Profits each year are the difference between revenues and costs.

Feedback loops in the model:

Positive feedback loops:

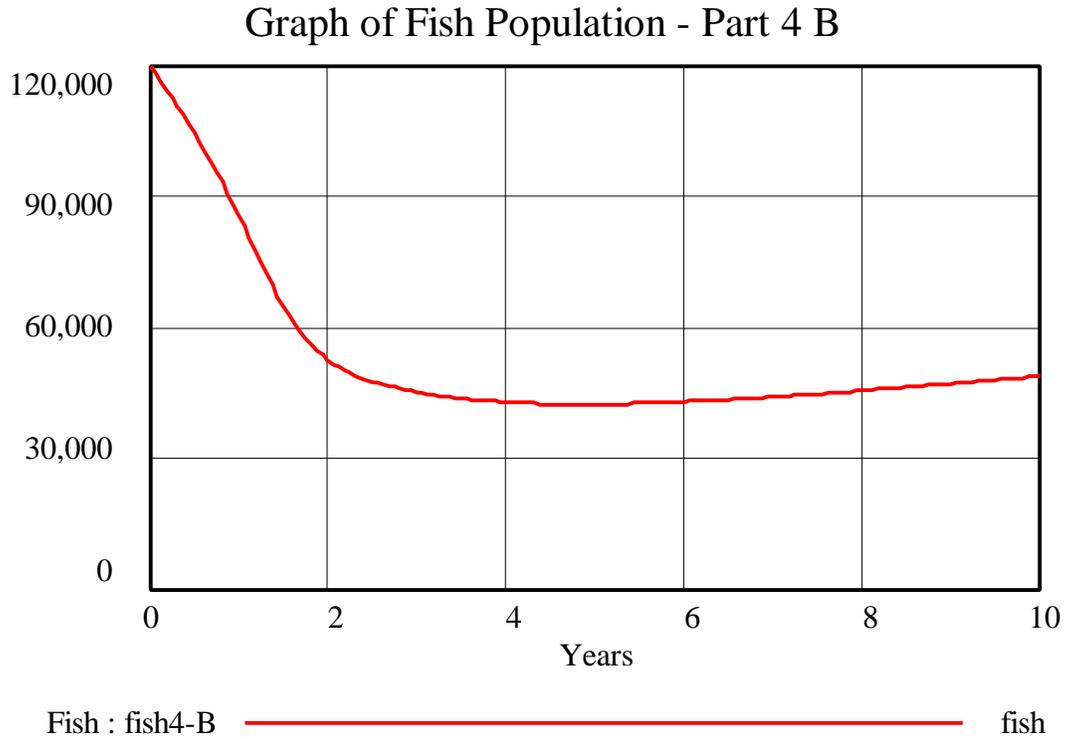
- As the number of fish increases, the number of fish hatched increases.
- As the number of boats increases, the yearly catch of fish increases, revenues increase, and the additional profits are invested in building more boats.

Negative feedback loops:

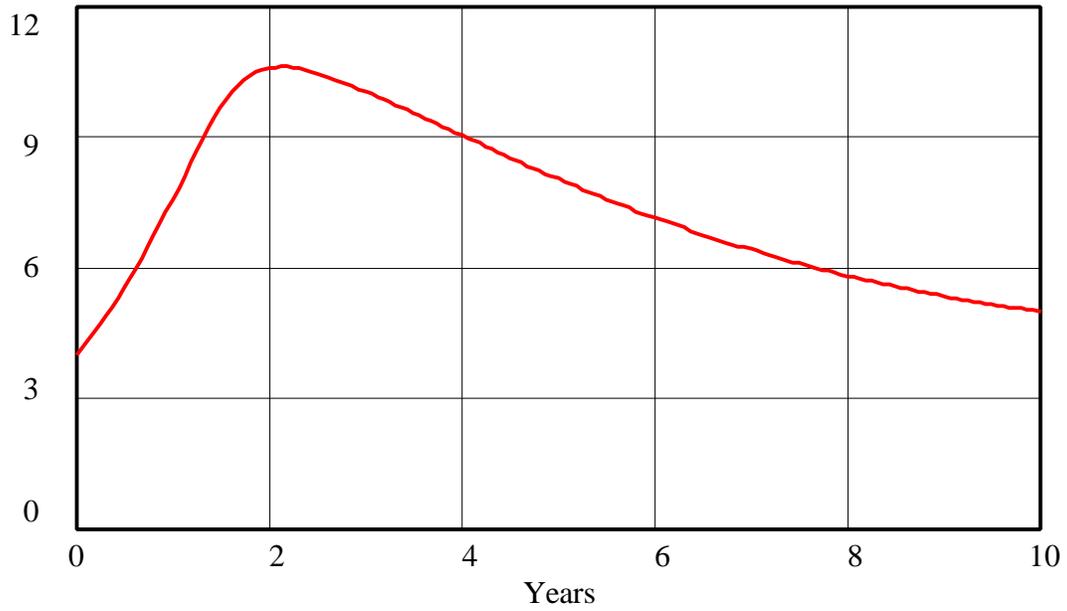
- As the number of fish increases, the number of fish deaths increases.
- As the number of fish increases, the density of fish increases, and more fish die due to crowding, reducing the number of fish.
- As the number of fish increases, the density of fish rises, and more fish are caught each year, reducing the number of fish.
- As the number of boats increases, operating costs increase, yearly profits are lower, and fewer additional boats are built.

B. Draw reference modes for the total number of boats on the lake, the fish population, the yearly profits, and the total profits. Simulate the model, assuming that the boats still use the old nets. In your assignment solutions document, include graphs of the behavior of the four above variables.

Model behavior:

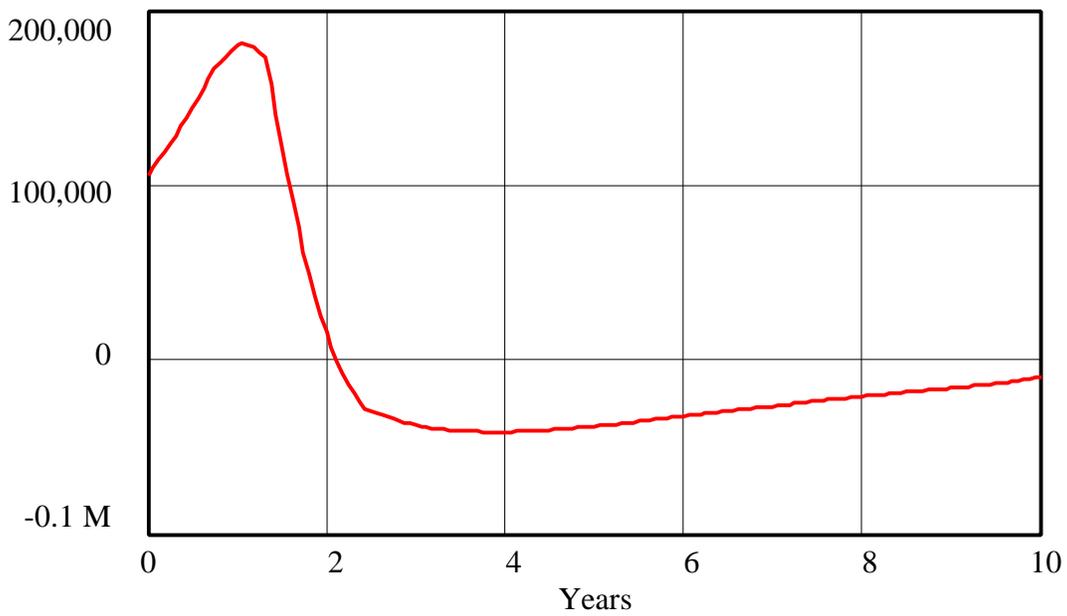


Graph of Boats - Part 4 B



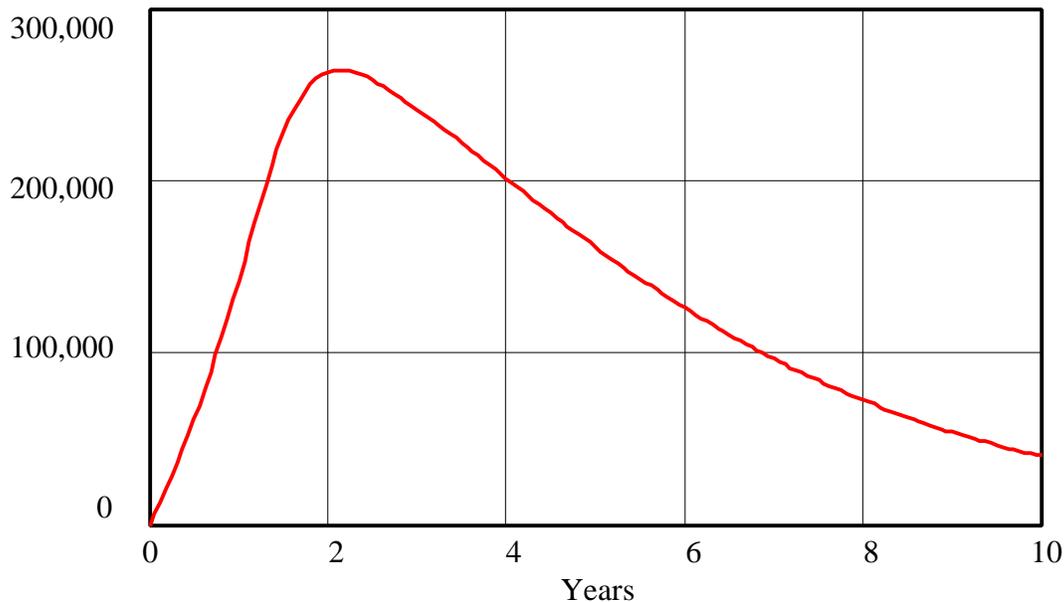
Boats : fish4-B ————— boat

Graph of yearly profits - Part 4 B



yearly profits : fish4-B ————— dollars/year

Graph of Total Profits - Part 4 B



Total Profits : fish4-B ————— dollars

C. Explain the dynamics driving the behavior of the four variables graphed above. What feedback loops dominate at different points in time?

The families begin to invest their fishing profits in the construction of new boats. The additional boats increase their yearly catch, decreasing the fish population, and providing revenues to build even more boats. The dominant feedback loop is the positive loop that encourages investment in more boats. The families continue to happily build more boats until the population of fish drops enough that density of fish becomes very small. The yearly catch of fish drops dramatically because fish are now difficult to find. The families, however, are still paying for upkeep on their large fleet of boats, even though those boats can no longer find many fish. Yearly profits plunge. Now the dominant feedback loops are the negative loops that restrict the harvesting of fish and the construction of new boats. As fishing tapers off, the population of fish stabilizes (and then slowly begins to grow a little) at a much lower equilibrium value.

D. Refer back to the appendix of Systems 1 by Draper Kauffman². What system note(s) does this model illustrate?

This model illustrates the behavior of the Tragedy of the Commons. The Tragedy of the Commons occurs when several individuals, each driven by personal gain, share a limited resource. Each family wants to maximize its individual profits. Yet, each family fails to

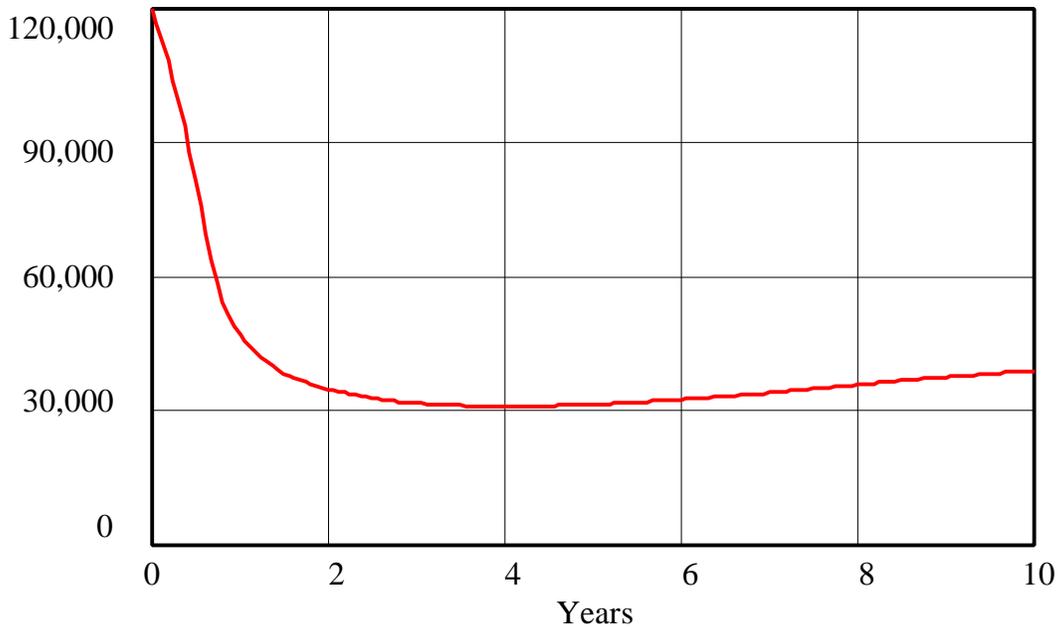
² Kauffman, Draper L. J., 1980. *Systems 1: An Introduction to Systems Thinking*.

take into account the actions of the other families. The resource is significantly depleted, to the detriment of all of the families involved.

E. Consider the scenario in which each family uses new nets that double the daily catch of fish. Draw reference modes for the total number of boats on the lake, the fish population, the yearly profits, and the total profits. Simulate the model. In your assignment solutions document, include graphs of the behavior of the four above variables. Use your understanding of the dynamics of the fishing system to explain how and why the behavior generated under this scenario differs from the behavior in part B.

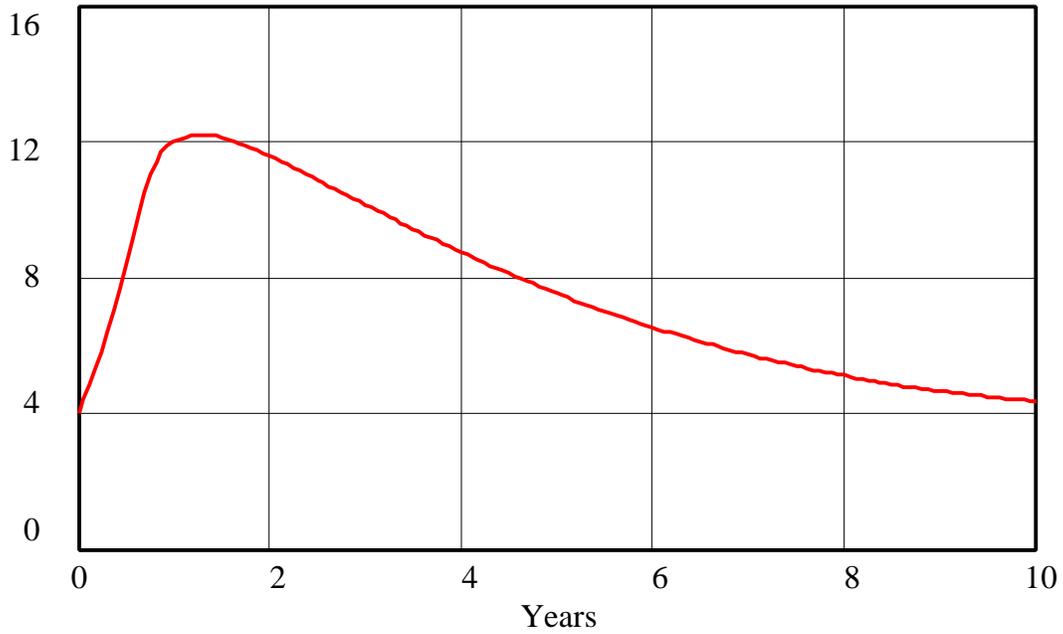
Model behavior:

Graph of Fish Population - Part 4 E



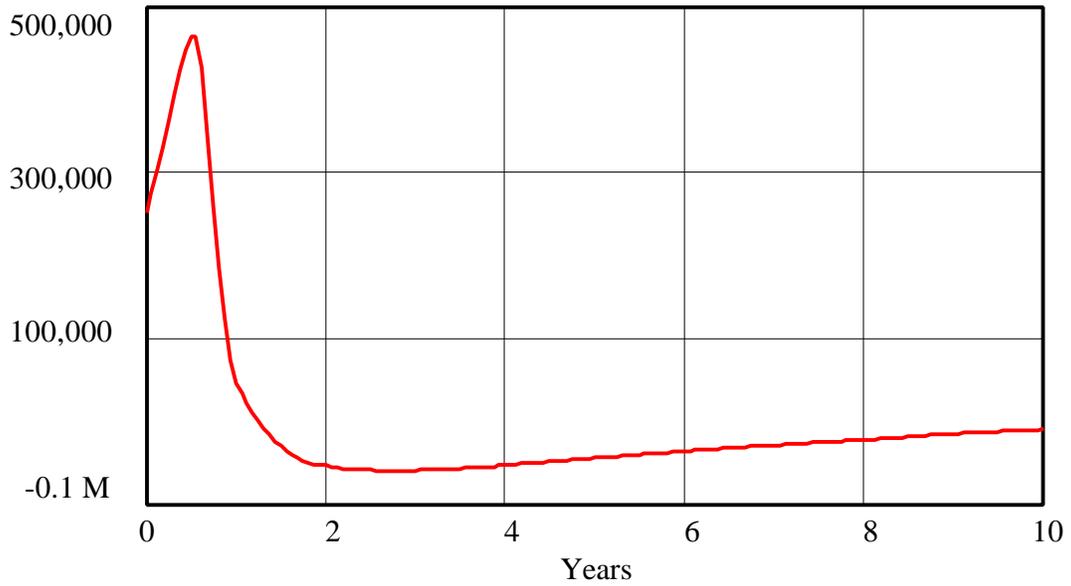
Fish : fish4-E ————— fish

Graph of Boats - Part 4 E



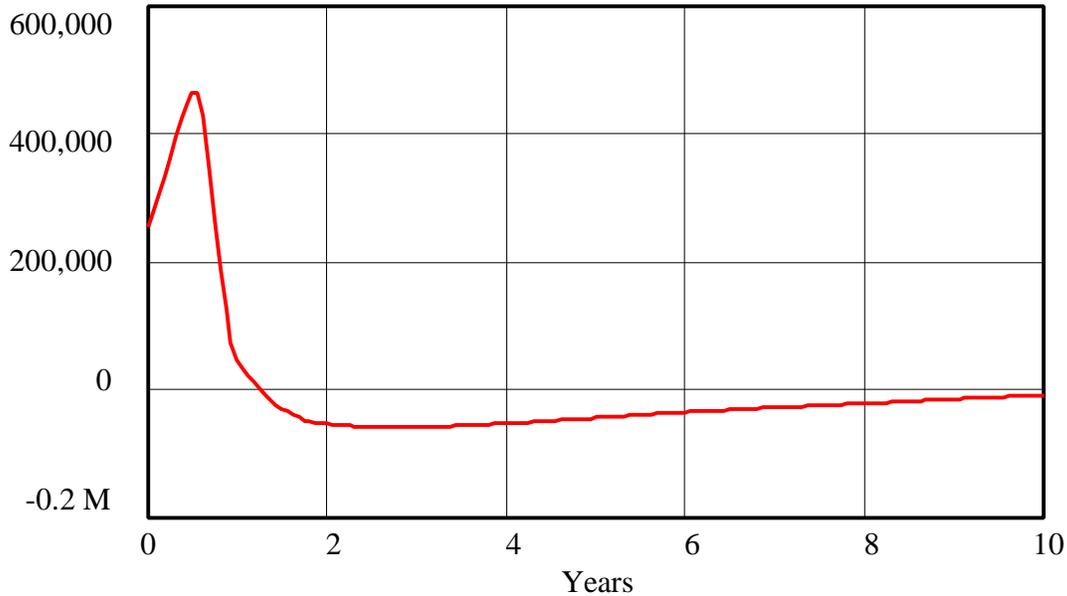
Boats : fish4-E ————— boat

Graph of yearly profits- Part 4 E



yearly profits : fish4-E ————— dollars/year

Graph of yearly profits - Part 4 E



yearly profits : fish4-E ————— dollars/year

The scenario with new nets precipitates the decline of the fish population. The new nets increase the yearly catch of the families, decreasing the fish population. In addition, the increased catch leads to increased revenues and investment in additional boats, which further reduces the fish population. The new nets, instead of being beneficial to the families, only worsen the crisis of the tragedy of the commons.

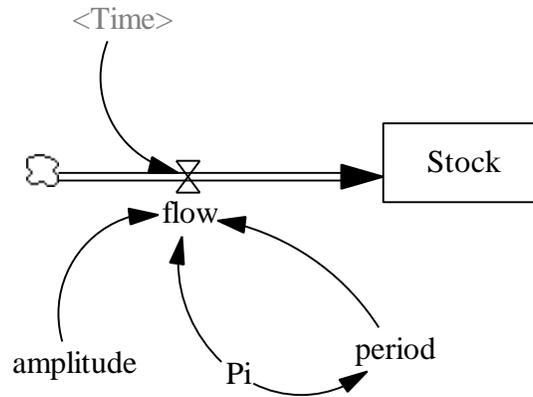
F. Propose a method to optimize the total profits of the families. How would you change the model to incorporate this policy design? (You do not need to change the model, just explain in terms of changes in parameters and additional structures how you would approach the problem. Of course, if you would like to model your policy suggestions and send us documented equations along with the output of your model and an explanation of the behavior that you observed, please take up the challenge.)

One way to fish sustainably is to limit the number of ships on the lake. If each family agrees to only have one or two fishing boats, the population of fish will not be depleted. Keeping the number of ships constant breaks the positive feedback loop through which the families invest their fishing profits in the purchase of new ships to further increase their yearly catch of fish.

2. Understanding Oscillatory Systems

This is the first in a series of exercises designed to help your understanding of oscillatory systems. Build the following model in Vensim PLE and then complete the exercises.

Make sure that the time step is smaller than 0.0625 (or small enough that the value of DT no longer has a substantial effect on the result of the simulation). The time horizon over which you simulate the model should be at least 16 units of time.



$$\text{amplitude} = 1$$

$$\text{flow} = \text{amplitude} * \text{SIN}(2 * \text{Pi} * \text{Time} / \text{period})$$

$$\text{period} = 2 * \text{Pi}$$

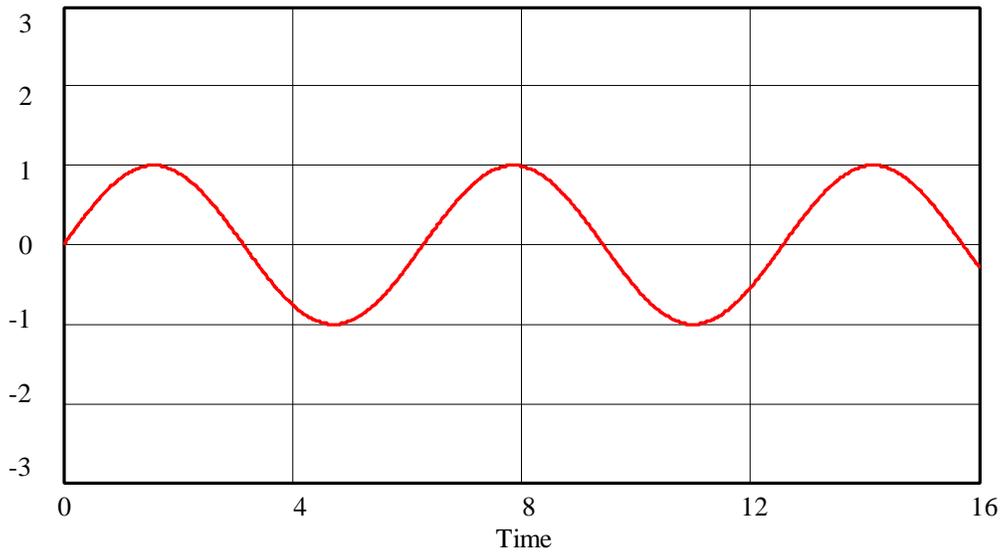
$$\text{Pi} = 3.14159$$

$$\text{Stock} = \text{INTEG}(\text{flow}, -1)$$

$$\text{TIME STEP} = 0.0078125$$

A. Graphically integrate the following flow. You do not have to submit your graph, but make sure it is correct.

Graph of flow

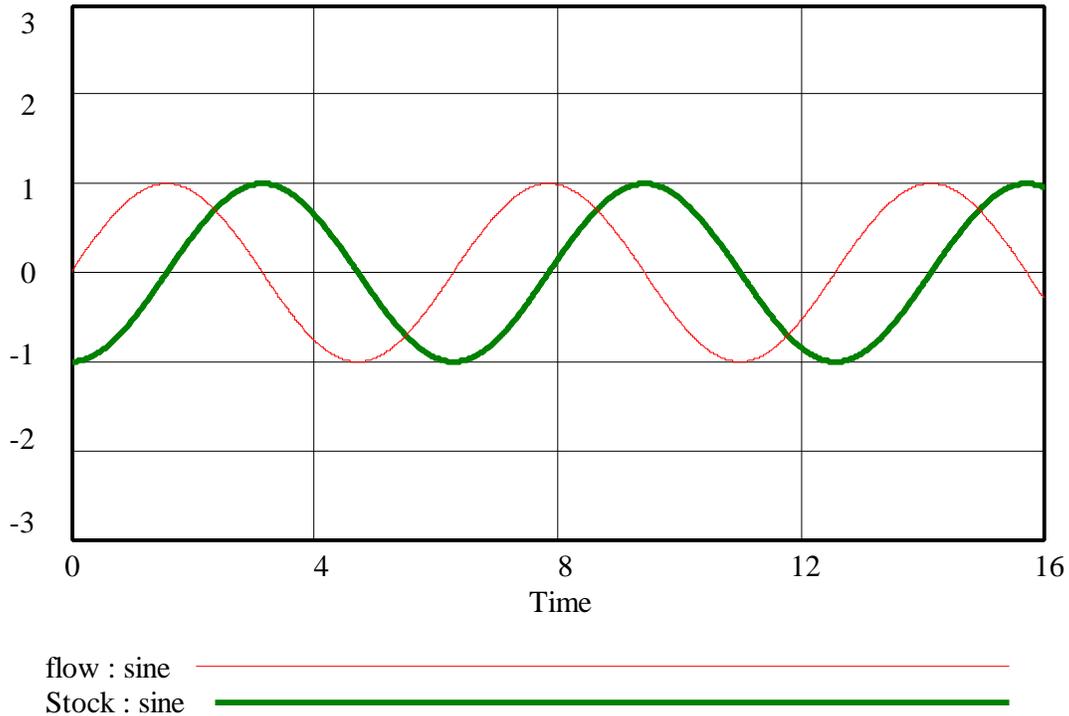


flow : sine



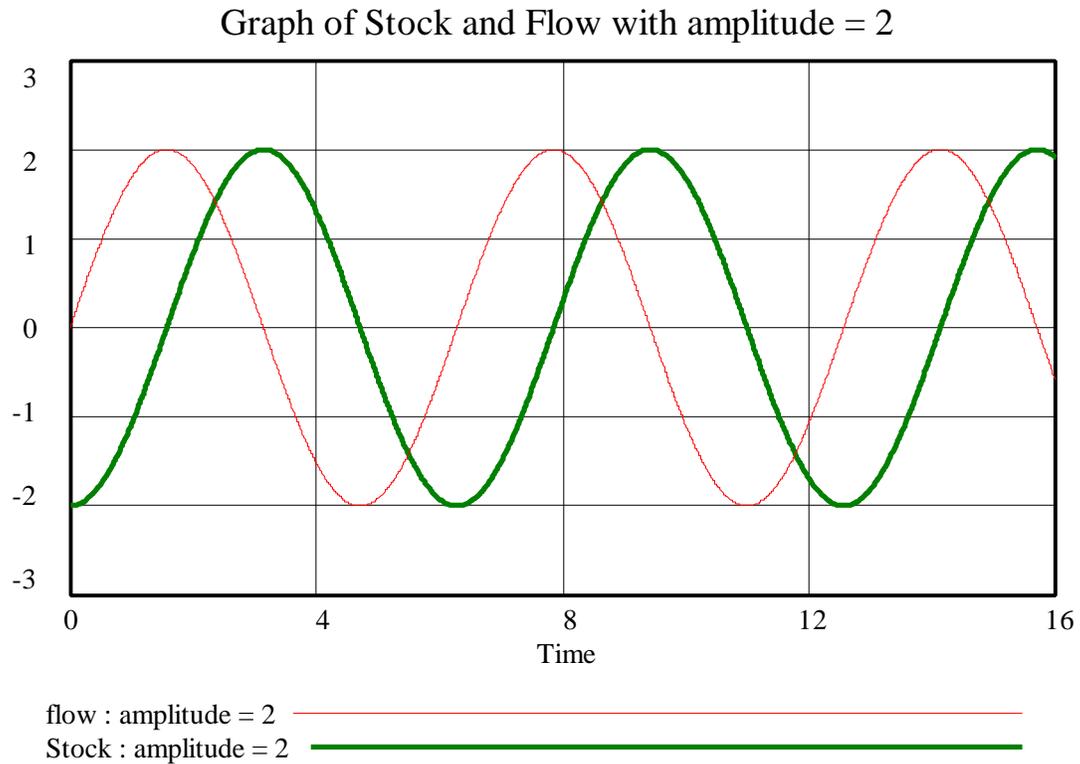
B. Simulate the model to confirm your graphical integration. In your assignment solutions document, include graphs of the behavior of the “flow” and of the “Stock.” Describe the relationship between the two graphs.

Graph of Stock and Flow



The graphs “flow” and “Stock” show oscillating behavior with the same period and the same amplitude. The “Stock,” however, lags one quarter of the period behind the “flow.” That is, the behavior of the “Stock” is the same as the “flow” behavior, but shifted to the right by one quarter of the period. Notice that the initial value of the “Stock” was chosen to be -1 in order to make sure the “Stock” oscillates around 0.

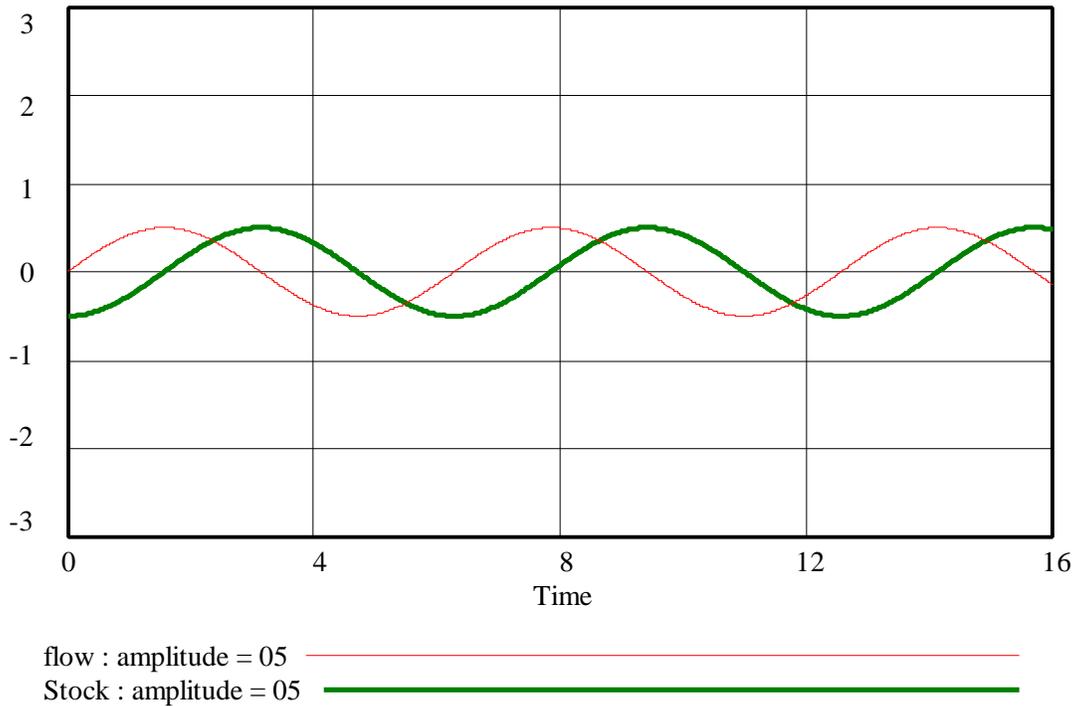
C. Create a new dataset and simulate the model with “amplitude” equal to 2. In your assignment solutions document, include graphs of the behavior of the “flow” and of the “Stock” in this simulation. You may wish to change the initial value of the “Stock.” Describe the relationship between the two graphs and compare the simulation to that from part B.



The two graphs again show oscillating behavior with the same period and the same amplitude, and the “Stock” lags one quarter of the period behind the flow. Compared to part A, the period of oscillation has not changed, but the amplitude of both “flow” and “Stock” is doubled. Notice again that we changed the initial value of the “Stock” to -2 to make the “Stock” oscillate around 0.

D. Repeat part C with “amplitude” equal to 0.5.

Graph of Stock and Flow with amplitude = 0.5



The two graphs again show oscillating behavior with the same period and the same amplitude, and the “Stock” lags one quarter of the period behind the “flow.” Compared to part A, the period of oscillation has not changed, but the amplitude of both “flow” and “Stock” is halved. Notice again that we changed the initial value of the “Stock” to -0.5 to make the “Stock” oscillate around 0.

E. What conclusions can you make about the relationship between an oscillating “flow” and its “Stock” as the amplitude of the “flow” changes?

As the amplitude of the “flow” changes, the amplitude of the “Stock” changes also. The “Stock” and the “flow” have the same amplitude of oscillation. The period of oscillation of the “flow” and of the “Stock” is not affected by changing the amplitude of the “flow.” Also, no matter what the amplitude of the “flow” is, the “Stock” always lags one quarter of the period behind the “flow.”