Guided Study Program in System Dynamics
System Dynamics in Education Project
System Dynamics Group
MIT Sloan School of Management[1]

Solutions to Assignment #7
Friday, November 13, 1998

Reading Assignment:

Please read the following:
- Collected Papers of Jay W. Forreste*r:[2]* *"Market Growth as Influenced by Capital Investment"*
- *Vensim PLE User's Guide: Section 7: Building a Function with Lookups (pp. 65 - 80)*

Please refer to Road Maps 3: A Guide to Learning System Dynamics (D-4503-4) and read the following.
- Study Notes in System Dynamics,*[3] by Michael Goodman: Section 3.12*

Exercises:

1. *Market Growth as Influenced by Capital Investment*

A. This paper by Prof. Forrester presents the "Market Growth Model" as an example of modeling a complex system. The first several sections of the paper describe the structure of systems in terms of feedback loops and levels and rates. Read these sections carefully because they contain guidelines that will be useful to you when you build your own models.

The paper then proceeds to build the market growth model. You do not need to be able to build the model on your own. You should, however, try to understand the model structure, and how the structure creates the behavior generated by the model. Pay close attention to the table functions (multipliers) in the model. In this assignment, you will in

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[2] Forrester, Jay W., 1975. *Collected Papers of Jay W. Forrester*, Waltham, MA: Pegasus Communications. 284 pp.

[3] Goodman, Michael R., 1974. *Study Notes in System Dynamics*, Waltham, MA: Pegasus Communications. 388 pp.

fact build a part of the market growth model, and you will learn how to create such table functions in Vensim PLE. In later assignments, you will eventually build the full model.

B. In the section "Capital Investment," Prof. Forrester describes the dynamics of changing the delivery delay goal of the company. Such behavior is often referred to as "eroding goals."

Please give two examples of systems in which goals are constantly readjusted to match actual performance, and describe the behavior of those systems.

Teaching can sometimes become an example of a system in which goals are constantly readjusted to match actual performance. When a teacher encounters a group of low-achievers with high failure levels, one way of helping the students is to lower the academic goals. Very often, as the pressure on them lowers, the students relax. The less the students are expected to learn, the less they exert themselves, and the less they learn, so goals are once again re-assessed and lowered even more, etc.

Project management identifies various planned activities that are to be performed in order to successfully introduce a new product on time. This goal (planned activities on the critical path) is constantly readjusted to ensure the launch date is met. Decisions are made to either shorten or eliminate planned activities, which increases the risk associated with the new product introduction.

An example of drifting goals is the system of cleaning up after our dog. The stock is the number of piles in the yard. The stock is increased by a constant rate, "deposits," and is drained by the cleanup rate. Our children take turns performing the duty, and they attempt to meet the goal of zero piles after each pile patrol. The cleanup rate is defined by the fraction picked up each week. Unfortunately for those of us who like to sit outside, the children have been allowing the goal of zero piles to drift upwards (thereby allowing something else to drift upwards, too!). The net outcome is that the number of piles in the yard has been growing.

On August 20, 1998, our newspaper (The Oregonian) printed a story on the front page about "Worsening traffic just up the road, new analysis says." The article says that with the problems of increasing traffic, population, road maintenance costs, etc. they will "redefine" what congestion is in the area. It goes on to say "… Metro and local governments have given up on the idea of trying to eliminate congestion in every part of the region." The analysis showed that reducing congestion to a level of service "D" (heavy but free-flowing with freeway speeds of 45 to 55 mph) would cost $13.5 billion over a period of 20 years. The next level of service is an "E" (cars would average 35 to 45 mph during busy travel times) with a cost of only $3.5 billion. "The new strategy assumes that people will tolerate peak-hour congestion in some areas…"

This is an interesting eroding goal. It will cost too much to have "no" congestion, so if they redefine the word, we will be able to tolerate the inevitable. The goal (old definition of congestion) will be readjusted to match what is going to happen traffic-wise in this area, and we will be "in balance" again. What a solution!

2.   Vensim PLE User's Manual: Section 7: Building a Function with Lookups

A.   *Follow the instructions to build the delivery delay model. In your assignment solutions document, please include the model diagram, documented equations, a graph of the delivery delay lookup function, and a graph of the behavior of "Backlog."[4] After changing the lookup function, submit the new lookup graph, and a graph of behavior of "orders" from the two simulations. Explain how the change in the lookup function changed the behavior of "orders."*

All answers are included in Section 7. Please note that we discourage the use of built-in functions, such as the MIN functions used in the equation for "shipments." Instead, a modeler should find better ways of achieving the same effect using simple (and usually more realistic) formulations. Lookup functions are often used in such cases. Later assignments will give you practice with this particular use of lookup functions.

B. The section "Normalizing the Lookup" on page 68 explains that the input and output values of a lookup function should be normalized.   Explain why you think that normalizing the values is important. Give separate reasons for the input and output values.

         Events are generally put into perspective and easier to understand when juxtaposed with states that come from common experience and are well understood. Often it is not a variable's actual value that matters, but the extent of the deviation or separation from a critical state that sets things into motion. For example, by itself, the "actual quality" of a product has no meaning. For the variable to make sense, you need to know the scale on which quality is measured. Giving a movie a three-star rating means nothing unless you know how many stars the critics would have given a movie that they adored, or how they rated another movie that you adored. Similarly, you want to know what the actual quality is *relative* to the expected quality, the desired quality, the minimum quality acceptable, or the best possible quality. Any of these values that set a scale to serve as a background for the variations in actual quality are called "normal" values.
         It is therefore common practice to model the input to a lookup function as a ratio of the current value of a variable to its "normal" value. Such a formulation guarantees that the input to the lookup function is dimensionless.
         Once the input is dimensionless, the output must also be dimensionless. The lookup function cannot take as its input a dimensionless number, and produce as its output a number with units. Whatever units go into the lookup function must come right back out. The dimensionless output of the lookup function then multiplies the "normal" value of the output variable, thus preserving its units. In addition, the same reasoning for using a normalized input also holds for the output.

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[4] Please refer to the document "GSP-VensimHelp.pdf" for tips on how to submit graphs of table functions.

C.   In system dynamics literature, the lookup functions are often called "table functions" or "multipliers." Why would you use a table function in a model? Describe two table functions that you would use in models of your choice. You do not need to build the models or the table functions, only describe the system and the table function in words.

Table functions are most often used to model a nonlinear relationship between two variables. Nonlinear relationships, such as the effect of delivery delays on orders, would be very difficult to model with an equation. An equation that accurately portrays the nonlinear relationship would be so complicated that very few people would be able to intuitively visualize the function that it represents. Graphical tables are easy to conceptualize, understand, and modify. Table functions allow a modeler to specify any nonlinear relationship that he believes exists between the two variables.

A common and simple example of a table function is the "effect of crowding" multiplier often used in modeling the death fraction of a rabbit population. The area in which the rabbits live can sustain a certain maximum number of rabbits. The input to the "effect of crowding" table function is then the ratio of the current rabbit population to the maximum sustainable population. When the number of rabbits is low in comparison with the maximum sustainable number, the death fraction is equal to its normal value because the rabbits are not crowded and do not need to compete for resources such as food or water. The table function then outputs the value of 1, and the actual death fraction equals the normal death fraction. As the number of rabbits starts to grow, however, crowding increases, and some of the rabbits die of starvation. When the number of rabbits reaches the maximum sustainable number, the output of the "effect of crowding" multiplier should be such that it makes the death fraction equal to the rabbits' birth fraction, thus leading to equilibrium. Hence, the table function would be a curve that increases at an increasing rate as the ratio of the rabbit population to the maximum sustainable population grows.

I have used lookup functions to describe nonlinear customer response to price increases of a planned new product. Basically, small price increases from the "expected" value don't impact customers as much as larger price increases. Similarly, price reductions have a nonlinear effect on the market. The table function was a great way to engage our marketing department in the discussion about just what they expected customer response would be to price changes for a new product.

A table function enables a nonlinear relationship between two variables. It is a means to introduce nonlinear change into the model. For example, in a model describing the impact of overtime (hours worked) on productivity and quality, the results of increased hours are nonlinear. At first, productivity increases and quality is relatively unchanged. After a certain amount of overtime, productivity begins to fall, and quality problems begin to increase. With enough overtime, productivity and quality can fall below the levels associated with a 40-hour workweek. Another example is the benefit resulting

from practicing a physical activity. If marathon runners increase the length of their runs, the impact increases their capacity and ability for some amount, but after they are running above a certain amount of miles per week, the training actually becomes detrimental. The conditioning of the runners is hurt, and injuries are more likely.

3. Study Notes in System Dynamics, Section 3.12

A. Build the basic linear pollution model, as shown in Figure 3-17 (a), and make sure the model generates the behavior shown in Figure 3-19.

Answers are in the book.

B. After reading the description of the nonlinear pollution model, build the complete model, as shown in Figure 3-21 (a). Notice, however, that the nonlinear table function for "pollution absorption time" shown in Figure 3-20, does not have a normalized output.[5] Following the instructions from section 7 of the Vensim PLE User's Manual, make sure that the lookup function in your model has normalized input and output values. To do this, you will need to define several additional variables, not shown in the model in Figure 3-21 (a). The "effect of pollution ratio on absorption time lookup" should look the same as the table function in Figure 3-20, and should be dimensionless. This variable corresponds to the "effect of delivery delay on orders lookup" in the delivery delay model. To help, the equations for the additional variables are given below:
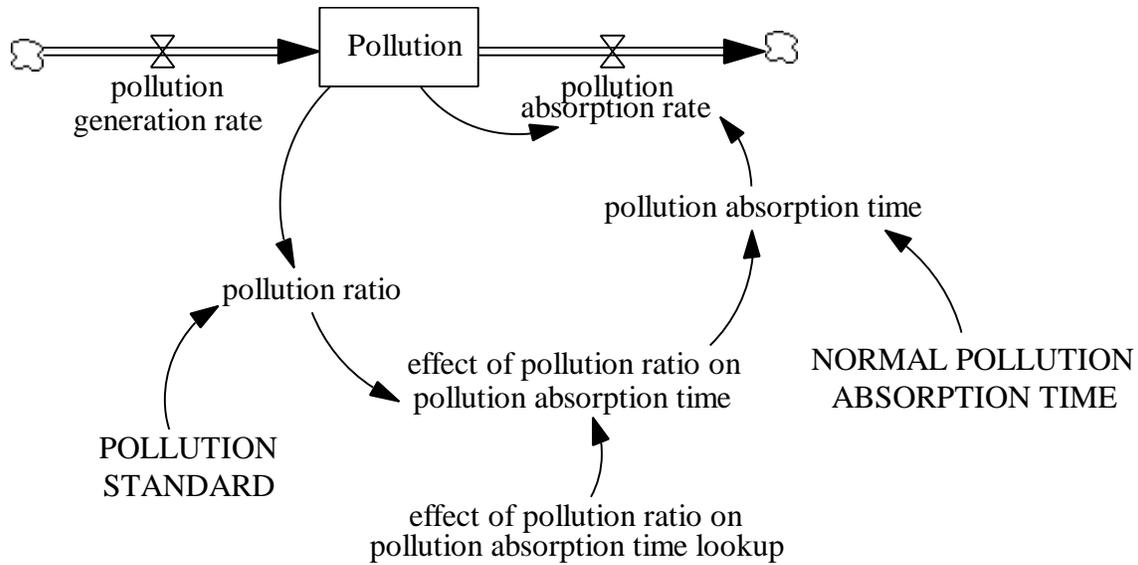
- *effect of pollution ratio on absorption time = effect of pollution ratio on absorption time lookup (pollution ratio)*
  Units: dmnl
  This variable corresponds to the "effect of delivery delay on orders" in the delivery delay model.
- *pollution absorption time = NORMAL POLLUTION ABSORPTION TIME * effect of pollution ratio on absorption time*
  Units: year
- *NORMAL POLLUTION ABSORPTION TIME = 1*
  Units: year

With the above equations, the model should be dimensionally consistent, and the lookup function has normalized input and output values.

In your assignment solutions document, please include the model diagram, documented equations, a graph of the lookup function, and graphs of the behavior of the "normal system" and the "overstressed system."

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[5] Also note that the input to the table function in Figure 3-20 should be "POLR pollution ratio," not "POLR pollution rate."

Model equations:

effect of pollution ratio on pollution absorption time = effect of pollution ratio on
    pollution absorption time lookup(pollution ratio)
        Units: dimensionless
        The output of the effect of pollution ratio on pollution absorption time lookup
        function.

effect of pollution ratio on pollution absorption time lookup ([(0,0)-(80,50)], (0,0.6),
    (1,1), (10,2.5), (20,5), (30,8), (40,11.5), (50,15.5), (60,20), (70,31), (80,50))
        Units: dimensionless
        The lookup function for the effect of pollution ratio on pollution absorption time.

NORMAL POLLUTION ABSORPTION TIME = 1
        Units: Year
        The average number of years to absorb a ton of pollutants, when the amount of
        pollution equals the standard amount of pollution.

Pollution = INTEG (+ pollution generation rate - pollution absorption rate, 0)
        Units: tons
        The amount of pollution in the air.

pollution absorption rate = Pollution / pollution absorption time
        Units: tons/Year
        The number of tons of pollutants absorbed every year.

pollution absorption time = NORMAL POLLUTION ABSORPTION TIME * effect of
    pollution ratio on pollution absorption time
        Units: Year
        The average number of years to absorb a ton of pollutants.

pollution generation rate = 2
   Units: tons/Year
   The number of tons of pollutants generated every year.

pollution ratio = Pollution / POLLUTION STANDARD
   Units: dimensionless
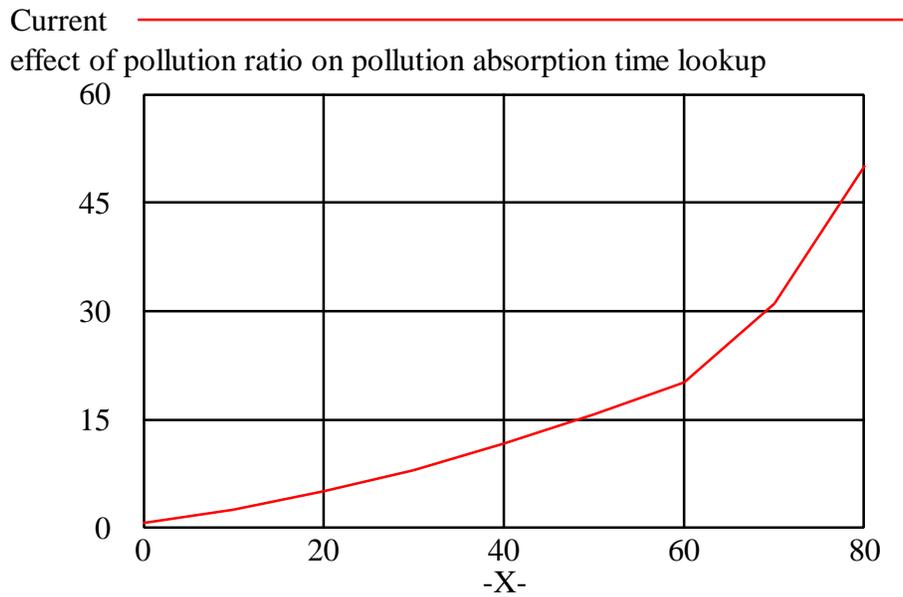   The ratio of the amount of pollution to the standard amount of pollution.

POLLUTION STANDARD = 1
   Units: tons
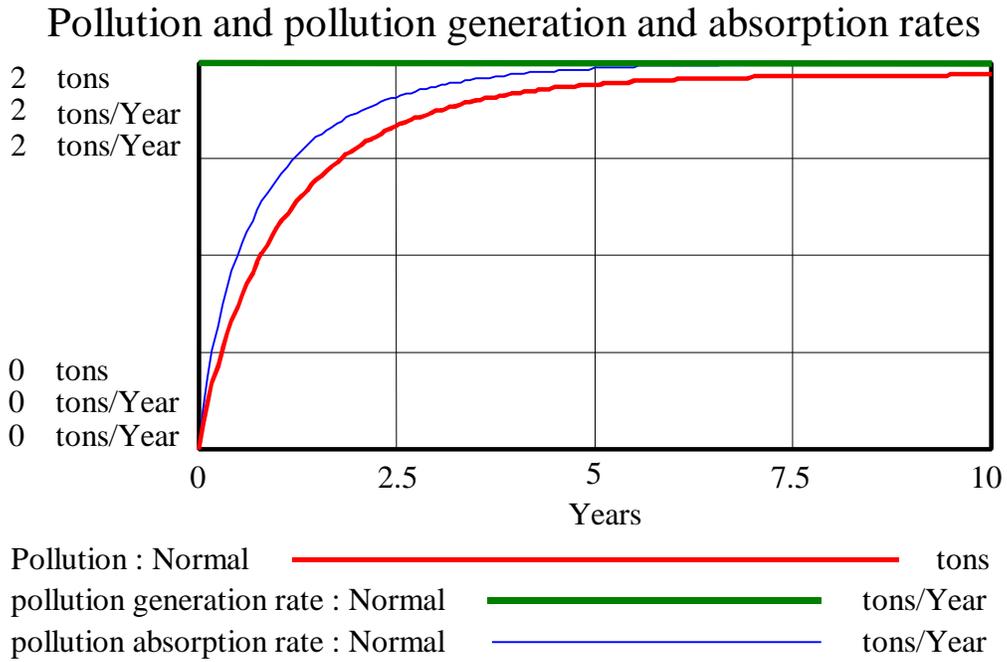   The standard amount of pollution in the air.

Note that the "effect of pollution ratio on pollution absorption time lookup" function should include the value (1,1) to indicate that when "Pollution" equals the "POLLUTION STANDARD," the "pollution absorption time" equals the "NORMAL POLLUTION ABSORPTION TIME."
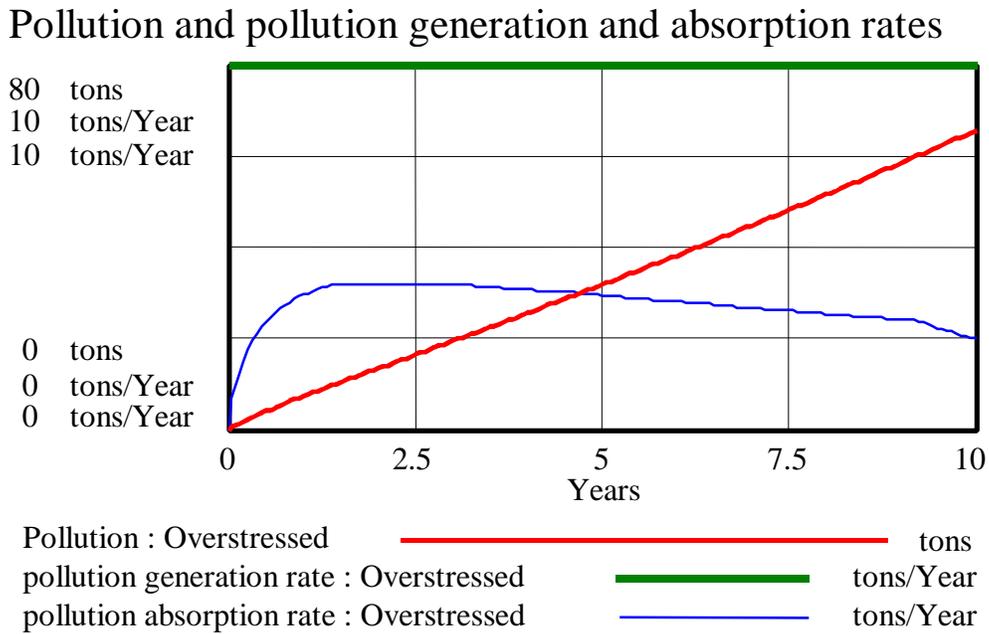
Graph of the lookup function:

Current

effect of pollution ratio on pollution absorption time lookup

Model behavior in the "normal system":

Pollution and pollution generation and absorption rates

2	tons
2	tons/Year
2	tons/Year

0	tons
0	tons/Year
0	tons/Year

Years

Pollution : Normal ——————————————— tons
pollution generation rate : Normal ——————————————— tons/Year
pollution absorption rate : Normal ——————————————— tons/Year

Model behavior in the "overstressed system":

Pollution and pollution generation and absorption rates

80	tons
10	tons/Year
10	tons/Year

0	tons
0	tons/Year
0	tons/Year

Years

Pollution : Overstressed ——————————————— tons
pollution generation rate : Overstressed ——————————————— tons/Year
pollution absorption rate : Overstressed ——————————————— tons/Year

4. Independent modeling exercise

This exercise will develop a simple model of the absorption of CFCs (chlorofluorocarbons) in the atmosphere.[6]

Some background:

CFCs are halogenated hydrocarbons that are used extensively as propellants in aerosol spray cans, coolants in refrigerators and air conditioners, solvents, and plastic foam products such as Styrofoam cups. The usefulness of CFCs lies in their nontoxic and nonflammable properties, as well as the ease with which they can be converted from a liquid to a gas and vice versa. CFCs became widely used after World War II.

Although CFCs are very useful in commercial and industrial applications, they pose a serious threat to the environment. In the 1970s, scientists discovered that after CFCs are released into the atmosphere, they pass through the troposphere and accumulate in the stratosphere. In the stratosphere, solar ultraviolet radiation breaks down the CFC molecules, releasing their constituent chlorine atoms. These then react with the ozone molecules, destroying the ozone layer.

The stratospheric ozone layer shields living organisms on Earth from the Sun's ultraviolet radiation. The radiation is associated with dangers such as skin cancer, cataracts, genetic disease, and weaker immune systems. Because of a growing concern over ozone depletion, in the late 1970s the United States, Canada, and the Scandinavian countries imposed a ban on the use of CFCs in aerosol-spray dispensers. In 1990, 93 nations agreed to end production of ozone-depleting chemicals by the end of the century, and in 1992, most of those same countries agreed to end their production of CFCs by 1996.

This exercise will study how the amount of CFCs in the atmosphere changes as the production of CFCs is restricted. When CFCs are released they first ascend into the troposphere. After an average of five years, the CFCs migrate to the stratosphere. The CFCs reside in the stratosphere on average 55 years before they are destroyed.

A. The model will contain the elements listed below. Identify each element as either stock, flow, or constant, and label its units. For each stock, determine its inflows and outflows.

- *time to rise to stratosphere*
- *CFC 11 destruction*
- *CFC 11 release pattern*
- *CFC 11 in stratosphere*
- *atmospheric CFC 11 releases*
- *lifetime of CFC 11 in stratosphere*
- *CFC 11 in troposphere*

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[6] The CFC model, system description, and data are based on information gathered in *Britannica Online*, *Biology: The Unity and Diversity of Life* by Starr and Taggart, and materials provided by Ventana Systems.

- *migration to stratosphere*

Name	Type	Units
time to rise to stratosphere	constant	years
CFC 11 destruction	flow	Mkg / year
CFC 11 release pattern	lookup	Mkg / year
CFC 11 in stratosphere	stock	Mkg
atmospheric CFC 11 releases	flow	Mkg / year
lifetime of CFC 11 in stratosphere	constant	years
CFC 11 in troposphere	stock	Mkg
migration to stratosphere	flow	Mkg / year

B. Using Vensim PLE, combine the elements to represent the structure of the system. In your assignment solutions document, please include the model diagram.
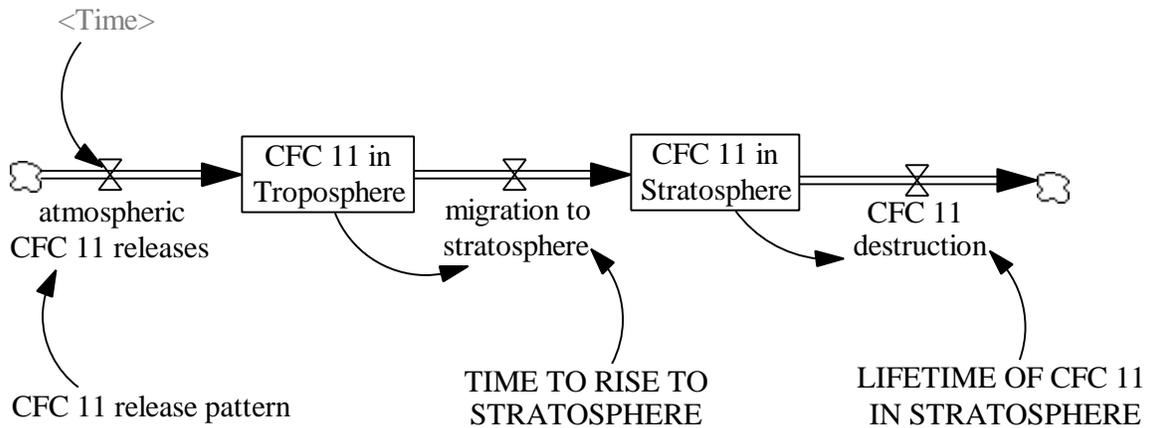
C. Using the system description and the table provided below, define the equations for each flow. Then supply the constants and initial values of the stocks. In your assignment solutions document, please include the documented equations and a graph of the lookup function.

Year	CFC production (Mkg/year)
1940	0
1945	0
1950	5
1955	20
1960	40

1965	*110*
1970	*200*
1975	*310*
1980	*250*
1985	*280*
1990	*220*
1995	*100*
2000	*0*
2005-2140	*0*

Hint: To define the rate at which CFC 11 is released into the atmosphere, you will need to use a lookup function. The input to the lookup will be the current time; in this case, you do not need to make the input dimensionless. You will need to use the shadow variable tool to add <Time> to the model diagram. To use the shadow variable tool, click once on the shadow variable icon, position the pointer on the diagram, and click once. You will then be able to choose "Time" from the list of all variables used in the model.

Model equations:

atmospheric CFC 11 releases =CFC 11 release pattern (Time)
        Units: mkg/Year
        The amount of CFC 11 released into the atmosphere every year.

CFC 11 destruction = CFC 11 in Stratosphere / LIFETIME OF CFC 11 IN
        STRATOSPHERE
        Units: mkg/Year
        The amount of CFC 11 destroyed every year.

CFC 11 in Stratosphere = INTEG (+migration to stratosphere - CFC 11 destruction, 0)
        Units: mkg
        The amount of CFC 11 in the stratosphere.

CFC 11 in Troposphere = INTEG (atmospheric CFC 11 releases - migration to
        stratosphere, 0)
        Units: mkg
        The amount of CFC 11 in the troposphere.

CFC 11 release pattern ([(1940,0) - (2140,400)], (1940,0), (1945,0), (1950,5), (1955,20),
        (1960,40), (1965,110), (1970,200), (1975,310), (1980,250), (1985,280),
        (1990,220), (1995,100), (2000,0), (2140,0))
        Units: mkg/Year
        The release pattern of CFC 11, based on historical data.

LIFETIME OF CFC 11 IN STRATOSPHERE =55
> Units: Year
> The average time CFC 11 stays in the stratosphere before destruction.

migration to stratosphere = CFC 11 in Troposphere / TIME TO RISE TO
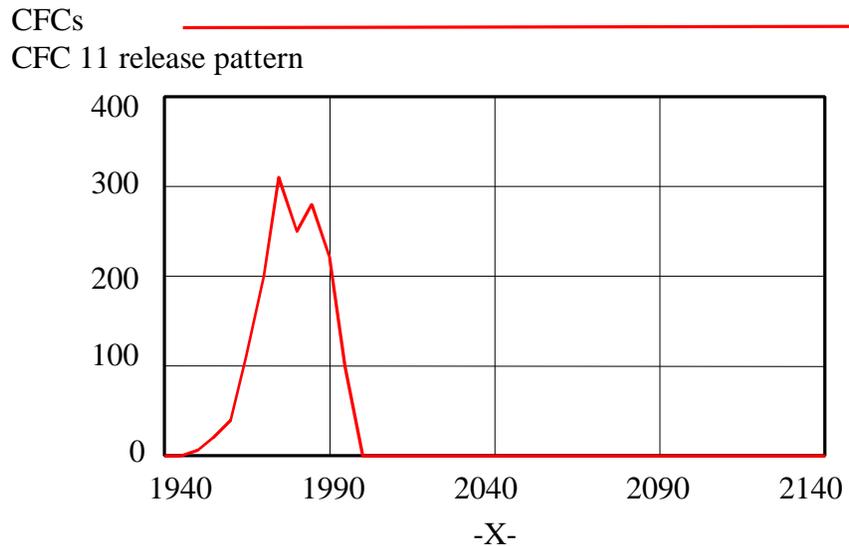> STRATOSPHERE
> Units: mkg/Year
> The amount of CFC 11 migrating from the troposphere to the stratosphere every year.

TIME TO RISE TO STRATOSPHERE = 5
> Units: Year
> The average time for CFC 11 to rise from the troposphere to the stratosphere.

The lookup function for CFC 11 release pattern:
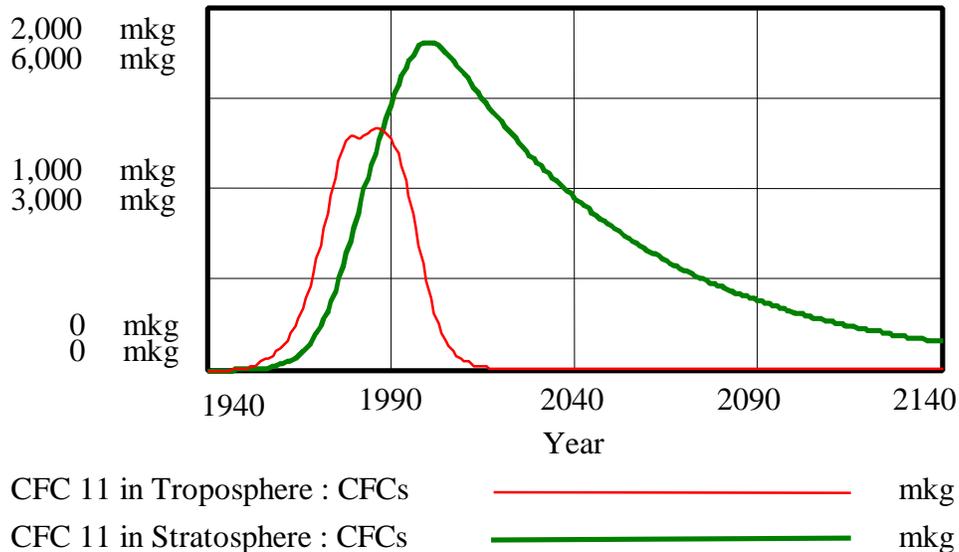
CFCs
CFC 11 release pattern

-X-

D. Before you simulate the model, draw reference modes for the stocks. You do not need to include these in your assignment solutions document, but you do need to draw them in order to be able to answer the next question.

E. Simulate the model from 1940 to 2140. In your assignment solutions document, please include graphs of the behavior of all stocks. Did the model generate the behavior you predicted? Why or why not?

Behavior of the stocks of the model:

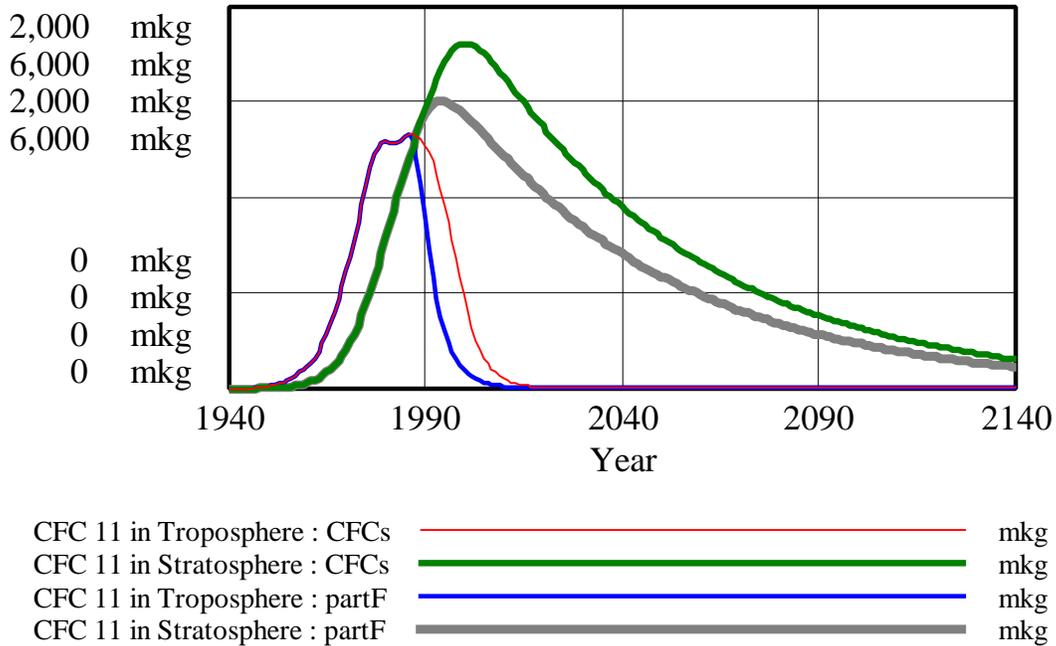## CFC 11 in Troposphere and CFC 11 in Stratosphere

2,000	mkg
6,000	mkg
1,000	mkg
3,000	mkg
0	mkg
0	mkg

1940   1990   2040   2090   2140

Year

CFC 11 in Troposphere : CFCs ————————————— mkg
CFC 11 in Stratosphere : CFCs ————————————— mkg

Immediately after the production of CFCs begins, the amount of "CFC 11 in Troposphere" starts to rise. Because it takes on average five years for CFC 11 to migrate into the stratosphere, the amount of "CFC 11 in Stratosphere" starts rising only after a delay. As the production of CFCs increases, both stocks continue to rise. Because the average "LIFETIME OF CFC 11 IN STRATOSPHERE" is longer than the "TIME TO RISE TO STRATOSPHERE," the stock "CFC 11 in Stratosphere" rises to a higher value than the stock "CFC 11 in Troposphere."

Then, as the production of CFCs is reduced, the amount of "CFC 11 in Troposphere" starts to decline. Because of the very long "LIFETIME OF CFC 11 IN STRATOSPHERE," however, the amount of "CFC 11 in Stratosphere" continues to rise. After the production of CFCs stops in year 2000, the stock "CFC 11 in Troposphere" soon decreases to 0. The stock "CFC 11 in Stratosphere," however, declines much more slowly, and will not even reach 0 by the end of the simulation, 2140, more than a hundred years after the production of CFCs ceased.

F. What would have happened if, in 1990, all countries had agreed to end production of CFCs immediately, instead of by the end of the century? Change the model to reflect the new scenario. Draw reference modes for each stock. Create a new dataset and simulate the model. In your assignment solutions document, submit graphs that contrast the behavior of the stocks from the two simulations. Did the model generate the behavior you predicted in the reference modes? Why or why not?

Behavior of the stocks in the base run and the scenario from part F:

Contrast of behaviors from the base run and part F

2,000	mkg
6,000	mkg
2,000	mkg
6,000	mkg
0	mkg
0	mkg
0	mkg
0	mkg

1940   1990   2040   2090   2140

Year

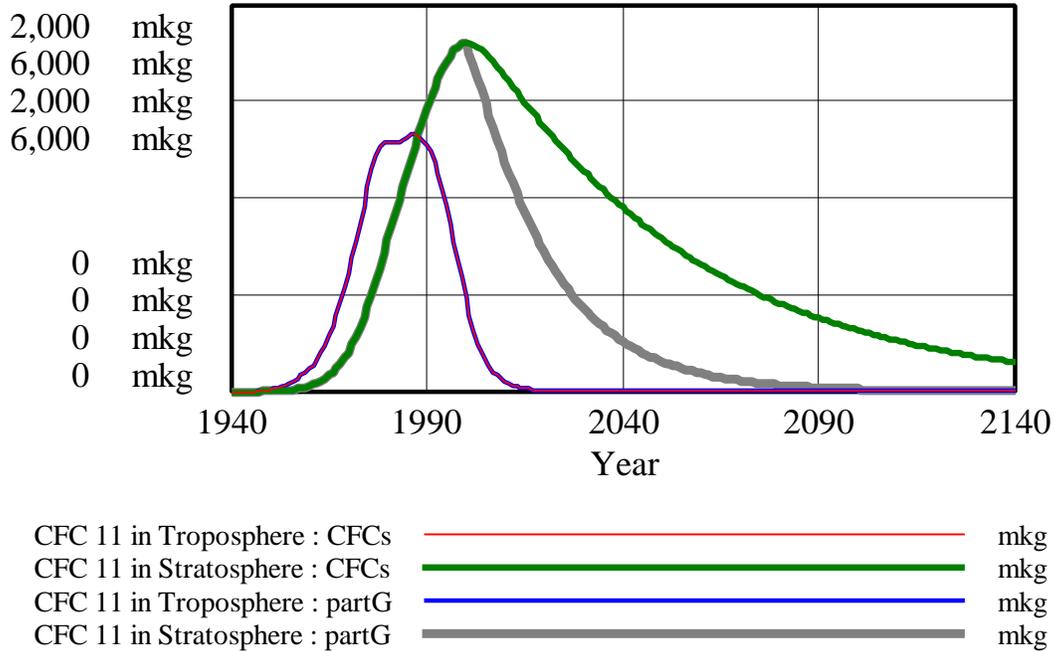|		
CFC 11 in Troposphere : CFCs		mkg
CFC 11 in Stratosphere : CFCs		mkg
CFC 11 in Troposphere : partF		mkg
CFC 11 in Stratosphere : partF		mkg

If the production of CFCs had ended in 1990, the stock "CFC 11 in Troposphere" would have declined faster because its inflow would have been reduced to zero earlier. The stock "CFC 11 in Stratosphere" would only have risen to a lower value and would have started declining sooner. Even in this scenario, however, there would still be a significant amount of "CFC 11 in Stratosphere" in 2140.

G. Return to the original data for CFC production. This time, however, imagine that in the year 2000, scientists develop a chemical that, when released into the stratosphere, decreases the average lifetime of CFCs from 55 to 20 years. Change the model to reflect this scenario and draw reference modes. Create a new dataset and simulate the model. Submit graphs that contrast this simulation with the original one. Did the model generate the behavior you predicted in the reference modes? Why or why not?

Behavior of the stocks in the base run and the scenario from part G:

Contrast of behaviors from the base run and part G

2,000 mkg	
6,000 mkg	
2,000 mkg	
6,000 mkg	
0 mkg	
0 mkg	
0 mkg	
0 mkg	

1940   1990   2040   2090   2140

Year

CFC 11 in Troposphere : CFCs		mkg
CFC 11 in Stratosphere : CFCs		mkg
CFC 11 in Troposphere : partG		mkg
CFC 11 in Stratosphere : partG		mkg

If in year 2000, scientists discover a way to reduce the "LIFETIME OF CFC 11 IN STRATOSPHERE," the behavior of "CFC 11 in Troposphere" would, of course, remain unchanged. Because "CFC 11 in Stratosphere" would be destroyed faster after year 2000, however, the behavior of "CFC 11 in Stratosphere" would change. Starting in 2000, the outflow "CFC 11 destruction" would be higher, so the stock "CFC 11 in Stratosphere" would decline faster. Hence, the stock would approach zero sooner, and by 2140, there would be almost no CFCs left in the stratosphere.

H. If in 1990, you were an advisor to the worldwide commission on CFC production, what policy would you recommend? Why?

The best policy that should be recommended is to stop CFC production as soon as possible because of the delays in obtaining results, mainly due to the long lifetime of CFCs in the stratosphere. With historical data as used in the first simulation, it will take approximately thirty years (around year 2020) to return to the situation in 1990 for the CFC level in the stratosphere. If CFC production halts completely in 1990, it will take only 13 years to reach again the 1990 values for CFC in the stratosphere. Even in this optimal case, however, it should be emphasized that although CFC production will be zero beginning in 1990, CFC levels in the stratosphere will still go on growing for more than a decade.

From the above graphs, it is clear that shorter lifetime of CFCs gives better results— CFCs disappear faster and to greater extent.

If I were an advisor to the worldwide commission on CFC production in 1990, I would recommend both to stop CFC production and scientific research that could shorten lifetime of CFC in stratosphere. There are several reasons:

- If countries are willing to stop production in year 1990 it should be done as soon as possible because they can change their mind after ten years (there could be major changes in the next ten years).
- We can hope that scientists can find a way to shorten the life of CFC production, but there is no guarantee that our expectations will be fulfilled.

I think that such research is necessary, but is also dangerous. If scientists find a way to shorten the lifetime of CFCs in the stratosphere, this will be justification for producers to stop production of CFC later (maybe in year 2200). Producers can even hope that scientists will find a way to immediately destroy CFCs in stratosphere, and they can fund such research, but I think that this will cause some additional problems that we cannot imagine.

Furthermore, in our hurry to solve the CFC problem, we may create negative side effects by using the new chemical.