Mistakes and Misunderstandings:

Time Constants and Decay Fractions

Peanuts

eating

EATING FRACTION
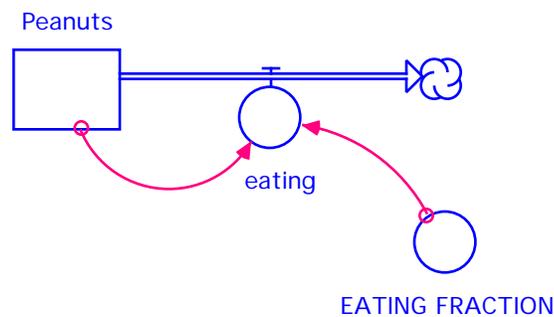
Prepared for the
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1. THE SYSTEM

This paper examines a mistake caused by misunderstanding the real meaning of a variable. The mistake leads to another conceptual mistake of separately defining two related variables. The examined model misrepresents a decay fraction leading to the introduction of a time constant to make the model dimensionally correct.

The model describes a system of eating peanuts. Assume that the number of peanuts on a plate diminishes only through eating and is not replenished. Initially, the plate contains one hundred peanuts. Each hour, Kevin eats a half of the remaining peanuts on the plate.

2. A FIRST ATTEMPT TO MODEL THE SYSTEM

Kevin begins building a simple model capturing the effect of eating peanuts on the number of peanuts left on the plate. He decides that he will eat only a half of the peanuts left on the plate every hour.

The stock in the model is "Peanuts," and the flow out of the stock is "eating." "Peanuts" can only be drained through "eating," and the stock of "Peanuts" is not replenished. Based on the given description, Kevin begins writing the rate equation for "eating." He decides that "FRACTION OF PEANUTS EATEN" is 0.5, or one half. In other words, Kevin eats only a half of the available peanuts. Consequently, Kevin writes the following equation:

eating = Peanuts * FRACTION OF PEANUTS EATEN   (1)

To examine whether the equation is correct, Kevin checks the units of "eating." Based on his initial formulation of variables, the stock of "Peanuts" has units of peanuts, and "FRACTION OF PEANUTS EATEN" is a dimensionless number. Thus, based on equation 1, "eating" should have units of peanuts. Kevin knows, however, that he eats peanuts over time, and that the variable of "eating" is a flow. Thus, "eating" should have units of peanuts per hour.

Kevin corrects his mistake by introducing a constant called "AVERAGE TIME PEANUT REMAINS ON PLATE," equal to one hour. In the new equation, "eating" equals "Peanuts" multiplied by "FRACTION OF PEANUTS EATEN" divided by "AVERAGE TIME PEANUT REMAINS ON PLATE."

eating = Peanuts * FRACTION OF PEANUTS EATEN / AVERAGE TIME
PEANUT REMAINS ON PLATE   (2)

Kevin again checks the model for dimensional consistency. In the description, "Peanuts" has units of peanuts, "FRACTION OF PEANUTS EATEN" is dimensionless, and "AVERAGE TIME PEANUT REMAINS ON PLATE" has units of hours. Thus, "eating" has the correct units of peanuts per hour. Figure 1 depicts a model corresponding to the above description. Appendix 6.1 gives documented equations of the model.
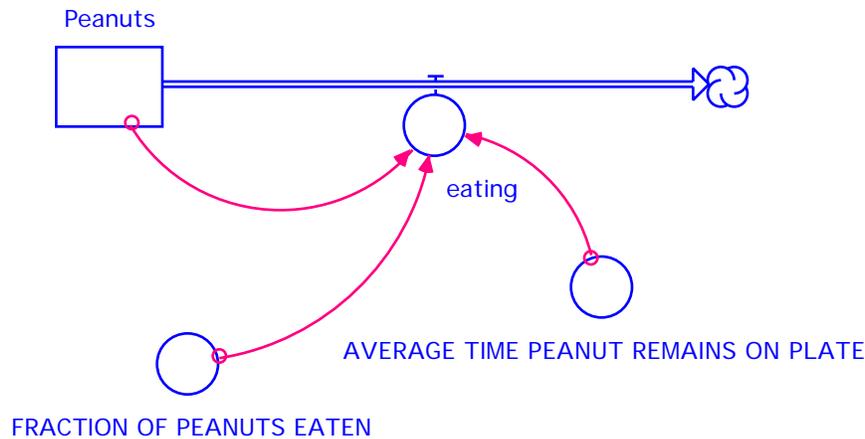

Figure 1: Model of peanut eating

3. MISTAKES AND MISUNDERSTANDINGS

The most important mistake in the model in Figure 1 is the incorrect definition of "FRACTION OF PEANUTS EATEN." Kevin thinks that he eats 0.5, or one half, of the remaining peanuts. Thus, in the model in Figure 1, "FRACTION OF PEANUTS EATEN" is 0.5, a dimensionless number. Kevin's definition of "FRACTION OF PEANUTS EATEN" is incorrect and does not make any real life sense. Kevin does not eat one half of the remaining peanuts; he eats one half of the remaining peanuts over a period of time. Thus, "FRACTION OF PEANUTS EATEN" should have units of peanuts per peanuts per hour, or 1/hour. In Kevin's original formulation of "eating," "eating" equals "Peanuts" times "FRACTION OF PEANUTS EATEN." If Kevin sets the units of "FRACTION OF PEANUTS EATEN" to 1/hour, then equation 1 is dimensionally consistent. Therefore, the "AVERAGE TIME PEANUT REMAINS ON PLATE" is not necessary in the equation.

Because Kevin made the mistake in the definition of "FRACTION OF PEANUTS EATEN," he had to introduce the constant "AVERAGE TIME PEANUT REMAINS ON PLATE" to make the model dimensionally consistent. His original misrepresentation of

"FRACTION OF PEANUTS EATEN" led to a conceptual mistake: separation of "FRACTION OF PEANUTS EATEN" and "AVERAGE TIME PEANUT REMAINS ON PLATE." The two constants, however, cannot be defined separately. "FRACTION OF PEANUTS EATEN" is the inverse of "AVERAGE TIME PEANUT REMAINS ON PLATE." If Kevin eats a half of the remaining peanuts every hour, then on average a peanut remains on the plate for 1/0.5 hours, or two hours. Thus, "AVERAGE TIME PEANUT REMAINS ON PLATE" and "FRACTION OF PEANUTS EATEN" contain the same information, and only one of them is required in the rate equation.

4. OVERCOMING OUR MISTAKES AND MISUNDERSTANDINGS

Figure 2 depicts a corrected model of peanut eating. Documented equations for the correct model are in Appendix 6.2.
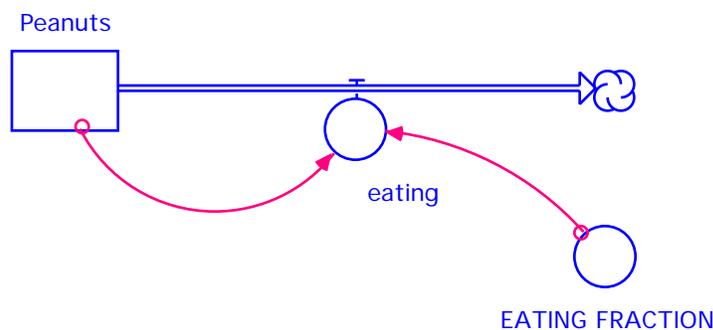

Figure 2: A corrected model of peanut eating

According to the model in Figure 2, "EATING FRACTION" represents the fraction of peanuts eaten per peanut on the plate every hour. "EATING FRACTION" has units of 1/hour. The stock of "Peanuts" is only drained by "eating," which depends on "Peanuts" and "EATING FRACTION." "Eating" equals "Peanuts" times the "EATING FRACTION":

eating = Peanuts * EATING FRACTION   (3)

Because "Peanuts" has units of peanuts, and "EATING FRACTION" has units of 1/hour, "eating" has units of peanuts per hour. In the corrected model in Figure 2, "EATING FRACTION" is modeled correctly, and the model is dimensionally consistent.

5. KEY LESSONS

1. All parameters in a model must be properly defined, and each must exhibit a real meaning. For example, in Kevin's model, "FRACTION OF PEANUTS EATEN" was misunderstood and incorrectly formulated. Kevin does not eat a fraction of remaining peanuts; he eats a fraction of remaining peanuts over time.

2. A decay fraction is the inverse of a time constant; thus, a decay fraction cannot be defined separately from a time constant. For example, "FRACTION OF PEANUTS EATEN" and "AVERAGE TIME PEANUT REMAINS ON PLATE" cannot be defined separately. In a correct model, only one of "AVERAGE TIME PEANUT REMAINS ON PLATE" or "EATING FRACTION" needs to be defined because the two constants are inversely proportional. In building a model of peanut eating, "EATING FRACTION" is more relevant than "AVERAGE TIME PEANUT REMAINS ON PLATE" because it is more intuitive to think about the fraction of peanuts eaten over time than the average time a peanut remains on the plate. In other cases, such as population models, people think in terms of average lifetime rather than fractional death rate.

3. A model that is dimensional consistent is not always correct. When Kevin first wrote the equation for "eating," he decided to add "AVERAGE TIME PEANUT REMAINS ON PLATE" in order to make the model dimensionally consistent. Even though the units in the model were consistent, the model in Figure 1 was incorrect.

6. APPENDIX

6.1 First (Incorrect) Model of Peanut Eating

Peanuts(t) = Peanuts(t - dt) + (- eating) * dt
INIT Peanuts = 100
DOCUMENT: Number of peanuts on the plate.
UNITS: Peanuts

OUTFLOWS:
eating = Peanuts * FRACTION_OF_PEANUTS_EATEN /
        AVERAGE_TIME_PEANUT_REMAINS_ON_PLATE
DOCUMENT: Number of peanuts eaten every hour.
UNITS: Peanuts/hour

FRACTION_OF_PEANUTS_EATEN = 0.5
DOCUMENT: Fraction of peanuts eaten from the plate.
UNITS: Dimensionless

AVERAGE_TIME_PEANUT_REMAINS_ON_PLATE = 1
DOCUMENT: Average time a peanut remains on the plate.
UNITS: hour

6.2 Corrected Model of Peanut Eating

Peanuts(t) = Peanuts(t - dt) + (- eating) * dt
INIT Peanuts = 100
DOCUMENT: Total number of peanuts on the plate.
UNITS: Peanuts

OUTFLOWS:
eating = Peanuts * EATING_FRACTION
DOCUMENT: Number of peanuts eaten every hour.
UNITS: Peanuts/hour

EATING_FRACTION = 0.5
DOCUMENT: Fraction of peanuts eaten every hour from the plate.
UNITS: 1/hour