EVERYDAY STOCKS and FLOWS
PROBLEM SETS for MIDDLE GRADES
by Alan Ticotsky

Stock/flow diagrams and computer modeling are wonderful tools to help us understand and solve complex problems and systems. But they should not be reserved only for those complex scenarios. Teaching young students to draw stock/flow diagrams to understand simpler problems helps them to begin thinking systemically. Using eleven familiar and common situations, this series of problem sets designed for middle school students makes the tools accessible and comprehensible.

DESCRIPTIONS OF PROBLEM SETS

Everyday Stocks and Flows

Description: Students are asked to graph changes over time and draw stock/flow diagrams of a set of everyday situations.

Skills:
• Drawing behavior-over-time graphs
• Drawing stock/flow diagrams

What Affects the Rate of a Flow?

Description: Following up the previous problem set, students are asked to graph changes over time of more complicated situations. After they draw stock/flow diagrams, they are asked to explain why the stock changed in the way it did.

Skills:
• Drawing behavior-over-time graphs
• Drawing stock/flow diagrams

Piggy Bank Problems

Description: Students read a short description of a girl's summer job. They calculate her earnings and then draw a graph. Next, they figure in her spending and find the net amount she saves.

Skills:
• Calculating a total given a rate over time
• Drawing a behavior-over-time graph with two variables
• Drawing stock/flow diagrams
Piggy Bank Problems 2

Description: Following up the previous problem set, students continue to analyze data about a girl’s summer job. The situation changes, and she also has to plan for working during the school year.

Skills:
- Completing a data table
- Calculating a total, given a rate over time
- Drawing a behavior-over-time graph with two variables
- Drawing stock/flow diagrams

Choosing Books to Read – Linear Modeling

Description: Students use different rates of reading (pages per minute) and calculate how long it will take to complete a set of books.

Skills:
- Drawing stock/flow diagrams
- Building linear models

Comparing Modes of Travel - Linear Modeling

Description: Students use different rates of travel (miles per hour) and calculate how long it will take to cover a given distance.

Skills:
- Drawing stock/flow diagrams
- Building linear models

Riding the MBTA Red Line

Description: Two riders on the metropolitan Boston public transit system count people entering and exiting their subway car. Students complete a data table, draw a stock/flow diagram, and graph the number of riders.

Skills:
- Completing a data table
- Drawing stock/flow diagrams
- Drawing a behavior-over-time graph with multiple variables

Gasoline Prices and the MBTA Red Line

Description: Following up the previous problem set, students examine data from the metropolitan Boston public transit system and analyze how rising gasoline prices affected ridership.

Skills:
- Calculating a percentage increase
- Drawing a behavior-over-time graph with two variables
- Creating a connection circle (process not explained)
Extending the *Making Friends* Game

**Description:** After students have played the game, *Making Friends*, they draw a stock/flow diagram of the game. Then they build a model and answer questions about the actual game and hypothetical situations.

**Skills:**
- Drawing stock/flow diagrams
- Building a linear model
- Building a model describing exponential growth

Paying for Parking

**Description:** Students learn how a parking garage charges for parking. They draw a stock/flow diagram and build a model to use to answer questions about one customer’s bill.

**Skills:**
- Drawing stock/flow diagrams
- Building a linear model
- Learning and using the ‘STEP’ function (process explained)

Shoveling the Driveway

**Description:** Students analyze different rates for clearing snow from a driveway. They draw a stock/flow diagram and build a model to use to answer questions about the results of different scenarios.

**Skills:**
- Drawing stock/flow diagrams
- Building a linear model
- Using the ‘STEP’ function (process not explained)
EVERYDAY STOCKS AND FLOWS

Sketch a behavior-over-time graph for each of the following situations. Be prepared to explain why you drew the line the way you did. Then draw a stock/flow map diagramming the situation you graphed.

1. Amount of water in a bathtub.

2. Money in a piggy bank.

3. Students in the classroom during the course of a day.

4. Students in the dining room.

5. Numbers of wins for a sports team you know.

6. Your hunger from the time you awake to the time you go to bed.
WHAT AFFECTS THE CHANGE OF A STOCK?

Graph the following situations. Then draw stock/flow maps, using the quantity you graphed as a stock.

1. People in a theatre attending a show.

2. Length of grass on a lawn.

3. Number of wins your favorite team has during a typical season.

4. Popularity of a song.

5. Number of people who see a movie.

6. Number of connected pieces in a jigsaw puzzle.

Look at your graphs. Are any of them straight lines? Write a paragraph for each situation describing why the stock changes the way it does.
PIGGY BANK PROBLEMS

Melinda gets paid $5.00 per hour to do childcare in the summer. She works Monday through Friday two hours per day. The job will last ten weeks. Melinda puts the money she earns into her piggy bank on Saturday morning.

1. How many hours does she work per week?

2. How much does she earn each week?

3. Draw a behavior-over-time graph of her earnings during the summer.

Every Saturday, before putting her money into her piggy bank, Melinda takes $3.00 out of her pay so she can buy herself a special Kids’ Meal at the Beach Shack. (The meal costs $3.00.)

4. How much does she have from her weekly earnings to put into the bank?

5. Graph the amount of money in Melinda’s piggy bank on the graph above (problem #3) using another color.

6. Draw a stock/flow map diagramming how the money in Melinda’s piggy bank changes over time.
PIGGY BANK PROBLEMS
Part Two

Beginning in week five, the Beach Shack raised the price of the Kids’ Meal to $3.50.

1. Complete the table below:

<table>
<thead>
<tr>
<th>Week</th>
<th>Money Melinda earned</th>
<th>Money Melinda spent</th>
<th>Money in piggy bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>50</td>
<td>3</td>
<td>47</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>3</td>
<td>94</td>
</tr>
<tr>
<td>3</td>
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<td>8</td>
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<td>9</td>
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<td></td>
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<tr>
<td>10</td>
<td></td>
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</tr>
</tbody>
</table>

Melinda’s work in the summer helped the families so much that she was asked to work during the school year. Make up a reasonable schedule for Melinda to work while she is in school.

2. How many hours per week should she work and how much should she charge per hour?

3. How much should she budget to spend per week? (The Beach Shack closes after Labor Day.)

4. Fill in the table below with the numbers you decided on for the school year.

<table>
<thead>
<tr>
<th>Week of school</th>
<th>Money Melinda earned</th>
<th>Money Melinda spent</th>
<th>Money in piggy bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>4</td>
<td></td>
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<td></td>
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<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
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<td>7</td>
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<td>8</td>
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<td>9</td>
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<td></td>
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<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Draw a behavior-over-time graph of the amount of money in Melinda’s piggy bank for the ten weeks of summer and the first ten weeks of school.

6. Use two different colors to graph the amount she earned and the amount she spent each week.
CHOOSING BOOKS TO READ
LINEAR MODELING

Fifth graders have been assigned to read three books during a unit about novels. The school librarian, Ms. Hobart, made a suggested reading list and divided it into three categories. Category A books are easiest to read, Category B are medium, and Category C are most challenging.

Phillip estimates that he can read one page of most novels in category A in about two minutes. Category B will take him about 2.75 minutes per page, and category C will require 3.5 minutes per page.

1. Draw a stock/flow map of Phillip’s reading.

2. Create a model on your computer so you can help Phillip predict how much time it will take to read each novel on the list. Remember to adjust the reading rate based on the degree of difficulty.

<table>
<thead>
<tr>
<th>LIST A TITLES</th>
<th># OF PAGES</th>
<th>TIME NEEDED</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Big Game</td>
<td>120</td>
<td></td>
</tr>
<tr>
<td>Journey to Mexico</td>
<td>134</td>
<td></td>
</tr>
<tr>
<td>Hamster’s Birthday</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>A Surprise for Jilly</td>
<td>107</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LIST B TITLES</th>
<th># OF PAGES</th>
<th>TIME NEEDED</th>
</tr>
</thead>
<tbody>
<tr>
<td>When Giants Ruled</td>
<td>161</td>
<td></td>
</tr>
<tr>
<td>Fred’s New Bike</td>
<td>119</td>
<td></td>
</tr>
<tr>
<td>Best Friends</td>
<td>128</td>
<td></td>
</tr>
<tr>
<td>Alien Invaders</td>
<td>172</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LIST C TITLES</th>
<th># OF PAGES</th>
<th>TIME NEEDED</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Stranger at the Dance</td>
<td>208</td>
<td></td>
</tr>
<tr>
<td>Alone in the Wilderness</td>
<td>257</td>
<td></td>
</tr>
<tr>
<td>The Greatest Team of All</td>
<td>189</td>
<td></td>
</tr>
<tr>
<td>Valley of the Geysers</td>
<td>281</td>
<td></td>
</tr>
</tbody>
</table>

3. Think about your reading rate. Estimate how long it will take you to finish the book you are currently reading. Explain how you arrived at your prediction.
COMPARING MODES OF TRAVEL
LINEAR MODELING

1. Draw four stock/flow maps to show distance traveled by these four modes: airplane, train, automobile, and truck.

2. Build a STELLA model of all four and assume the following rates of travel:

<table>
<thead>
<tr>
<th>TYPE OF VEHICLE</th>
<th>AVERAGE MILES PER HOUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>airplane</td>
<td>300</td>
</tr>
<tr>
<td>train</td>
<td>50</td>
</tr>
<tr>
<td>automobile</td>
<td>45</td>
</tr>
<tr>
<td>truck</td>
<td>40</td>
</tr>
</tbody>
</table>

3. Run your model for six hours and graph the vehicles on the same screen. How many miles would each cover in six hours?

4. Run the model long enough so that each vehicle travels 600 miles. How many hours does it take for each to cover the distance?

5. Your company wants to send a 10-pound package from suburban Boston to an office in downtown Atlanta, Georgia. The distance is about 926 miles. You need to decide how to send the package.

   • How long will each method of travel take?

   • What else do you need to know?

   • What are the advantages and disadvantages of each type of travel?

   • With your team, choose the method you favor. Create a proposal to persuade the company decision makers to take your advice.
RIDING THE MBTA RED LINE

Teddy and Phillipa often take the “T” from Alewife to Park Street. One day they kept track of how many passengers rode in their subway car. They split up the job and counted people getting on and off at each station. After Porter Square, they stopped calculating the totals so they wouldn’t lose count of people getting on and off.

<table>
<thead>
<tr>
<th>STATION</th>
<th>PASSENGERS ARRIVING ON THE TRAIN</th>
<th>RIDERS ENTERING CAR</th>
<th>RIDERS EXITING CAR</th>
<th>PASSENGERS DEPARTING ON TRAIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alewife</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>Davis</td>
<td>16</td>
<td>12</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>Porter</td>
<td>25</td>
<td>10</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Harvard</td>
<td>18</td>
<td>10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central</td>
<td>12</td>
<td></td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>Kendall/MIT</td>
<td>9</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charles/MGH</td>
<td>9</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Park Street</td>
<td>17</td>
<td>13</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Identify the flow(s) in this exercise.

2. Identify the stock(s).

3. Draw a stock/flow map of this situation.

4. Complete the table, filling in the boxes above that Teddy and Phillipa left blank.

5. Graph the number of passengers in the subway car on a behavior-over-time graph.

6. Using a different color on the same pad, graph the number of riders getting on.

7. Using a third color, graph the number of riders getting off.
## GASOLINE PRICES AND THE MBTA RED LINE

<table>
<thead>
<tr>
<th>STATION</th>
<th>PASSENGERS ENTERING STATION</th>
<th>20% INCREASE</th>
<th>PEOPLE LIKELY TO BE IN TEDDY AND PHILLIPA’S CAR IN MAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alewife</td>
<td>9,567</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Davis</td>
<td>10,891</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Porter</td>
<td>8,089</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Harvard</td>
<td>20,212</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Central</td>
<td>11,736</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Kendall/MIT</td>
<td>11,214</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charles/ MGH</td>
<td>7,855</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table above shows an average daily count from the turnstiles at seven Red Line stations during the middle of the decade beginning in 2001. In 2008, gasoline prices rose steadily during the first half of the year, climbing to more than $4 per gallon. By May 2008, the MBTA recorded an average ridership increase of 20% on the subways.

1. Why do you think more people rode the “T?”

2. Fill in the column in the table with the heading “20% increase.”

3. Graph the data from both columns on the same behavior-over-time graph. Use a different color for each line.

4. Using your last problem sheet (“Riding the MBTA Red Line”), fill in the column headed “People likely to be in Teddy and Phillipa’s car in May.” Explain how you did the calculations.

5. Draw a connection circle to show how gasoline prices and ridership on the “T” may be related. Add other important elements you think might affect how many people ride the subway.
EXTENDING THE *MAKING FRIENDS* GAME

1. Draw a stock/flow map of the way the *Making Friends* game was played the first time. Remember that each member of the team chose a new friend one time only.

2. Make a STELLA model on your computer and simulate the game as you did in class. Have the computer graph the Friends team. Copy that graph here.

3. How many turns did it take until all the students in the class were on the team?

4. Imagine a school with 850 students. Simulate playing the *Making Friends* game with everyone. How many turns would it take until they were all on the team?

5. Draw a stock/flow of the way the *Making Friends* game was played the second time. Remember that each person on the team chose a new friend every round.

6. Make a STELLA model on your computer of the second version of the game and simulate how it was played in class. Copy the graph here.

7. Use your model of *Making Friends* game 2 to learn how many turns it would take to bring all the people below onto the Friends team. Start with two friends.
   - Town of Carlisle, about 5,000 people.
   - State of Massachusetts, about 6,000,000 people.
   - United States of America, about 300,000,000 people.
PAYING FOR PARKING

1. Bruno’s Garage is open 24 hours per day. Some people park there while they work or shop during the day and others park in the evening when they go out or visit friends. Some people park overnight. Bruno charges by the hour. Draw a stock/flow map with one flow to show how the parking charge accumulates.

2. Bruno’s Garage has two different rates for parking. Between 9 AM and 3 PM, the hourly rate is three dollars. At all other times, the rate is two dollars per hour. Cindy wants to compute how much it will cost her to park at Bruno’s from 7 AM to 2 PM one day. What will she have to pay?

3. Build a STELLA model on your computer to model Cindy’s parking fee. To use only one flow, you will need to change the rate during the time the model runs. To do this, you can use the “STEP” function in STELLA. Double click on the flow and enter the starting rate Cindy pays. Then add a “+” and find the list of built-in functions. Click on “STEP” and you will see parentheses to fill in. Two numbers need to be in the parentheses; the first number needs to be the amount by which the rate changes.*

   • How much does the rate change? Put that amount in the parentheses.

   • The second number in the parentheses needs to tell the computer when to change the rate.

   • After how many hours of parking does Cindy begin to pay the changed rate?

4. Run the model and graph the stock and the flow. Draw them here and explain the behavior.

*Hint: When you use “STEP” in a flow, think of the phrase “behavior over time” to remember how to enter numbers in the parentheses. The first number tells how much the behavior of the flow will change. The second tells the time of the change, measured from the beginning of the run.
SHOVELING THE DRIVEWAY

Lisa’s family has a long driveway that measures 72 meters long from the road to her house. When it snows fewer than three inches, Lisa and her younger brother Bernard clear it off. Lisa can clear 1.2 meters per minute. Bernard can clear 0.8 meters per minute.

1. Draw a stock/flow map to represent this situation.

2. How long would it take Lisa to clear the driveway if she worked alone?

3. How long would it take Bernard if he worked alone?

4. How long would it take them when they work together? You may use a STELLA model to solve this problem.

5. One day, Bernard needed to finish his homework before helping Lisa shovel the driveway. It took Bernard 32 minutes to complete his homework and get out to the driveway job. How long did it take them to finish? You may use a STELLA model.

You probably used the “STEP” function if you modeled this problem on your computer. Write the step function you used to represent the flow in your STELLA model.
EVERYDAY STOCKS AND FLOWS

Sketch a behavior-over-time graph for each of the following situations. Be prepared to explain why you drew the line the way you did. Then draw a stock/flow map diagramming the situation you graphed.

1. Amount of water in a bathtub.

```
Water in bathtub

flow of water from faucet  flow of water down drain
```

2. Money in a piggy bank.

```
Money in the piggy bank

money put in  money taken out
```

3. Students in the classroom during the course of a day.

```
Students in classroom

students entering  students leaving
```
4. Students in the dining room.

![Diagram of students in dining room]

5. Numbers of wins for a sports team you know.

![Diagram of total wins]

6. Your hunger from the time you awake to the time you go to bed.

Students may get creative here. “Hunger” as a stock can be increased by a flow named “getting hungry.” A flow named “eating food” may be an outflow that reduces the stock “Hunger.” Students should have fun graphing this stock and trying to describe flows that affect it.
WHAT AFFECTS THE CHANGE OF A STOCK?

Graph the following situations. Then draw stock/flow maps, using the quantity you graphed as a stock.

1. People in a theatre attending a show.
   Some people arrive early, some show up at the last minute. While the show is playing, the stock is at its maximum. After the show, the stock drains steadily and quickly. Some students may include an intermission when the stock can drain, or include the leaving and re-entering of people during the performance.

   ![People in theatre](image)

2. Length of grass on a lawn.
   In temperate climates, grass grows most quickly in the spring and fall. In hot summer weather, it often slows down. The amount of rainfall and frequency of watering also affect the rate of growth. The number of times the lawn is mowed, and the height of the mower blades will also determine the length of the grass.

   ![Length of grass](image)

3. Number of wins for your favorite team during a typical season.
   The graph never declines. It will plateau during a losing streak and climb steadily when the team does well.

   ![Total victories](image)
4. Popularity of a song.
At first only a few people may know about the song. Word of mouth and repeated play on media exposes more people to it, and familiarity can drive popularity. Inevitably, people want to hear something new and they may tire of the song, sending its popularity down. Some popular songs become “classics” and plateau at a level, while others fade into obscurity.

5. Number of people who see a movie.
Some movies follow a pattern similar to the example in #4 about popular music. Their popularity and attendance grow over time. Other movies are so widely anticipated that crowds see them as soon as possible. This example might be a good one to use to graph not only the stock but also the flow over time.

6. Number of connected pieces in a jigsaw puzzle.
Many people try to build the border first since they can separate the pieces with straight edges. There may be a period of sorting when few if any pieces are connected. After the border is constructed, there may be a slowdown matching interior pieces. Finally, toward the completion, work may speed up as the number of pieces dwindles.
Some students may think of the loose pieces as a stock and represent the building in a different way:

**Pieces with no connections**

![Diagram of pieces with no connections](image)

Look at your graphs. Are any of them straight lines? Write a paragraph for each situation describing why the stock changes the way it does.

It’s unlikely that any of these graphs would be straight lines, indicating a steady rate of change. Flows change stocks, so when the rates of flows are not constant, the graphs of the accompanying stocks will not be straight lines.
PIGGY BANK PROBLEMS

Melinda gets paid $5.00 per hour to do childcare in the summer. She works Monday through Friday two hours per day. The job will last ten weeks. Melinda puts the money she earns into her piggy bank on Friday evening.

1. How many hours does she work per week? 10

2. How much does she earn each week? $50.00

3. Draw a behavior-over-time graph of her earnings during the summer. The x-axis will be labeled week 0, 1, 2, … The y-axis will measure the accumulated earnings, a straight diagonal line, rising from 0, steadily higher, at $50 per week.

Every Saturday morning, before going to the beach, Melinda takes $3.00 out of her piggy bank so she can buy herself a special Kids’ Meal at the Beach Shack. (The meal costs $3.00.)

4. How much does she leave from her weekly earnings in her bank? $47.00

5. Graph the amount of money in Melinda’s piggy bank on the graph above (problem #3) using another color. This line will have a slightly less steep slope as it increases by $47 per week.

6. Draw a stock/flow map diagramming how the money in Melinda’s piggy bank changes over time.

Money in piggy bank

money being earned

money being spent

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PIGGY BANK PROBLEMS
Part Two

Beginning in week five, the Beach Shack raised the price of the Kids’ Meal to $3.50.

1. Complete the table below:

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<th>Money Melinda spent</th>
<th>Money in piggy bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$50</td>
<td>$3</td>
<td>$47</td>
</tr>
<tr>
<td>2</td>
<td>50</td>
<td>3</td>
<td>94</td>
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<td>6</td>
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<td>3.50</td>
<td>281</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>3.50</td>
<td>327.50</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>3.50</td>
<td>374</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>3.50</td>
<td>420.50</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>3.50</td>
<td>467</td>
</tr>
</tbody>
</table>

Melinda’s work in the summer helped the families so much that she was asked to work during the school year. Make up a reasonable schedule for Melinda to work while she is in school.

2. How many hours per week should she work and how much should she charge per hour? Students may decide she should get a raise. Maybe Melinda’s school and social schedule will reduce her to working fewer days per week.

3. How much should she budget to spend per week? (The Beach Shack closes after Labor Day.) Will she find other reasons to spend?

4. Fill in the table below with the numbers you decided for the school year.

<table>
<thead>
<tr>
<th>Week of school</th>
<th>Money Melinda earned</th>
<th>Money Melinda spent</th>
<th>Money in piggy bank</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
5. Draw a behavior-over-time graph of the amount of money in Melinda’s piggy bank for the ten weeks of summer and the first ten weeks of school. Depending on Melinda’s changes in hours worked, amount earned per hour, and amount spent, the graph will change after the summer.

6. Use two different colors to graph the amount she earned and the amount she spent each week.
CHOOSING BOOKS TO READ  
LINEAR MODELING

Fifth graders have been assigned to read three books during a unit about novels. The school librarian, Ms. Hobart, made a suggested reading list and divided it into three categories. Category A books are easiest to read, Category B are medium, and Category C are most challenging. Phillip estimates that he can read one page of most novels in category A in about two minutes. Category B will take him about 2.75 minutes per page, and category C will require 3.5 minutes per page.

1. Draw a stock/flow map of how many pages Phillip can read over time.

![Stock/Flow Map]

2. Create a model on your computer so you can help Phillip predict how much time it will take to read each novel on the list. Remember to adjust the reading rate based on the degree of difficulty.

<table>
<thead>
<tr>
<th>LIST A TITLES</th>
<th># OF PAGES</th>
<th>TIME NEEDED</th>
</tr>
</thead>
<tbody>
<tr>
<td>The Big Game</td>
<td>120</td>
<td>240 minutes</td>
</tr>
<tr>
<td>Journey to Mexico</td>
<td>134</td>
<td>268</td>
</tr>
<tr>
<td>Hamster's Birthday</td>
<td>86</td>
<td>172</td>
</tr>
<tr>
<td>A Surprise for Jilly</td>
<td>107</td>
<td>214</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LIST B TITLES</th>
<th># OF PAGES</th>
<th>TIME NEEDED</th>
</tr>
</thead>
<tbody>
<tr>
<td>When Giants Ruled</td>
<td>161</td>
<td>443 minutes</td>
</tr>
<tr>
<td>Fred's New Bike</td>
<td>119</td>
<td>327</td>
</tr>
<tr>
<td>Best Friends</td>
<td>128</td>
<td>352</td>
</tr>
<tr>
<td>Alien Invaders</td>
<td>172</td>
<td>473</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LIST C TITLES</th>
<th># OF PAGES</th>
<th>TIME NEEDED</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Stranger at the Dance</td>
<td>208</td>
<td>728 minutes</td>
</tr>
<tr>
<td>Alone in the Wilderness</td>
<td>257</td>
<td>900</td>
</tr>
<tr>
<td>The Greatest Team of All</td>
<td>189</td>
<td>662</td>
</tr>
<tr>
<td>Valley of the Geysers</td>
<td>281</td>
<td>984</td>
</tr>
</tbody>
</table>
3. Think about your reading rate. Estimate how long it will take you to finish the book you are currently reading. Explain how you arrived at your prediction. All students will estimate how fast they are reading their current book. After they divide the number of remaining pages by the reading rate per page, they can estimate how many minutes of reading are left. Be sure to emphasize that not only do people read at different rates in different books, but that readers can choose to vary their rates. Faster is not necessarily better!
COMPARING MODES OF TRAVEL
LINEAR MODELING

1. Draw four stock/flow maps to show distance traveled by these four modes: airplane, train, automobile, and truck.

   ![Distance traveled diagram]

   vehicle traveling

2. Build a STELLA model of all four and assume the following rates of travel:

<table>
<thead>
<tr>
<th>TYPE OF VEHICLE</th>
<th>AVERAGE MILES PER HOUR</th>
</tr>
</thead>
<tbody>
<tr>
<td>airplane</td>
<td>300</td>
</tr>
<tr>
<td>train</td>
<td>50</td>
</tr>
<tr>
<td>automobile</td>
<td>45</td>
</tr>
<tr>
<td>truck</td>
<td>40</td>
</tr>
</tbody>
</table>

3. Run your model for six hours and graph the vehicles on the same screen. How many miles would each cover in six hours?
   - airplane = 1,800 miles
   - train = 300 miles
   - automobile = 270 miles
   - truck = 240 miles

4. Run the model long enough so that each vehicle travels 600 miles. How many hours does it take for each to cover the distance?
   - airplane = 2 hours
   - train = 12 hours
   - automobile = 13.5 hours
   - truck = 15 hours

5. Your company wants to send a 10 pound package from suburban Boston to an office in downtown Atlanta, Georgia. The distance is about 926 miles. You need to decide how to send the package. How long will each method of travel take?
   - airplane = 3.2 hours (airport to airport)
   - train = 18.5 hours (terminal to terminal)
   - automobile = 20.5 hours
   - truck = 23.25 hours

   What else do you need to know?

   What are the advantages and disadvantages of each type of travel?

   With your team, choose the method you favor. Create a proposal to persuade the company decision makers to take your advice.

   Students will have to consider factors other than speed of the vehicles: cost, how far the destination may be from the airport and rail terminal, how crucial is fast delivery in this case, etc. In their proposal have them make up information about the situation to support their plan.
RIDING THE MBTA RED LINE

Teddy and Phillipa often take the “T” from Alewife to Park Street. One day they kept track of how many passengers rode in their subway car. They split up the job and counted people getting on and off at each station. After Porter Square, they stopped calculating the totals so they wouldn’t lose count of people getting on and off.

<table>
<thead>
<tr>
<th>STATION</th>
<th>PASSENGERS ARRIVING ON THE TRAIN</th>
<th>RIDERS ENTERING CAR</th>
<th>RIDERS EXITING CAR</th>
<th>PASSENGERS DEPARTING ON TRAIN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alewife</td>
<td>0</td>
<td>16</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>Davis</td>
<td>16</td>
<td>12</td>
<td>3</td>
<td>25</td>
</tr>
<tr>
<td>Porter</td>
<td>25</td>
<td>10</td>
<td>8</td>
<td>27</td>
</tr>
<tr>
<td>Harvard</td>
<td>27</td>
<td>18</td>
<td>10</td>
<td>35</td>
</tr>
<tr>
<td>Central</td>
<td>35</td>
<td>12</td>
<td>14</td>
<td>33</td>
</tr>
<tr>
<td>Kendall/MIT</td>
<td>33</td>
<td>9</td>
<td>15</td>
<td>27</td>
</tr>
<tr>
<td>Charles/MGH</td>
<td>27</td>
<td>9</td>
<td>12</td>
<td>24</td>
</tr>
<tr>
<td>Park Street</td>
<td>24</td>
<td>17</td>
<td>13</td>
<td>28</td>
</tr>
</tbody>
</table>

1. Identify the flow(s) in this exercise.
The riders entering the car are an inflow, and the riders existing the car are an outflow.

2. Identify the stock(s).
Passengers on the train

3. Draw a stock/flow map of this situation.

   **Passengers on the train**

   \[\text{riders entering} \rightarrow \text{passengers on the train} \rightarrow \text{riders exiting}\]

4. Complete the table, filling in the boxes above that Teddy and Phillipa left blank.

5. Graph the number of passengers in the subway car on a behavior-over-time graph.

6. Using a different color on the same pad, graph the number of riders getting on.

7. Using a third color, graph the number of riders getting off.
### GASOLINE PRICES AND THE MBTA RED LINE

<table>
<thead>
<tr>
<th>STATION</th>
<th>PASSENGERS ENTERING STATION</th>
<th>20% INCREASE</th>
<th>PEOPLE LIKELY TO ENTER TEDDY AND PHILLIPA’S CAR IN MAY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alewife</td>
<td>9,567</td>
<td>11,480</td>
<td>14</td>
</tr>
<tr>
<td>Davis</td>
<td>10,891</td>
<td>13,069</td>
<td>19</td>
</tr>
<tr>
<td>Porter</td>
<td>8,089</td>
<td>9,707</td>
<td>12</td>
</tr>
<tr>
<td>Harvard</td>
<td>20,212</td>
<td>24,254</td>
<td>22</td>
</tr>
<tr>
<td>Central</td>
<td>11,736</td>
<td>14,083</td>
<td>14</td>
</tr>
<tr>
<td>Kendall/MIT</td>
<td>11,214</td>
<td>13,456</td>
<td>11</td>
</tr>
<tr>
<td>Charles/ MGH</td>
<td>7,855</td>
<td>9,426</td>
<td>11</td>
</tr>
</tbody>
</table>

The table above shows an average daily count from the turnstiles at seven Red Line stations during the middle of the decade beginning in 2001. In 2008, gasoline prices rose steadily during the first half of the year, climbing to more than $4 per gallon. By May 2008, the MBTA recorded an average increase of 20% ridership on the subways.

1. Why do you think more people rode the “T”?
   People wanted to use less gasoline so they took public transportation. The cost of driving had gone up so it made the cost of the “T” more attractive.

2. Fill in the column in the table with the heading “20% increase.”
   Students can multiply “Passengers entering station” by 1.2 to calculate increased ridership.

3. Graph the data from both columns on the same behavior-over-time graph. Use a different color for each line.
   If students draw line graphs, they are graphing the flow of passengers onto trains at each station. There would be two parallel lines. Graphs could also be bar graphs.

4. Using your last problem sheet (“Riding the MBTA Red Line”), fill in the column headed “People likely to enter Teddy and Phillipa’s car in May.” Explain how you did the calculations.
   Students might multiply the numbers in the column “Riders entering car” from their previous work by 1.2 to calculate a likely increase of 20%.
   Others may multiply the new data on this page in the column “20% Increase” by some small factor like 0.001 to get a typical number of riders.

5. Draw a connection circle to show how gasoline prices and ridership on the “T” may be related. Add other important elements you think might affect how many people ride the subway.

See Connection Circle suggestions in *The Shape of Change*. 
EXTENDING THE **MAKING FRIENDS GAME**

1. Draw a stock/flow map of the way the *Making Friends* Game was played the first time. Remember that each member of the team chose a new friend one time only.

   ![Diagram](image)

2. Make a STELLA model on your computer and simulate the game as you did in class. Have the computer graph the Friends team. Copy that graph here.
   
   The graph will be a straight diagonal line.

3. How many turns did it take until all the students in the class were on the team?
   
   Answers will depend on how many friends were added each turn and how many students are in the class.

4. Imagine a school with 850 students. Simulate playing the *Making Friends* Game with everyone. How many turns would it take until they were all on the team?
   
   A lot! 850 divided by the # of new friends each turn.

5. Draw a stock/flow of the way the *Making Friends* Game was played the second time. Remember that each person on the team chose a new friend every round.

   ![Diagram](image)

   The arrow is a connector indicating that the number of new friends added each turn equals the number of friends already in the stock.

6. Make a STELLA model on your computer of the second version of the game and simulate how it was played in class. Copy the graph here.
   
   This graph will grow exponentially because the number of friends doubles each round.

7. Use your model of *Making Friends* Game 2 to learn how many turns it would take to bring all the people in #7 - #9 onto the Friends team. Start with two friends.
   
   - Town of Carlisle, about 5,000 people: About 11 turns (4,096 friends)
   - State of Massachusetts, about 6,000,000 people: 22 turns
   - United States of America, about 300,000,000 people: 28 turns
PAYING FOR PARKING

1. Bruno’s Garage is open 24 hours per day. Some people park there while they work or shop during the day and others park in the evening when they go out or visit friends. Some people park overnight. Bruno charges by the hour. Draw a stock/flow map with one flow to show how the parking charge accumulates.

   ![Stock/flow diagram]

   Total due for parking

   ![Flow diagram]

   hourly fee for parking

2. Bruno’s Garage has two different rates for parking. Between 9 AM and 3 PM, the hourly rate is three dollars. At all other times, the rate is two dollars per hour. Cindy wants to compute how much it will cost her to park at Bruno’s from 7 AM to 2 PM one day. What will she have to pay?

   Total = $19
   7 – 9 AM = $2/hour * 2 hours = $4
   9 AM – 2 PM = $3/hour * 5 hours = $15

3. Build a STELLA model on your computer to model Cindy’s parking fee. To use only one flow, you will need to change the rate during the time the model runs. To do this, you can use the “STEP” function in STELLA.

   Double click on the flow and enter the starting rate Cindy pays. Then add a “+” and find the list of built-in functions.

   Click on “STEP” and you will see parentheses to fill in. The first number needs to be the amount by which the rate changes.

   • How much does the rate change?
   Increases $1. Because there is a plus before the “STEP” function, a positive sign in the parentheses is not necessary.

   The second number in the parentheses needs to tell the computer when to change the rate.

   • After how many hours of parking does Cindy begin to pay the changed rate?
   After 2 hours. The flow would be written: 2+STEP(1,2)

4. Run the model and graph the stock and the flow. Draw them here and explain the behavior.

   Note – Unless students know how to set the scales on the graph to be the same, the graphs might be run one at a time to avoid confusion.

   Hint: When you use “STEP” in a flow, think of the phrase “behavior over time” to remember how to enter numbers in the parentheses. The first number tells how much the behavior of the flow will change. The second tells the time of the change, measured from the beginning of the run.
SHOVELING THE DRIVEWAY

Lisa’s family has a long driveway that measures 72 meters long from the road to her house. When it snows fewer than three inches, Lisa and her younger brother Bernard clear it off. Lisa can clear 1.2 meters per minute. Bernard can clear 0.8 meters per minute.

1. Draw a stock/flow map to represent this situation.

   Snow removed from driveway

   Lisa removing snow

   Bernard removing snow

Students may represent the stock as “Snow on driveway.” The flows would then have to be outflows.

2. How long would it take Lisa to clear the driveway if she worked alone?
   60 minutes

3. How long would it take Bernard if he worked alone?
   90 minutes

4. How long would it take them when they work together? You may use a STELLA model to solve this problem.
   36 minutes

5. One day, Bernard needed to finish his homework before helping Lisa shovel the driveway. It took Bernard 32 minutes to complete his homework and get out to the driveway job. How long did it take them to finish? You may use a STELLA model.
   49 minutes

You probably used the “STEP” function if you modeled this problem on your computer. Write the step function you used to represent the flow in your STELLA model.

0+STEP(0.8,32) This means Bernard had a snow removal rate of 0 until minute 32 when he finished his homework and his rate increased to 0.8 meters per minute.