The Mammoth Game:  
*The Shape of Change*

The text of
Lesson 3: The Mammoth Game
From the books

*The Shape of Change*
and

*The Shape of Change: Stocks and Flows*

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Illustrated by Nathan Walker
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Introduction

In this activity, teams of students play a dice and graphing game to track the population growth and decline of a herd of twenty mammoths. By changing variables and probabilities with the dice, students can explore theories of extinction and speculate about which factors contributed to the wooly mammoths’ demise. Potential interdisciplinary links include science topics such as extinctions and population rates, and social studies investigation of ice age cultures. Math concepts include graphing, probability, percentages and fractions, and exponential decay.

Materials

- 20 dice per team (plus a few extra for the teacher)
- Cardboard boxes for dice rolling
- Markers of the same two colors for each student
- One copy of the Mammoth Game Rules for each team (page 9)
- Copies of two worksheets for each student: Keeping Track of Your Herd (page 11) Graph of Your Mammoth Population (page 12)

How It Works

Scientists believe wooly mammoths were once plentiful on the North American continent but became extinct about 11,000 years ago. Opinions vary as to the cause of their demise. Was the warming climate responsible, or was an as yet undiscovered disease the primary culprit? Did predators hunt mammoths to extinction?

Although scientists have not reached consensus, most agree that the arrival of a significant number of humans put more pressure on a mammoth population already stressed by a warming climate. Skillful human hunters may have reduced the already vulnerable mammoths to numbers that spiraled to extinction.

Students simulate the effect of human hunting upon a declining population by playing two versions of the Mammoth Game. One version will track the mammoth population without human hunting and graph the extinction curve. The second game will add people as a factor and students will see the rate of extinction increase. Displaying and comparing the graphs helps students see the patterns of behavior more clearly.

This is a simulation. We want to understand why mammoths went extinct, but because we cannot study real mammoths in the classroom, we will use dice to represent them.
Procedure
1. Generate a list of mammoth extinction theories with students. This conversation in class can be very rich.

2. Tell students that they will pretend to track the population of a herd of twenty mammoths over time. Then, they will graph the population.

3. Distribute 20 dice per team of students. Each die will represent one mammoth, so the starting population is 20. Students record this on the Keeping Track of Your Herd worksheet (page 11) under Game 1. (Keep a few extra dice available in case a herd population rises above 20 during the game.)

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<th>Year</th>
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4. Give one copy of the Mammoth Game Rules (page 10) to each team or use an overhead projector to explain the rules. Each time the set of dice is rolled, one simulated year goes by. The number on each die determines the fate of the individual mammoth it represents. To begin play, shake and roll all the dice into the cardboard box. Sort the dice using the rules in the box below.

**Rules for Game 1**
1 = a calf is born
2 = the mammoth is killed by a predator
3 = the mammoth dies of starvation
4 = the mammoth keeps living another year
5 = the mammoth keeps living another year
6 = the mammoth keeps living another year

5. Accuracy is very important, so spend enough time establishing procedures. Each student should track population on his or her own table, but all team members should agree on the numbers.
   - Sort, count and record the number of mammoths remaining after the first year.
   - For the second year, roll the dice again, using only those mammoths that survived the first year plus any new calves. Record the results.
   - Play and record for 20 “years” or until the mammoths become extinct.

The Mammoth Game 3
Note: Because they want the mammoths to survive, younger students may be tempted to cheat and change the dice results. Explain that this is a simulation, not a contest. The object is similar to a science experiment – if you create certain conditions, what is the result?

6. Depending on the age of the students and your classroom routines, you can either assign jobs to team members or let them choose tasks among themselves. For example, one student might remove dead mammoths, another adds new calves, a third is the official counter, and so on. Rotate jobs to involve everyone. Cooperative team learning works best when students understand what their roles are.

7. After students finish playing the game, ask them to plot the results on the *Graph of Your Mammoth Population* worksheet (page 12).
   - Each student should graph the data using the *same color* pencil or marker.
   - Graphing can be difficult for younger students, so be sure they are plotting points correctly before connecting them to make a line graph.
   - It works best to play the game *first* and then draw the graph.

Students draw *line graphs* so that they can more easily discern patterns of behavior over time.
Bringing the Lesson Home

Post one graph from each team on the wall for easy comparison and discussion. Questions like these will arise. Help students use the game to build critical thinking skills and deeper understanding.

? What do the graphs tell us about what happened to the mammoths?
   All of the herds went extinct.

? When did your mammoths go extinct? Why did that happen?
   These are brainstorming questions that stimulate student thinking. In Game 1, predators and starvation caused deaths. There could have been other causes too, such as disease.

? If some baby mammoths were born each year, why did the population still decline?
   The population declined because more mammoths were dying than being born each year.

? What is the general pattern of the graphs? Depending on the level of the students: What is the rate of change? What is the slope?
   The graphs show a steep downward curve at first that levels out as the mammoths approach extinction. Older students can discuss slope. Younger students use descriptions like "steeper" and "flatter" to describe the rates of change.

? What is similar about all the graphs? What is different? Why?
   All the graphs decline in the same general pattern. The lines vary somewhat because the dice rolled differently for different teams; in real life, different herds could have different luck too – bad weather, less food, illness, etc.

? Why is the line curved? What does the curved line say about what was happening to the population? Why is the line steeper in some places?
   The line is steeper at first because there were more mammoths to die at the beginning. As the herd shrank, the death rate applied to fewer and fewer animals until there were none left. This pattern is called exponential decay. The line is curved because the number of deaths varied, depending on the number of mammoths left.

? At what point was the herd half its original size?
   The half-life is about 4 years.

? Would the animals still become extinct if you started with 100 mammoths?
   The size of the herd would not affect the general pattern. Under the same death rate, the herd would be half its size by the same time and extinct by the same time.
Playing Game 2

1. Change one of the “the mammoth keeps living another year” fates to “the mammoth is killed by a hunter.” Introducing human hunters into the game allows players to compare what happens to the mammoth population when hunting pressure is applied.

<table>
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<th>Rules for Game 2</th>
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<tr>
<td>1 = a calf is born</td>
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<td>2 = the mammoth is killed by a predator</td>
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<td>3 = the mammoth dies of starvation</td>
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<td>4 = the mammoth is killed by a hunter</td>
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<td>5 = the mammoth keeps living another year</td>
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2. Ask students to predict what might happen in Game 2 and explain their reasoning. It does not matter if a prediction is wrong or right. Reflections help students reflect on their thinking as the game progresses.

? **How many dice numbers represent deaths in Game 2?**

*The death fraction in Game 2 is 3/6 or 1/2. In Game 1 it was 2/6 or 1/3. On average, when the dice are rolled, a larger fraction of the herd will die each year in Game 2.*

3. Play game two following the new rules. Graph the results in a second color on the same graph used for Game 1, as shown on the next page.
The Mammoth Game - Sample Student Graph

Bringing the Lesson Home

As before, post the student graphs for easy comparison and discussion prompted by questions like these.

? What happened this time? When did your mammoths go extinct? Is this what you predicted? Why?

*Mammoths went extinct even more quickly than before because more mammoths were dying each year and not enough babies were being born.*

? Is there a general pattern again? Why is it steeper in some places than others?

Again, the graphs show a step downward curve that levels off as the mammoths approach extinction. The population decreased at a faster rate at the beginning because the death rate applied to more mammoths at first. By the end, there were fewer mammoths left to die.
Why is the line curved? Why isn’t the line straight?
The line is curved because the number of deaths varied depending on how many mammoths were left. The line would be straight if a constant number of mammoths were born and died every year.

How are the lines for Game 1 and Game 2 alike? How are they different?
Students may use words such as steep, flat, and slope to describe the lines. Explore these concepts. Be sure to relate the shape of the line to the rate of population decline. Both lines show exponential decay, but Game 2 had a higher death rate, so the mammoths died off more quickly.

What difference did the hunters make?
Broaden the discussion to explore extinction theories. Why might one herd survive somewhat longer than another? It is likely that herds faced different conditions. In earlier times, mammoths were able to rebound after various disasters. Could the new human hunters have been enough of a threat to mammoths to push them to extinction?

Encourage students to step back and take a broader look. What does the Mammoth Game tell us about how a population changes?

What makes a population decline?
Deaths exceed births.

What makes a population grow?
Births exceed deaths.

Can a population stay the same?
Yes, if births and deaths are equal.

Does this happen only to mammoths? Can you think of other cases?
The same principles apply to all populations.
- Populations of bacteria in a test tube
- Populations of fish in a pond or deer in a forest
- Populations of people in the world or in a country or town (including migration.)
Feedback

Births cause a population to grow larger. A larger population results in even more births, causing the population to grow even larger, and so on. At the same time, deaths cause a population to decrease, and a smaller population results in fewer deaths. These are called feedback loops.

Notes

1 The Mammoth Game was adapted from the teacher’s guide to Newton’s Apple, Show Number 1509, Twin Cities Public Television, St Paul, MN, 1997.

For a simple system dynamics computer model of the Mammoth Game with complete instructions for using it with students in the classroom, see “The Mammoth Extinction Game” by Stamell, Ticotsky, Quaden and Lyneis, 1999, available through the Creative Learning Exchange at www.clexchange.org.


*The Call of Distant Mammoths*, by Peter D. Ward (Copernicus, 1997) explores theories of mammoth extinction and relates them to modern species – an excellent resource for adults.
MAMMOTH GAME
RULES

1. Each die represents one mammoth.

2. Each roll of the dice represents one year.

3. Roll all the dice at once into the box. The numbers on the dice will tell you what happened to each mammoth.

GAME 1
1 = A calf is born
2 = The mammoth is killed by a predator
3 = The mammoth dies of starvation
4 = The mammoth keeps living another year
5 = The mammoth keeps living another year
6 = The mammoth keeps living another year

4. Do what the numbers tell you to do:
   If a calf is born, add one die to the herd.
   If a mammoth dies, remove that die from the herd.
   If a mammoth keeps living, just leave that die in the game for the next round.

5. Continue to play for 20 years (20 rounds.) Record how many mammoths are in your herd at the end of each year on your Keeping Track of your Herd worksheet.

6. Change the rules for Game 2.

GAME 2
1 = A calf is born
2 = The mammoth is killed by a predator
3 = The mammoth dies of starvation
4 = The mammoth is killed by a human hunter
5 = The mammoth keeps living another year
6 = The mammoth keeps living another year
**KEEPING TRACK OF YOUR HERD**

Record the number of mammoths remaining in your herd after each year.

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<thead>
<tr>
<th>Year</th>
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GRAPH OF YOUR MAMMOTH POPULATION

Mammoths in Herd

Time (Years)
The Mammoth Game

This lesson builds on the classroom activities described in *The Shape of Change*, by Rob Quaden, Alan Ticotsky and Debra Lyneis, 2004, The Creative Learning Exchange. You can download the text of the original single lesson or get the graphics and layout in the complete book from the CLE at www.clexchange.org.

The Shape of Change

In Lesson 3 of *The Shape of Change*, students explored theories of extinction as they played a dice game and graphed the population growth and decline of their herds of mammoths. See Pages 27-38 in *The Shape of Change* for the complete lesson.

Overview

Building a stock/flow map of the Mammoth Game adds new elements beyond the map of the Making Friends game in the previous lesson. Making Friends is based on reinforcing feedback which results in exponential growth. In addition to producing reinforcing feedback, the Mammoth Game also produces balancing feedback which results in exponential decay.

Seeing the structure

1. After playing the Mammoth Game, develop a stock/flow map with the students. Ask students to name the stock, the quantity that can increase and decrease over time. Students should have no difficulty suggesting mammoths.

   ![Stock/Flow Map]

   Mammoths

2. Ask students to describe what happened to the mammoths in the game and how that can be shown on the map. With each roll of the dice, new mammoths were born and old mammoths died. Again, students should have no difficulty explaining that the stock of mammoths was increased by births and decreased by deaths each year.

   ![Stock/Flow Map]

   Births

   Mammoths

   Deaths
(The “clouds” signify the boundaries of the system. We are interested in how the population changed with births and deaths. We are not concerned with the mammoths before they were born or after they died.)

*Note: Think of a bathtub again. The accumulation of water in the tub changes as water flows in through the faucet and out through the drain. The size of the mammoth population is controlled by the flows of births and deaths.*

3. While this map is a good start, note that it implies a constant flow of births and a constant flow of deaths. However, this is not what happened in the game; the flow of births was determined, in part, by the number of mammoths. It might be helpful to give an example from the game: when you have 20 dice, there will be more rolls of “one,” on average, than when you have only 10 dice. Or, thinking about mammoths, there will be a greater number of baby mammoths born each year in a large herd than in a small herd. Ask students how to represent this on the stock flow map.

(Notice that this is like the structure of Making Friends when the number of friends already on the team influenced the number of new friends added each round.)

Similar reasoning can be applied to the flow of deaths. The number of deaths each year also depended on the number of mammoths.
4. The stock of mammoths influencing the inflow of births is an example of a reinforcing loop leading to exponential growth as we saw in Making Friends. But the stock of mammoths did not increase in this game. In fact, it declined to extinction.

Examine the outflow of deaths more closely with students.

The feedback between the stock of mammoths and the outflow is an example of a balancing loop. With all else equal, an increase in the number of mammoths will lead to an increase in the number of deaths. However, this increase in the number of deaths will lead to a decrease in the number of mammoths. Use the arrow technique to make this more explicit.

Start by increasing the number of mammoths and show this by drawing an arrow pointing upward near the stock:

Ask students how this affects the number of deaths. An increase in the number of mammoths causes an increase in the number of deaths above what it would have been otherwise, a change in the same direction.
What happens to the stock next? The **increase** in deaths leads to a **decrease** in the number of mammoths below what it would have been. Add an arrow pointing downward:

![Diagram](image)

Ask students to add the next arrow. A **decrease** in the number of mammoths leads to a **decrease** in the number of deaths:

![Diagram](image)

This **decrease** in deaths in turn leads to a relative **increase** in the number of mammoths: the number of mammoths is higher than it would have been if the number of deaths had not changed because mammoths are dying at a slower rate.

![Diagram](image)

Notice that the arrows reverse during each “cycle.” No matter how long we repeat the process, the arrows will keep turning up and down, “balancing” back and forth.

*In a balancing feedback loop, any change reverses direction each time around the loop.*
5. Like the situation in Making Friends, the flow of births and the stock form a \textit{reinforcing loop}. An \textit{increase} in the stock of mammoths leads to an \textit{increase} of the inflow, and a \textit{decrease} in the stock leads to a \textit{decrease} of the inflow. In other words, the change from the stock to the inflow is in the same direction.

However, in the Mammoth Game, the flow of deaths and the stock form a \textit{balancing loop}. All else equal, an \textit{increase} in the outflow of deaths causes a \textit{decrease} in the number of mammoths, while a \textit{decrease} in the outflow causes a relative \textit{increase} in the stock. As we saw using the arrow technique, any change reverses direction each time around the loop.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{mammoth_diagram.png}
\caption{A balancing loop results in graphs that decline at a decreasing rate, a behavior called \textit{exponential decay}. In this case, the graph shows that the number of mammoths approached zero – they became extinct. (Balancing loops can also cause stocks to grow toward a goal; more on that later.)}
\end{figure}
6. Refer back to the stock/flow map and ask students to explain the difference between the two mammoth games in terms of the stock/flow map. All the elements of the first map are needed in a map for the second game. However, in the second game there were more ways for the mammoths to die. Put another way, the fraction of mammoths that died was higher. This can be reflected in the map by adding the Death Fraction.

![Stock Flow Map]

The death fraction shows in a more precise way that the number of deaths is influenced by more than the stock of mammoths. It depends on the number of mammoths and on the fraction of mammoths that dies every year.

? **How does this explain the similarity as well as the difference between the two mammoth games.**

*In the first game, 2 out of 6 mammoths died each year on average; in the second game with the addition of human hunters, 3 out of 6 mammoths died each year. Both games exhibit exponential decline because there were more deaths than births each year, but in the second game, the fraction of deaths was larger, resulting in a quicker decline of the population.*

7. Students should see that the symmetry between births and deaths can also be extended to the birth and death fractions. The flow of births is influenced by the stock of mammoths as well as a birth fraction. In the case of the game, that fraction was 1/6.

The birth fraction represents factors other than total population that influence the birth rate, like the percentage of the population that is female, the percentage of females of child-bearing age, etc. The higher the birth fraction, the more births.
This map is a complete mental model of the Mammoth Game. The stock of mammoths is changed over time by flows of births and deaths each year. The flows in turn are influenced by the stock. This sets up two feedback loops: one reinforcing loop and one balancing loop. It is the relative strength of the two loops that determines the fate of the population over time.

The causal loop on the right tells us that an increase in mammoth births causes a relative increase in the mammoth population, which further increases the number of births—changes in the same direction labeled “S.” This is a reinforcing feedback loop much like Making Friends; alone, it would cause exponential growth.

Meanwhile, an increase in the population causes an increase in the number of deaths. This increase then leads to a relative decrease in the population—a change in the opposite direction labeled “O.” This is a balancing feedback loop causing exponential decay. Both loops influence the population simultaneously.
What causes the mammoth population to increase?
If the birth fraction is higher than the death fraction, then there will be more births than deaths each year. The reinforcing feedback loop will dominate and the population will grow exponentially.

What causes the mammoth population to decrease?
If the death fraction is higher than the birth fraction, then there will be more deaths than births each year. The balancing feedback loop will dominate and the population will decay exponentially approaching zero.

Can the population stay the same size?
Yes, a population can remain stable at a constant level if the birth fraction equals the death fraction at any value. The total population does not change because an equal number of mammoths are born and die each year.

Does this map apply only to mammoth populations?
These principles apply to all populations.

How does the stock/flow map give us a better understanding of how a population changes.
The stock/flow map shows us that any population is an accumulation over time that is increased by births and decreased by deaths. Since a stock can be changed only by its flows, understanding how a population changes means taking a closer look at the birth and death flows (and migration flows in some populations). In the Mammoth Game we saw that increasing the outflow of deaths caused the population to decline more rapidly. Other things being equal, decreasing the inflow of births would also cause the population to decline.

Different birth and death rates would play out simultaneously to cause a population to grow, decline or stabilize over time. Seeing the structure helps us to understand the behavior we observe.

ENDNOTE
1 These relationships can be quantified and simulated in a system dynamics computer model. Students can experiment with different birth and death rates to observe how the mammoth population would change over time under varying conditions. For a simple system dynamics computer model of the Mammoth Game with complete instructions for using it with students, see “The Mammoth Extinction Game” by Stamell, Ticotsky, Quaden and Lyneis (1999), available for free from the Creative Learning Exchange at www.clexchange.org.