A Cooling Cup of Coffee:
An Introduction to
Constant Outflow and Negative Feedback

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BY

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<u>A Cooling Cup of Coffee</u>: An Introduction to Constant Outflow and Negative Feedback

by

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Abstract

One can visualize and comprehend a one-level constant outflow system, but in reality, few systems like these exist. *A Cooling Cup of Coffee: An Introduction to Constant Outflow and Negative Feedback* illustrates that a cup of hot coffee does not cool at a constant rate of outflow. A cooling cup of coffee system contains negative feedback and tries to reach an equilibrium. The coffee cooling rate is dependent on both the coffee temperature and the room temperature. The difference in temperatures causes the heat from the coffee to flow into the atmosphere, thus, cooling the coffee. The following STELLA[2] models and simulations show that a one-level constant outflow system is not a correct representation of reality, while the revised system containing negative feedback is more realistic.

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[2] STELLA (Systems Thinking, Experiential Learning Laboratory, with Animation) is a registered trademark of High Performance Systems, Inc.
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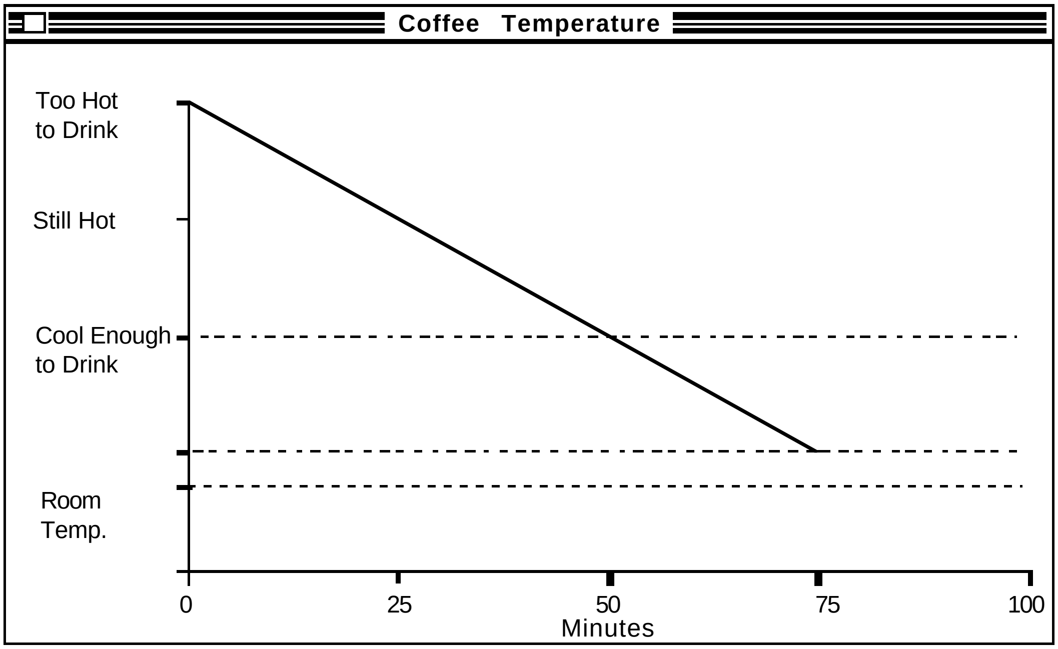
Introduction

Imagine a hot cup of coffee sitting on a kitchen table. After waiting a few minutes, it cools enough for one to drink it. However, many probably cannot explain for certain how or why cooling occurs. Why does the coffee cool? Does it cool constantly, a little bit each minute, or does the temperature follow an asymptotic decreasing pattern?

A Cooling Cup of Coffee: An Introduction to Constant Outflow and Negative Feedback explains what really happens to this hot cup of coffee. This coffee example introduces the ideas of constant rates, negative feedback, and stabilizing behavior to facilitate an understanding of some of the fundamental concepts of system dynamics.

Misconceptions About Constant Rates

Let's examine what happens to the cup of hot coffee while we wait for it to cool. Some people will automatically answer, "Well, the coffee temperature cools by a few degrees each minute and then you drink it." This answer is expressed in Figure #1. People are aware that the coffee is very hot, and that the coffee temperature is much higher than the room temperature. This causes the coffee to cool a little each minute until it reaches a drinkable temperature.



Figure#1 In this graph, a cup of coffee starts off at a very high temperature and cools for 7.5 minutes. At that point, the coffee is very cool and is consumed. This graph of temperature is one way of thinking about the coffee system. A better way, however, is to model in terms of heat.

Although many tend to think of the "hotness" of coffee in terms of temperature as in the graph above, the correct way to measure "hotness" is actually in terms of heat. Temperature is a *measure* of heat, and also the size and physical nature of the body containing the heat. However, the coffee loses heat, not temperature, because heat can flow from a body of high temperature to a body of low temperature.[3] Temperature can not physically flow. Figure #2 represents the model of our cooling cup of coffee in terms of heat.

What would happen to a cup of coffee, cooling at a constant rate, if you left it on the table and forgot about it for a few hours? Figure #3 represents this constant rate cooling cup of coffee system.

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[3] Michael Shayne Gary. "D-4272-7 Mistakes and Misunderstandings: Examining Dimensional Inconsistency." 1992. System Dynamics in Education Project. MIT. 1 Amherst St. Cambridge, MA.

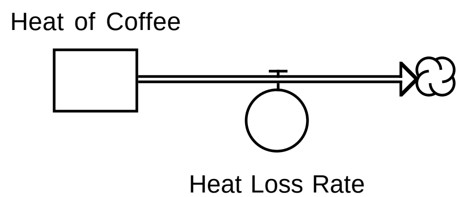


Figure #2 This STELLA II model represents a cooling cup of coffee that loses a constant amount of heat each minute. The coffee loses heat and not temperature because heat can physically flow while temperature cannot. Temperature depends on the amount of heat and the nature of the body containing the heat.

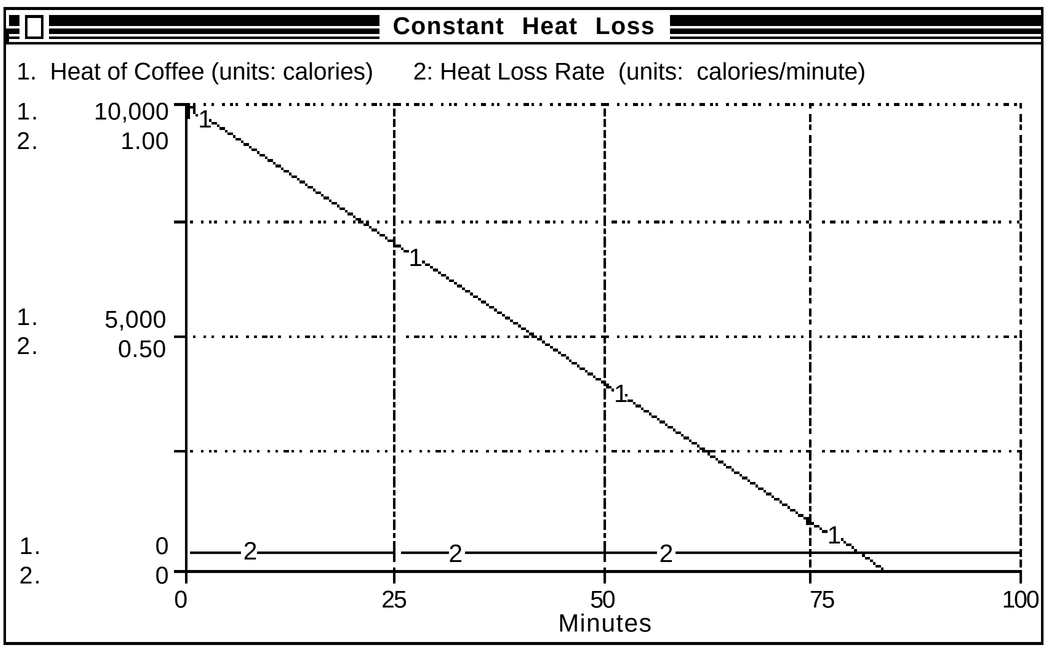


Figure #3 In this graph, the number one line represents the amount of heat in the coffee. The coffee loses a constant number of calories a minute. The heat outflow is a constant rate represented by the number two line.

Because the oversimplified model shown above contains a constant **Heat Loss Rate**, without anything to discontinue the cooling, the **Heat of the Coffee** will continue to lose heat even after it has reached room temperature. See Figure #3. In this model, there is no connector between **Heat of Coffee** and **Heat Loss Rate**. Therefore, when the heat in the coffee is equal to the heat at room temperature, there is nothing in the model to tell the **Heat Loss Rate** to stop. It is intuitive that a cooling cup of coffee does reach an equilibrium at room temperature. Thus, the previous model is lacking something. The difference in the coffee and room temperatures must affect the rate, and these factors are missing from the model. The model shown in Figure #2 may seem correct, but it is not.

We know that the previous model is not complete, not only because it does not make sense intuitively, but also because it defies laws of thermodynamics. If we allow the model to continue running, it would continue to remove heat from the coffee even after it reaches the freezing point of water. From personal experience, we know that leaving a hot cup of coffee in room temperature cannot result in a frozen cup. Furthermore, as the model continues to run, the coffee would reach a point called *absolute zero*. Absolute zero may be more commonly known as zero degrees Kelvin, or -273 degrees Celsius. Temperatures cannot fall below this point, because this is when all heat has been removed.

These problems arise because the coffee cup model with constant heat loss is not a complete representation of what actually occurs. In reality, the difference between the coffee temperature and room temperature causes heat to flow from the coffee to the atmosphere. Thus, we must create a new model that includes these factors.

Negative Feedback Loops

Nothing in life increases or decreases at the same rate forever; most things tend to move toward an equilibrium. Negative feedback loops generate this stabilizing or goal-seeking behavior. In the previous example shown in Figure #2, the **Heat of Coffee** was lowered by the **Heat Loss Rate**, yet the rate remained constant. This relationship contains no feedback. The Stella causal diagram in Figure #4 below shows the interconnections between the level and the rate. As the **Heat of Coffee** goes down, the **Heat Loss Rate** also goes down. This is represented by the plus sign, showing that both move in the same direction. As the rate goes down, the **Heat of Coffee** lowers, *but to a lesser degree*. In other words, the slope of the **Heat of Coffee** gets flatter.

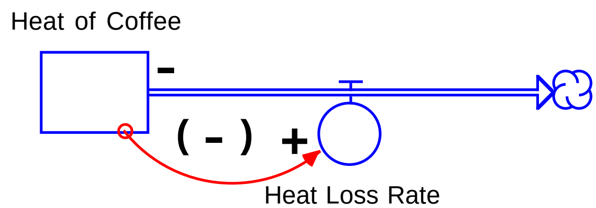


Figure #4 This is a STELLA causal loop diagram. This figure represents the structure that produces the stabilizing behavior that works towards equilibrium. The rate is dependent on the Heat of Coffee. Now, the Heat of Coffee cannot fall below the equilibrium point.

A More Realistic Model

The model in Figure #2 was very simple, with one stock and one outflow. However, we saw that the constant rate model was too simplistic, and thus it was incorrect. Most things and actions in this world are affected or depend on other factors. Figure #4 shows **Heat of Coffee** affecting **Heat Loss Rate**, and vice versa. While Figure #4 is a better representation of reality than the constant outflow model, it does little to show exactly how **Heat of Coffee** affects the **Heat Loss Rate**. A more complete model is shown in Figure #5.

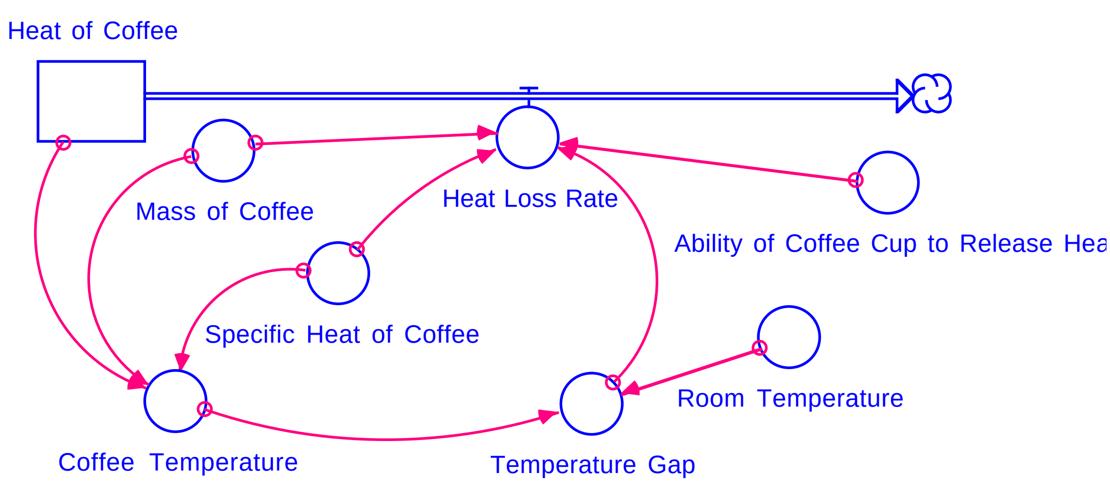


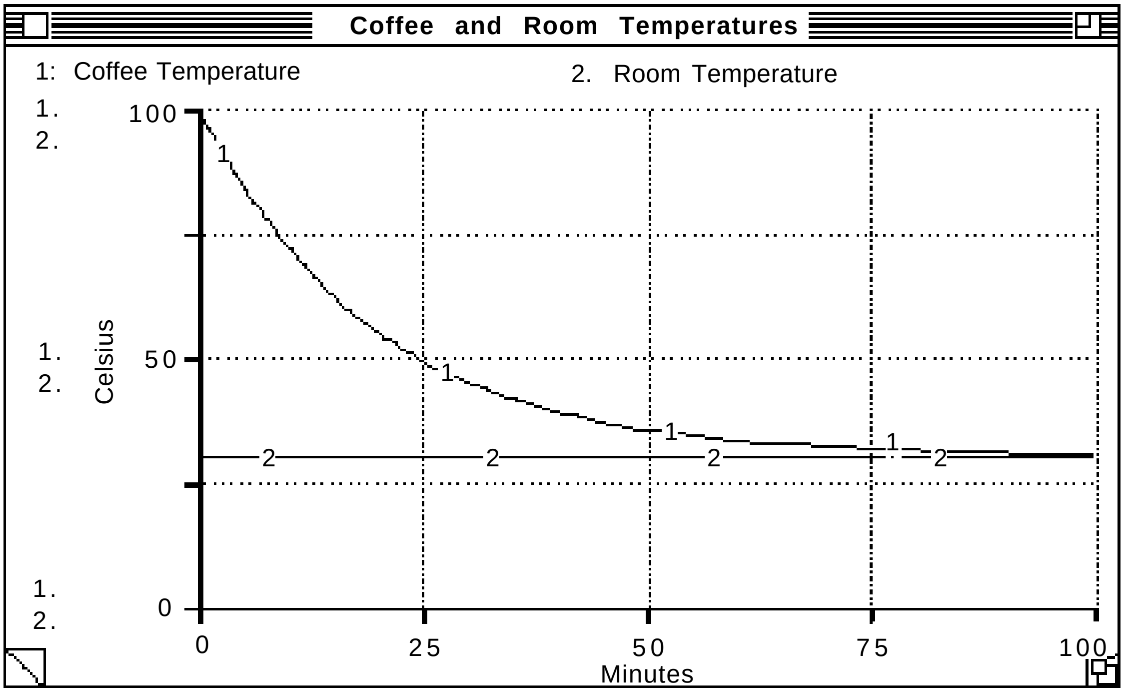
Figure #5 This is a revised and more realistic STELLA model which displays the negative feedback which is present in the real system. The Heat of Coffee and the Heat Loss Rate are the level and rate respectively. The Temperature Gap is the discrepancy in the model that drives the rate. The Coffee Temperature converts the heat units to temperature units. The Mass of Coffee, and Specific Heat of Coffee are converters that help translate the absolute measure of the Heat stock into its relative measure of Temperature. The Ability of Coffee Cup to Release Heat is another convertor.

Although this model is more complete, it still has only one level, **Heat of the Coffee.** This level is the number of calories in the coffee above those at zero degrees Celsius. In this model, the heat is initially at 10,000 calories. If there are 10,000 calories of heat in an enormous tank of coffee, the coffee would be rather cool. However, if the same number of calories were in a small coffee cup, the coffee would be much hotter. This is a hot cup of coffee.

The **Specific Heat** of the coffee is known as the amount of energy it takes to heat up one gram of water by one degree. In this case, the specific heat is equal to one.[4] The **Mass** has also been set equal to 100 grams.

In this model, the stock and flow are not independent of each other. The outflow, **Heat Loss Rate**, is driven by the discrepancy between the **Room Temperature** and the **Coffee Temperature**. The **Temperature Gap** causes the heat to flow from the coffee to the atmosphere, because heat tends to flow from high temperature to low temperature. In other words, the **Heat Loss Rate**, dependent on the **Temperature Gap**, lowers the **Heat of the Coffee** and therefore the **Coffee Temperature**. This causes the **Temperature Gap** to decrease because the coffee is now cooler, but not as cool as the room temperature. Although the heat still continues to drop, it does so at a lower rate. See Figure #6. The coffee temperature falls by a lesser amount each time period until it gradually reaches equilibrium at room temperature.

[4] Because coffee, is very close to water, the specific heat of the coffee is set equal to the specific heat of water. The specific heat of water is one.



Figure#6A In this graph, the temperature drops asymptotically until it reaches equilibrium at room temperature.

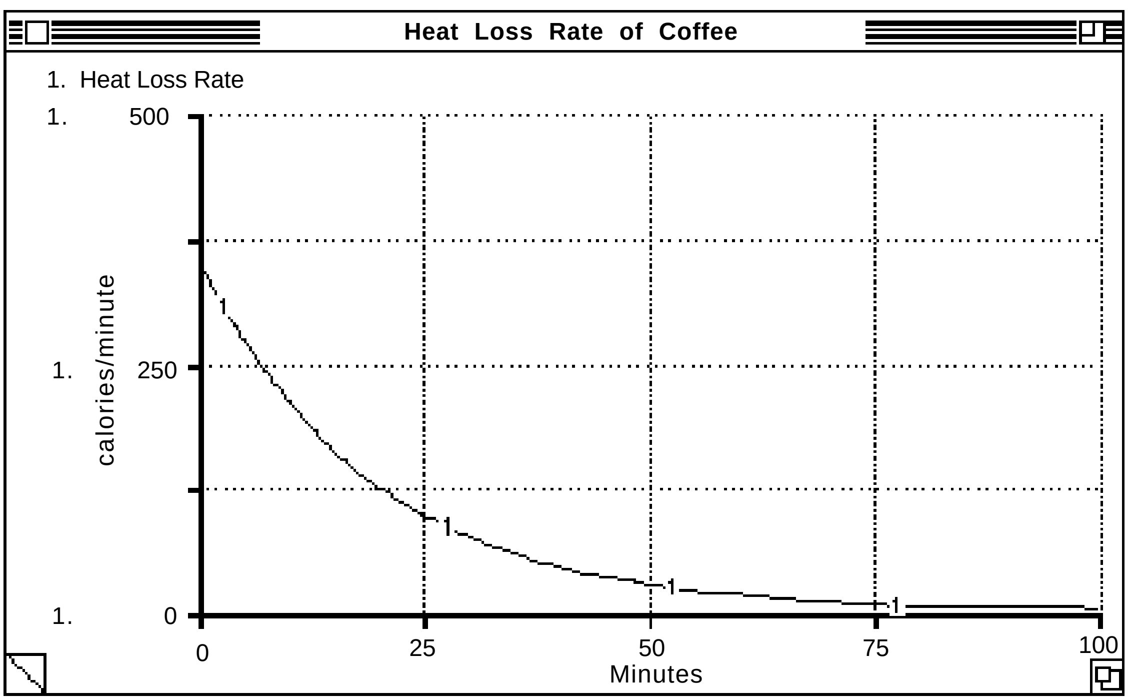


Figure #6B In this graph, the heat loss rate also drops asymptotically until the Heat of Coffee is in equilibrium with the heat in the atmosphere. The units calories / minute represent the number of calories lost each minute.

Because the **Temperature Gap** is the driving force of the **Heat Loss Rate**, the gap affects the rate the most when the **Coffee Temperature** is at its initial one hundred degrees Celsius. See Figure #6A. As shown in Figure #6B, the **Heat Loss Rate** is also the greatest initially.

The **Coffee Temperature** can be calculated through a simple conversion from heat to temperature. To derive temperature from **Heat of Coffee**, a conversion of units from calories to degrees is necessary. The equation in which temperature can be calculated is:

Temperature = Heat/ (Specific Heat*Mass)[5]

[5] Watkins, Emiliani, Chiaverina, Harper, LaHart. General Science. Harcourt Brace Jovanovich. Orlando, Florida. 1989. p. 223-5.

The units of the different factors in this equation are as follows:

<u>Temperature</u> : Celsius degrees

<u>Heat</u>: calories

<u>Mass:</u> grams

$$\textbf{Specific Heat}: \quad \frac{\text{calorie}}{\text{gram * Celsius degrees}}$$

Since checking dimensional consistency is a method of confirming an equation, the units work out as shown below:

$$\text{Celsius degrees} = \frac{\text{calorie}}{(\text{calorie}/(\text{gram* Celsius degrees})) * \text{gram}}$$

$$\text{Celsius degrees} = \text{Celsius degrees}$$

Another factor that affects the cooling rate of the coffee is the coffee cup itself. Some materials tend to retain more heat than others causing the coffee to cool slower. In addition, more heat will escape if the coffee cup has a large surface area exposed to the air, compared to the volume of the coffee. For example, two 100 gram cups of coffee are placed in the same room. Both cups are made of the same ceramic material. The first remains in the cup while the second is poured onto a plate. The second would cool faster, since more coffee is in direct contact with the cooler air. In this model, these factors are simplified into the converter called **Ability of Coffee Cup to Release Heat**. This coffee cup factor directly influences the rate of cooling. If this converter is equal to one, then the coffee cup releases the heat in the coffee very, very quickly. If the convertor is set equal to zero, no heat is released.

Conclusion

When considering a hot cup of coffee in a room at normal temperature, it is easy to believe that the coffee will cool. However, it is not quite as easy to discern how or why this behavior happens. Many tend to believe that the coffee temperature will drop at a constant rate by a few degrees each minute. Or in other words, they believe that the heat of the coffee has a constant outflow. This is incorrect, not only because the one-level constant outflow model goes into negative heat, which is an impossibility but also because intuitively, most things in the world contain negative feedback loops that seek an equilibrium. In the revised model, the heat in the coffee escapes at a rate that is dependent on the difference between the room and coffee temperatures. The negative feedback loop in this model caused the coffee temperature to reach its equilibrium at room temperature.

Supplementary Ideas

This negative feedback loop and the resulting behavior does not apply only to a cooling cup of coffee. A draining bathtub and the half-life of radioactive elements have the same underlying structure. One possible exercise to teach this concept would be to experiment by heating a tea kettle, and then letting it cool while measuring the temperature every few minutes. By plotting the recorded temperatures, the student can see the behavior of the cooling for himself.

Equations for Revised Model

Heat_of_Coffee(t) = Heat_of_Coffee(t - dt) + (- Heat_Loss_Rate) * dt
INIT Heat_of_Coffee = 10,000

DOCUMENT: Heat of Coffee is the amount of heat that the cup of coffee contains above the amount of heat at zero degrees Celsius. Initially, it has 10,000 calories for a 100 grams of water, which is equivalent to a cup of coffee starting at 100 degrees Celsius.
units: calories

Heat_Loss_Rate = (Ability_of_Coffee_Cup_to_Release_Heat * Temperature_Gap * Specific_Heat_of_Coffee * Mass_of_Coffee)

DOCUMENT: Heat Loss Rate is the amount of heat that flows out of the coffee per minute. It is dependent on the Temperature Gap between the Coffee Temperature and the Room Temperature. Since the Temperature Gap is in units of Celsius degrees, the temperature units must be converted to the equivalent amount of heat in calories at that specific temperature. The conversion is done by multiplying by the the specific heat (=1 calorie/(gm*Celsius)), and the mass (=100 gm).
units: calories/ minute

Ability_of_Coffee_Cup_to_Release_Heat = .05

DOCUMENT: The Ability of the Coffee Cup to Release Heat is a simplified coefficient representing how much the coffee cup influences the release of heat.
units: dimensionless/ minute

Coffee_Temperature = (Heat_of_Coffee) / (Specific_Heat_of_Coffee * Mass_of_Coffee)

DOCUMENT: Coffee Temperature= Coffee Heat/ (Specific Heat*Mass). This equation is a simple conversion from calories to degrees Celsius.
units: degrees Celsius

Mass_of_Coffee = 100

DOCUMENT: The Mass of Coffee is a 100 gm cup of coffee.
units: grams

Room_Temperature = 30

DOCUMENT: The Room Temperature stays constant at 30 degrees Celsius.
units: degrees Celsius

Specific_Heat_of_Coffee = 1

DOCUMENT: The specific heat represents the amount of energy needed to raise a gram of substance one degree Celsius. the specific heat of water is one.

units: calorie / (gm*Celsius)

Temperature_Gap = Coffee_Temperature - Room_Temperature

DOCUMENT: The Temperature Gap is the difference between the Coffee Temperature and the Room Temperature.

units: degrees Celsius