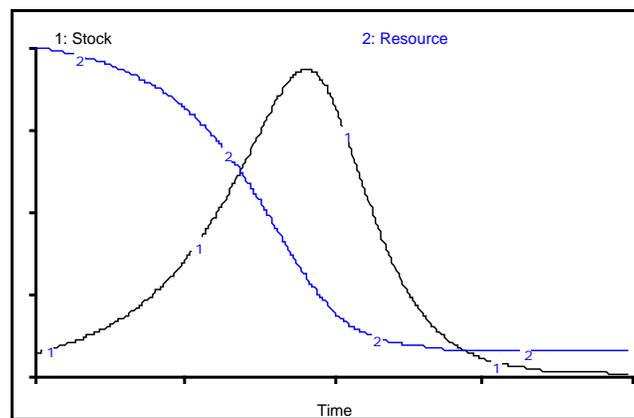# Generic Structures:

Overshoot and Collapse

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
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Table of Contents

1. INTRODUCTION

"Goldfield, Arizona was a busy bustling little town perched atop a small hill between the mighty Superstition Mountains to the east and the Goldfield Mountains to the west. The first gold-strike was made in 1892, and it took just a little under a year before the town came to life. One rich strike after another supported the town and approximately 4000 people who lived in the district for five rich years. The town boasted 3 saloons, a boarding house, general store, blacksmith shop, brewery, meat market and a school house. Just when it looked like the town would outgrow Mesa,[1] the vein faulted, the grade of ore dropped and the town died a slow painful death."[2]

Goldfield, Arizona is today one of the many ghost towns that litter the Western United States. The history of this town is an example of the behavior of overshoot and collapse. When gold was discovered in the nearby hills, the town sprung up as miners flocked in, eager in their rush for gold. The population grew rapidly as new miners arrived, and stores, saloons, and schools followed. Meanwhile, however, the miners were extracting the gold from the hills. After a few years, gold became harder and harder to find. When the miners came home with empty pans, they decided to move on, to follow the trail of gold. They abandoned their small town and, within months, Goldfield, Arizona lay deserted, a ghost town.

Overshoot and collapse is a behavior in which a stock, in the case of Goldfield, the population of the town, relies upon a depletable resource, gold. When the gold is abundant, the population grows quickly. As the population grows, however, it depletes the supply of gold. When gold becomes scarce, the population drops rapidly as the miners leave to try their luck elsewhere.

This paper introduces a structure that produces the behavior of overshoot and collapse and examines other behaviors that the structure can generate. Several examples illustrate the study of systems containing the structure. The exercises at the end of the paper provide the reader with an opportunity to see how knowledge about structure and behavior can be transferred between different systems.

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[1] Mesa is a city in Mariscopa county, south-central Arizona. Its site was settled in 1878 by Mormons. (From the Encyclopædia Britannica.)
[2] From: http://mmm.arizonaguide.com/goldfieldghosttown/adventure.html

2. OVERSHOOT AND COLLAPSE: THE BEHAVIOR

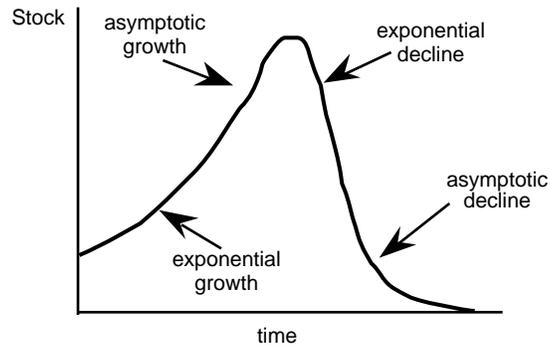Overshoot and collapse is a common behavior of complex systems. Figure 1 shows its characteristic behavior.

Figure 1: Overshoot and collapse

At first, the system's behavior seems satisfactory—the stock of interest, such as the rapidly growing population of Goldfield, Arizona, grows exponentially. Exponential growth, however, is not sustainable. The increasing stock soon starts to cause the depletion of a necessary and often nonrenewable resource, such as gold. The stock's growth slows down. Then, after some time, the declining resource causes the growth to stop completely. Because of the lack of the resource, the stock starts decreasing and quickly collapses—as when all the gold miners abandon Goldfield.

The following examples present two systems that exhibit the behavior of overshoot and collapse. Each example first examines the structure of the system and then explains how the structure produces the observed behavior.

2.1 Example 1: Petroleum Extraction

The first example examines the number of wells in a petroleum field. Companies drill wells to tap into the petroleum reserves under the ground. As more wells extract petroleum, the reserves start to decline, so the drilling of new wells slows down. In addition, the lower level of petroleum reserves reduces the amount of petroleum that each well can extract. As the extraction per well falls, some of the wells stop extracting and close. Eventually, the recoverable reserves are depleted, no more extraction is possible, and all wells close.

Figure 2 shows a possible model of the system; documented equations are in section 7.1 of the Appendix.

Figure 2: Model of petroleum extraction

The number of "Wells" on a field increases by an inflow of "drilling wells" dependent on the number of "Wells" and on a variable "drilling fraction." The "drilling fraction" is the product of a "NORMAL DRILLING FRACTION" and a multiplier called "effect of reserves on drilling fraction." The multiplier expresses a positive relationship between the fraction of "Petroleum Reserves" remaining and the "drilling fraction."[3] The fraction of "Petroleum Reserves" remaining is the ratio of the current level of "Petroleum Reserves" to the "INITIAL PETROLEUM RESERVES." A decrease in the ratio means that less "Petroleum Reserves" remain, so less petroleum can be extracted, and the incentive to drill new wells is lower. Hence, the "drilling fraction" decreases.

The stock of "Wells" also has an outflow, "closing wells," which is the product of a "NORMAL CLOSING FRACTION" and an "effect of extraction on closing fraction." The "effect of extraction on closing fraction" is a nonlinear function showing the effect of "extraction per well," as compared to "MAXIMUM EXTRACTION PER WELL," on the "closing fraction." A decrease in the ratio of "extraction per well" to "MAXIMUM EXTRACTION PER WELL" means that extraction is becoming more difficult, and some owners choose to close the wells. The "closing fraction" therefore increases. The actual "extraction per well" is the product of "MAXIMUM EXTRACTION PER WELL" and

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[3] A positive relationship between two variables means that the variables move in the same direction.

an "effect of reserves on extraction per well" multiplier. The multiplier reflects a positive relationship between petroleum extraction and reserves. As the fraction of "Petroleum Reserves" remaining decreases, extraction becomes more difficult, so the "extraction per well" decreases.

Finally, because petroleum is a nonrenewable resource, the stock of "Petroleum Reserves" has no inflow. The outflow from "Petroleum Reserves" is the total amount of "extraction" per year, which is the product of "extraction per well" and the number of "Wells."

Figure 3 shows the behavior generated by the petroleum extraction model.
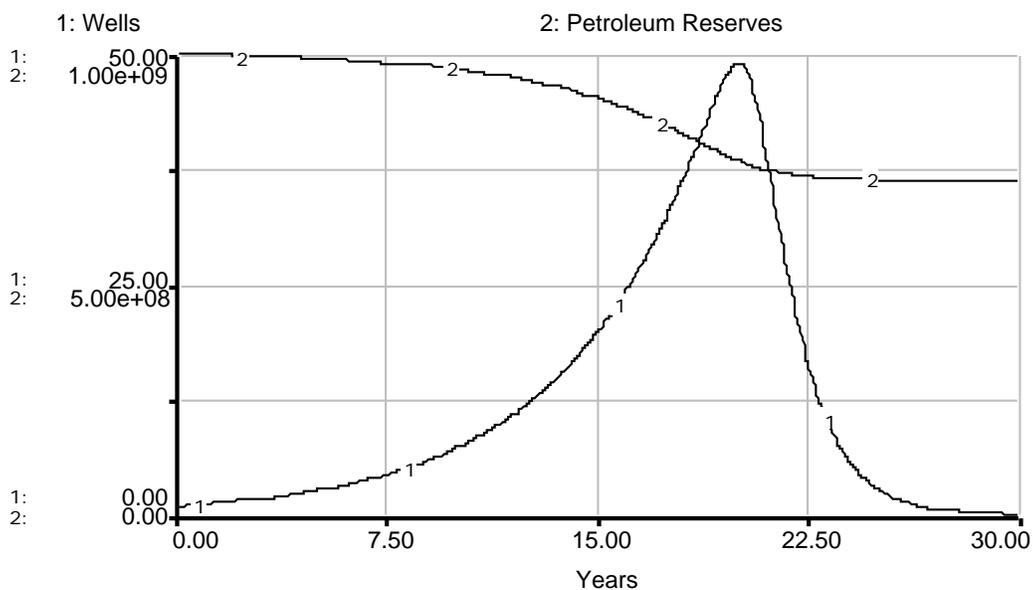

Figure 3: Behavior of "Wells" and "Petroleum Reserves"

Initially, the number of "Wells" is low but growing exponentially, starting the depletion of "Petroleum Reserves." As "Petroleum Reserves" fall, extraction becomes more difficult. Hence, "extraction per well" decreases, causing an increase in the "closing fraction" through the "effect of extraction on closing fraction." In addition, the declining "Petroleum Reserves" cause a decrease in the "drilling fraction," further slowing down the growth of "Wells." Eventually, the "closing fraction" becomes larger than the "drilling fraction," and the outflow of "closing wells" exceeds the inflow of "drilling wells." The number of "Wells" starts declining. Because the number of "Wells" is still large, "Petroleum Reserves" continue to decline. The number of "Wells" decreases rapidly. As long as any "Wells" remain, the decline of "Petroleum Reserves" continues. Finally, after 25% of the "Petroleum Reserves" have been depleted, extraction

becomes nearly impossible.[4] So the "Wells" close even though 75% of the initial amount of "Petroleum Reserves" still remains.

2.2 Example 2: Deer Population

The second example examines the behavior of a deer population with a limited amount of vegetation available. Figure 4 shows a model of the system. Documented equations are in section 7.2 of the Appendix.

Figure 4: Model of the deer population

Notice that the structure of the model is similar to the structure of the petroleum extraction model from section 2.1 with only a few differences. Petroleum is a nonrenewable resource: it has no inflow, only an outflow. On the other hand, the "Vegetation" in the deer population model is able to regenerate and grow. The stock of "Vegetation" is increased by "growth" and decreased by "consumption." The inflow of

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[4] Petroleum pumps take advantage of the pressure difference between the bottom of the well and the section of the rock deep underground that contains the petroleum. As petroleum is extracted, the pressure difference decreases. Typically, by the time 25% of the reserves have been extracted, the pressure on both ends of the pump equalizes, and any further extraction would require expensive artificial means of maintaining the pressure difference.

"growth" is determined by the amount of "Vegetation" and by a constant fraction of "GROWTH PER UNIT OF VEGETATION."

Another difference between the two models is that the "drilling fraction" in Example 1 depends on the amount of "Petroleum Reserves" remaining, while the "birth fraction" in the deer population model is assumed to be independent of the amount of "Vegetation" left.

A comparison of the two models shows that the underlying structure of the two systems is similar. Likewise, one can expect the models to produce similar behaviors. The deer population model generates the behavior shown in Figure 5.
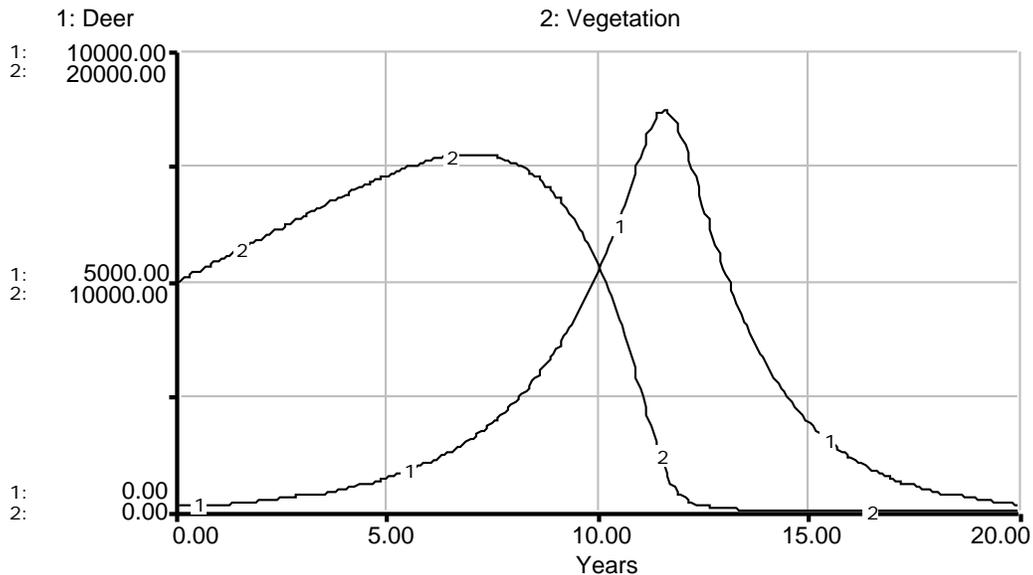
Figure 5: Behavior of "Vegetation" and "Deer"

The behavior of the deer population model is similar to the behavior of the petroleum extraction model. Initially, the amount of "Vegetation" is sufficient to support all of the "Deer." The number of "Deer" is so low that the "growth" of "Vegetation" is higher than "consumption," so "Vegetation" increases. The increase in "Vegetation" is possible because, unlike "Petroleum Reserves," the stock of "Vegetation" has an inflow. Consequently, the "effect of vegetation on death fraction," and hence the "death fraction" is low, so deer "deaths" are fewer than "births." The population of "Deer" increases exponentially. The increase in "Deer," however, means more demand for food, and soon the "consumption" rate becomes higher than the natural rate of "growth," so "Vegetation" starts to decline. A lower amount of "Vegetation" means less food per deer, which, through the "effect of vegetation on death fraction," increases the "death fraction." The "death fraction" becomes equal to the "BIRTH FRACTION" around year 12. The "Deer" population no longer grows. Thus, after a period of exponential growth,

the population peaks and starts declining. Because the number of "Deer" is still high, "consumption" is also high, so "Vegetation" keeps decreasing. The depletion of "Vegetation" continues as long as "consumption" by the "Deer" is higher than "growth." The "Deer" population, just like the number of "Wells" in the previous example, collapses.

A similar behavior may occur repeatedly within a system. After most of the deer die, "consumption" becomes small, and the natural rate of "growth" will cause the "Vegetation" to start growing again. If at least a few deer managed to survive the temporary shortage of food, the population might start growing again, and the behavior of overshoot and collapse could repeat in the future.

3. THE GENERIC STRUCTURE

3.1 Model Diagram

The two models discussed in the examples from section 2 share the same basic structure that generates their similar behavior. Figure 6 shows the generic structure which produces overshoot and collapse.
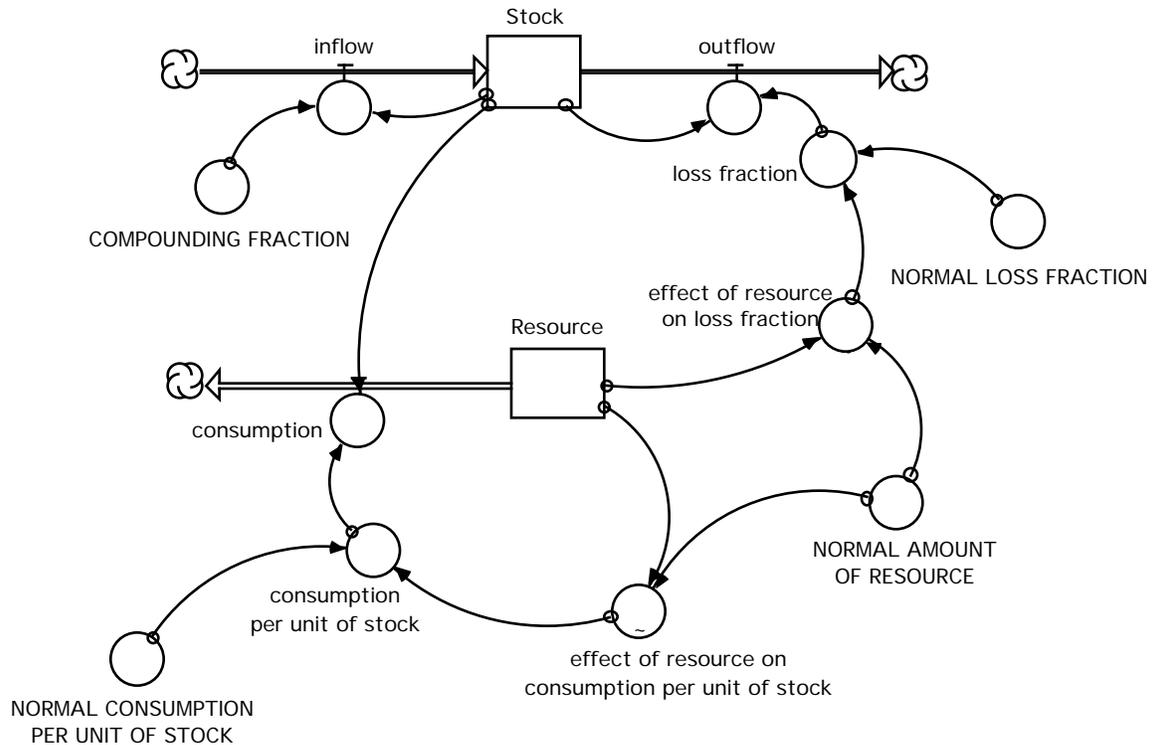
Figure 6: Generic structure producing overshoot and collapse
(with a nonrenewable resource)

The structure is composed of several feedback loops. The only positive feedback loop in the system is the loop that joins the "Stock" to the "inflow": an increase in the "Stock" causes and increase in the "inflow," which in turn causes a further increase in the "Stock." As a counterpart to the positive feedback loop, the system also contains a simple negative feedback loop between the "Stock" and the "outflow": as the "Stock" increases, the "outflow" increases, causing a decrease in the "Stock."

The system also contains negative feedback loops that control the growth of the "Stock." One of the loops shows a negative relationship between the amount of the "Stock" and the amount of the "Resource." An increase in the "Stock" increases

"consumption," thus decreasing the "Resource." A decrease in the "Resource," however, increases the "loss fraction" through the "effect of resource on loss fraction," and thus the "outflow" increases and the "Stock" decreases. In addition, if the amount of "Resource" increases,[5] the "consumption per unit of stock" also increases through the "effect of resource on consumption per unit of stock." Thus, the total "consumption" increases, causing a decrease in the "Resource."

The structure contains two stock variables. The first one is referred to as "Stock" because it is the stock of interest to the modeler. In the two examples in section 2, the number of "Wells" and the population of "Deer" correspond to the "Stock." The second stock variable is called "Resource" because it is a resource necessary for the existence of the "Stock." The "Resource" corresponds to "Petroleum Reserves" and "Vegetation" in the two examples.

The "Stock" increases by an "inflow," the product of the "Stock" and a usually constant "COMPOUNDING FRACTION." Examples of an "inflow" include "drilling wells" and "births" in the preceding examples. The "outflow" from the "Stock" is determined by the amount of the "Stock" and a variable "loss fraction." In the petroleum extraction model, the "outflow" corresponds to "closing wells," and in the deer population example, the "outflow" corresponds to "deaths." The "loss fraction," the equivalent of the "closing fraction" and the "death fraction," is the product of a "NORMAL LOSS FRACTION" and an "effect of resource on loss fraction" multiplier. The multiplier is a table function expressing the negative relationship between the "loss fraction" and the fraction of the "Resource" remaining, which is the ratio of the "Resource" to the "NORMAL AMOUNT OF RESOURCE." As the ratio declines, the "Resource" becomes less available to the "Stock," so the "loss fraction" increases.

The "Resource" is depleted by "consumption," the product of the number of units of the "Stock" and the "consumption per unit of stock." In the generic structure, "consumption" corresponds to "extraction" of petroleum and "consumption" of vegetation in the examples from section 2. The "consumption per unit of stock" depends on the "NORMAL CONSUMPTION PER UNIT OF STOCK" and on an "effect of resource on consumption per unit of stock." The "effect of resource on consumption per unit of stock" is a table function showing that as the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE" decreases, the "Resource" is less available to the "Stock," so "consumption per unit of stock" also decreases.

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[5] Note that an increase in the amount of "Resource" is impossible if the "Resource" is nonrenewable. The analysis of the feedback loop, however, is simpler with the assumption of an initial increase in the "Resource."

In some cases, as in the deer population model in section 2.2, the "Resource" may be increased by regeneration, as "Vegetation" is increased by "growth." The addition of an inflow to the "Resource" affects the behavior of the model to some degree. In the deer population model, "Vegetation" was growing initially, and thus the population of "Deer" was able to grow to a larger number than it would have grown without the "growth" of "Vegetation." The overall behavior of overshoot and collapse, however, was still present in the system. As long as the regeneration inflow to the "Resource" is not significantly stronger than the outflow of "consumption," regeneration will not have a considerable effect on the qualitative behavior of the system.

3.2 Model Equations

The following are the equations for the generic structure producing overshoot and collapse. The constant numerical values used to run the simulation were chosen arbitrarily and do not carry any meaning.

☐ **Resource(t)** = Resource(t - dt) + (- consumption) * dt

INIT Resource = *a constant*

DOCUMENT: The resource that the stock consumes. It corresponds to the "Vegetation" and the "Petroleum Reserves" in the examples above. It can either be nonrenewable, as "Petroleum Reserves," or renewable, as "Vegetation." Note that the addition of a flow renewing the resource is a structural change to the model.

Units: units of resource

☞ OUTFLOWS:

consumption = Stock * consumption_per_unit_of_stock

DOCUMENT: The rate at which the resource is consumed or depleted. It corresponds to "extraction" and "consumption" in the examples above.

Units: units of resource/unit of time

☐ **Stock(t)** = Stock(t - dt) + (inflow - outflow) * dt

INIT Stock = *a constant*

DOCUMENT: The stock that consumes the resource. It corresponds to "Wells" and "Deer" in the examples above.

Units: units of stock

☞ INFLOWS:

inflow = Stock * COMPOUNDING_FRACTION

DOCUMENT: The inflow is the fraction of the stock that flows into the system per unit of time. It corresponds to "drilling wells" and "births" in the examples above.

Units: units of stock/unit of time

☞ OUTFLOWS:

outflow = Stock * loss_fraction

DOCUMENT: The outflow is the fraction of the stock that flows out of the system per unit of time. It corresponds to "closing wells" and "deaths" in the examples above.

Units: units of stock/unit of time

○ **COMPOUNDING_FRACTION** = *a constant*

DOCUMENT: The "COMPOUNDING FRACTION" along with the "Stock" determines the "inflow" to the "Stock." It corresponds to the "NORMAL DRILLING FRACTION" and "BIRTH FRACTION" in the examples above. It can either be constant, as the "BIRTH FRACTION" in the deer population model, or can be affected by the percentage of "Resource" remaining, as the "NORMAL DRILLING FRACTION" in the petroleum extraction model.

Units: 1/unit of time

○ **consumption_per_unit_of_stock** = NORMAL_CONSUMPTION_PER_UNIT_OF_

STOCK * effect_of_resource_on_consumption_per_unit_of_stock

DOCUMENT: Consumption of the resource by the stock per unit of time. It depends on the "NORMAL CONSUMPTION PER UNIT OF STOCK" as well as on the "effect of resource on consumption per unit of stock." It corresponds to the "extraction per well" and "consumption per deer" in the examples above.

Units: (units of resource/units of stock)/unit of time

○ **NORMAL_AMOUNT_OF_RESOURCE** = *a constant*

DOCUMENT: The normal amount of resource. It corresponds to the "INITIAL PETROLEUM RESERVES" and "NORMAL AMOUNT OF VEGETATION" in the examples above.

Units: unit of resource

○ **loss_fraction** = NORMAL_LOSS_FRACTION * effect_of_resource_on_loss_fraction
  DOCUMENT: The fraction of the stock that is lost per unit of time. It depends on the "NORMAL LOSS FRACTION" and on the "effect of resource on loss fraction." It corresponds to the "closing fraction" and "death fraction" in the examples above.
  Units: 1/unit of time

○ **NORMAL_CONSUMPTION_PER_UNIT_OF_STOCK** = *a constant*
  DOCUMENT: The amount of resource that a unit of stock consumes per unit of time while the resource is readily available. It corresponds to the "MAXIMUM EXTRACTION PER WELL" and "NORMAL CONSUMPTION PER DEER" in the examples above.
  Units: (units of resource/units of stock)/unit of time

○ **NORMAL_LOSS_FRACTION** = *a constant*
  DOCUMENT: The fraction of the stock that is lost given a "NORMAL AMOUNT OF RESOURCE." It corresponds to the "NORMAL CLOSING FRACTION" and "NORMAL DEATH FRACTION" in the examples above.
  Units: 1/unit of time

⊘ **effect_of_resource_on_consumption_per_unit_of_stock** =
  GRAPH (Resource/NORMAL_AMOUNT_OF_RESOURCE)
  (0.00, 0.00), (0.1, 0.365), (0.2, 0.6), (0.3, 0.75), (0.4, 0.855), (0.5, 0.91), (0.6, 0.955), (0.7, 0.98), (0.8, 0.99), (0.9, 1.00), (1, 1.00)
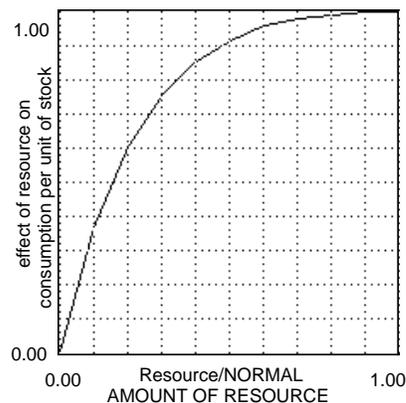
  DOCUMENT: The effect of the remaining percentage of the resource on the consumption per unit of stock. It corresponds to the "effect of reserves on
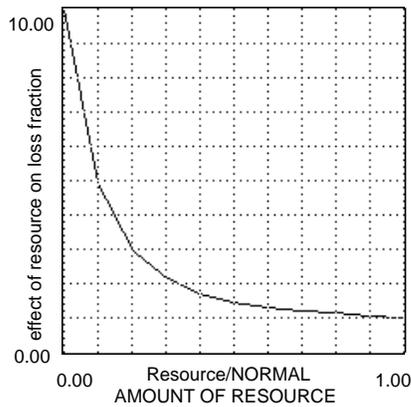
extraction per well" and "effect of vegetation on consumption per deer" in the examples above.[6]

Units: dimensionless

⊘ **effect_of_resource_on_loss_fraction** =

GRAPH(Resource/NORMAL_AMOUNT_OF_RESOURCE)

(0.00, 10.0), (0.1, 4.90), (0.2, 3.00), (0.3, 2.20), (0.4, 1.70), (0.5, 1.45), (0.6, 1.30), (0.7, 1.20), (0.8, 1.12), (0.9, 1.05), (1, 1.00)

DOCUMENT: The effect of the remaining percentage of the resource on the loss fraction. It corresponds to the "effect of reserves on closing fraction" and "effect of vegetation on death fraction" in the examples above.[7]

Units: dimensionless

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[6] The shape and assumptions of the "effect of resource on consumption per unit of stock" table function will be explained in section 4.1.

[7] The shape and assumptions of the "effect of resource on loss fraction" table function will be explained in section 4.2.

3.3 Model Behavior

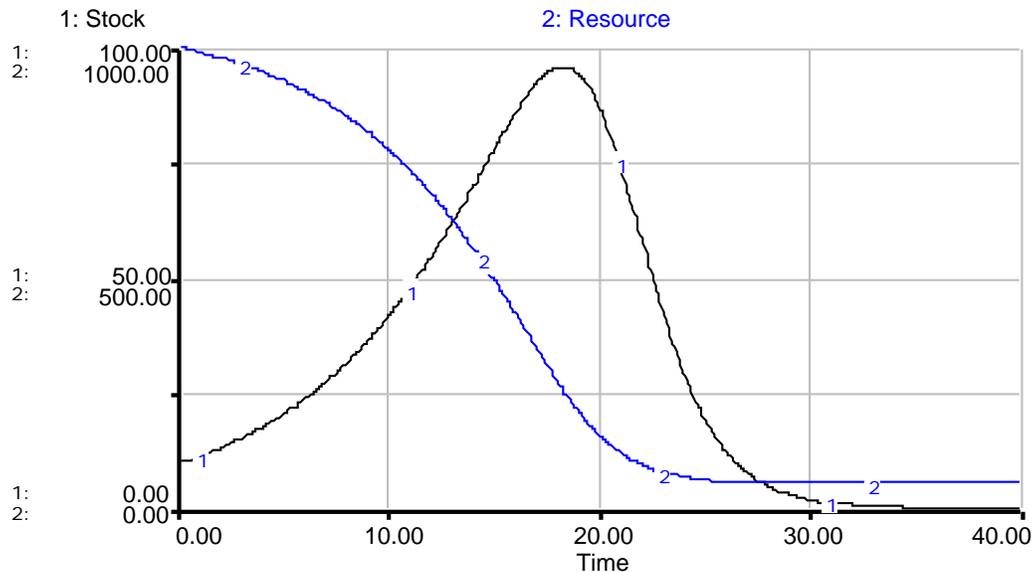Figure 7 shows the behavior produced by the generic structure.

Figure 7: Overshoot and collapse behavior produced by the generic structure

Initially, the positive feedback loop composed of the "inflow" and the "Stock" drives the exponential growth of the "Stock." The exponential increase in the "Stock" causes "consumption" to also increase exponentially. Consequently, the "Resource" decreases exponentially, causing the "loss fraction" to increase exponentially. The *slope* of the net flow[8] into the "Stock" begins to decrease to zero.

Soon, the increasing "loss fraction" makes the "outflow" increase faster than the "inflow," but the "inflow" is still larger than the "outflow." The slope of the net flow is then negative: the net flow decreases but is still positive. A graph showing the behavior of the net flow is shown in Figure 8.

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[8] The net flow is defined as "inflow" - "outflow." The net flow is not part of the model; however, for better understanding of the structure, it is useful to create a converter called "net flow" in the model, and to observe its behavior on a graph.
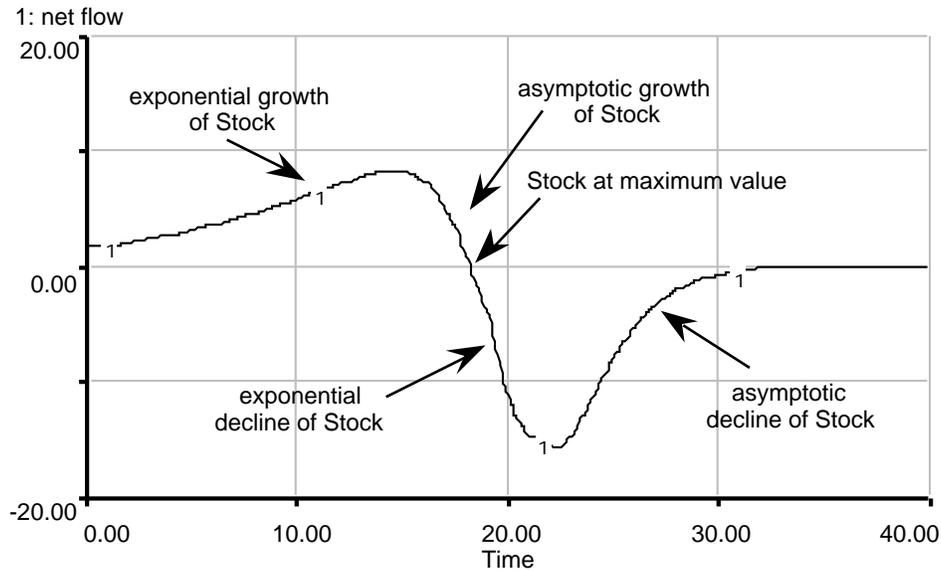
Figure 8: Net flow into the "Stock" of the generic structure

Meanwhile, as the growth rate slows because of the decreasing net flow, the "Stock" stops growing exponentially; its growth becomes asymptotic. The "Resource" continues being depleted through "consumption," and the "loss fraction" continues to grow. Then, the value of the "loss fraction" reaches the value of the "gain fraction," and the "outflow" reaches the "inflow." Thus, the net flow is zero (it is still decreasing), and the "Stock" peaks. In Figure 9, the dashed line at time 18 indicates that the "Stock" reaches its maximum when the "outflow" equals the "inflow" (net flow is equal to 0).
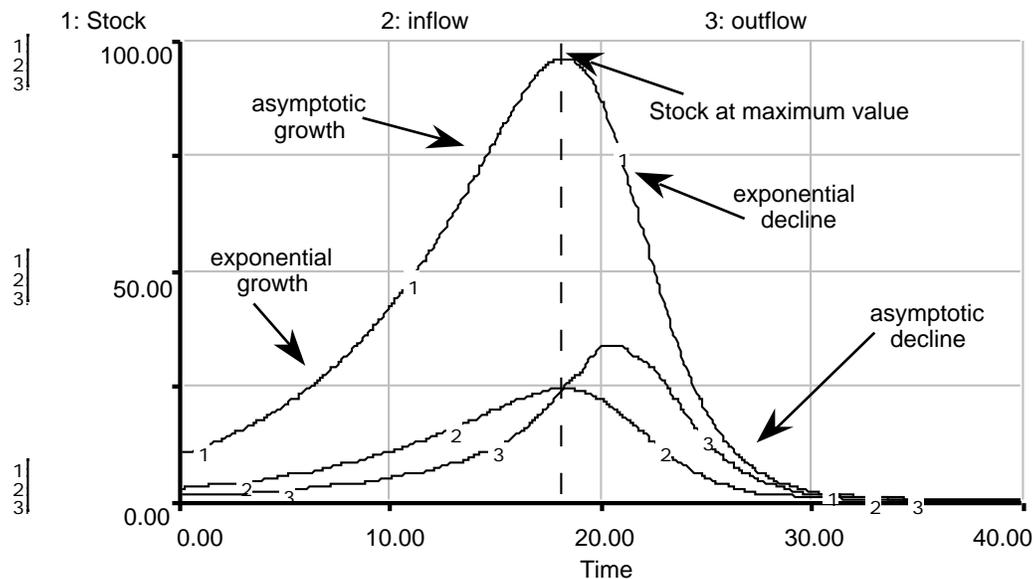

Figure 9: Behavior of "Stock," "inflow," and "outflow" of the generic structure

The lack of "Resource" causes the "loss fraction" to keep increasing, and the "outflow" exceeds the "inflow" (the net flow becomes negative). As soon as the net flow becomes negative, the "Stock" reverses direction and starts decreasing. The "outflow" still increases because of the growing "loss fraction," while the "inflow" has started to decrease as a result of the decrease in the "Stock." The slope of the "Stock," which is equal to the net flow into the "Stock," becomes increasingly negative. The "Stock" is declining exponentially.

However, as the "Stock" starts decreasing, the decrease in "Resource" is no longer exponential, but asymptotic. As a result, the "loss fraction" also stops increasing exponentially and switches to asymptotic growth. Consequently, the "outflow" starts decreasing because of the decrease in "Stock," and the net flow starts increasing again. The net flow still stays negative, however, causing a further, although slower, asymptotic decline of the "Stock." The net flow eventually approaches zero, and the "Stock" stabilizes at zero.

It is crucial to realize that the overshoot and collapse behavior occurs because the "loss fraction" is not constant but varies with the amount of "Resource." As the "Resource" is being consumed, the "loss fraction" increases and so does the "outflow." As soon as the "outflow" exceeds the "inflow," the collapse of the "Stock" begins and continues until the "Stock" reaches zero.

The goal value of the "Stock" and the only possible equilibrium in the generic structure is zero. The "Resource" cannot reach equilibrium as long as the "Stock" is greater than zero. If the initial amount of "Resource" is sufficiently large to permit growth of the "Stock," then even a slight initial disturbance of the "Stock" away from zero starts off the "consumption" of the "Resource." The slight initial disturbance produces the overshoot and collapse behavior that leads the "Stock" to stabilize at zero again.

4. TABLE FUNCTIONS IN THE GENERIC STRUCTURE

The generic structure producing overshoot and collapse presented in section 3 contains two table functions: the "effect of resource on consumption per unit of stock"

and the "effect of resource on loss fraction." [9] This section explains the shape of the table functions and their effect on model behavior.

4.1 "Effect of resource on consumption per unit of stock" table function

The "effect of resource on consumption per unit of stock" table function is an increasing curve with a decreasing slope, with values ranging from 0 to 1 on both axes. Figure 10 shows the graph of the table function.
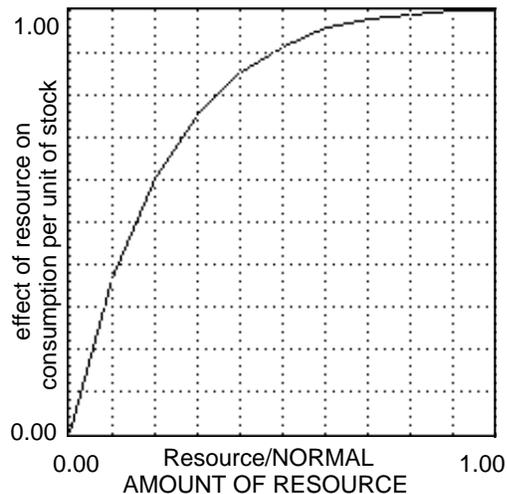

Figure 10: The "effect of resource on consumption per unit of stock" table function

The slope of the function is positive because of the positive relation between the "consumption per unit of stock" and the fraction of the "Resource" remaining, which is the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE." The higher the ratio, the higher the "consumption per unit of stock," so the table function gives a high value. As the ratio decreases, so does "consumption per unit of stock," so the table function outputs a lower value.

In addition, the curve of the table function increases with a decreasing slope. If the "Resource" is still relatively high as a fraction of the "NORMAL AMOUNT OF RESOURCE," "consumption per unit of stock" is not greatly affected. The "effect of

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[9] In some models, additional table functions might be necessary to represent the feedback mechanisms of the system. For example, the petroleum extraction model from section 2.1 contains a table function of the "effect of reserves on drilling fraction" because it can be argued that the remaining amount of "Petroleum Reserves" not only affects the rate of "closing wells," but also the rate of "drilling wells." For simplicity, however, the table function of the "effect of resource on compounding fraction" was omitted from the generic structure.

resource on consumption per unit of stock" is then close to 1. In Figure 10, for example, the table function outputs values greater than 0.9 for values of the ratio greater than 0.5. As the "Resource" decreases further, however, the "effect of resource on consumption per unit of stock" becomes stronger, so the table function declines more steeply.

The table function ranges from 0 to 1 for both the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE" and for the "effect of resource on consumption per unit of stock." When the "Resource" is equal to the "NORMAL AMOUNT OF RESOURCE" (the ratio is equal to 1), "consumption per unit of stock" is equal to the "NORMAL CONSUMPTION PER UNIT OF STOCK," and the table function has a value of 1. As the ratio decreases, however, the "Resource" becomes more difficult to obtain, so "consumption per unit of stock" becomes smaller than the "NORMAL CONSUMPTION PER UNIT OF STOCK." The table function then outputs a value between 0 and 1. When the "Resource" is completely depleted (the ratio is equal to 0 because "Resource" is equal to 0), "consumption per unit of stock" must also be equal to 0, so the table function outputs the value of 0. If the amount of "Resource" is greater than the "NORMAL AMOUNT OF RESOURCE" (the ratio is greater than 1), the table function will output the value of 1 by default because it is assumed that "consumption per unit of stock" is never greater than the "NORMAL CONSUMPTION PER UNIT OF STOCK," even if the "Resource" is overabundant.

The reader may try to change the shape of the table function to see that, as long as the table function is increasing, the exact shape does not determine the behavior of the model. Even if the table function is linear or has an increasing slope, the model still exhibits overshoot and collapse.

4.2 "Effect of resource on loss fraction" table function

The "effect of resource on loss fraction" table function is a curve whose negative slope decreases in magnitude. The curve ranges from 0 to 1 for the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE," and from 1 to a higher value for the "effect of resource on loss fraction." Figure 11 shows the graph of the table function.
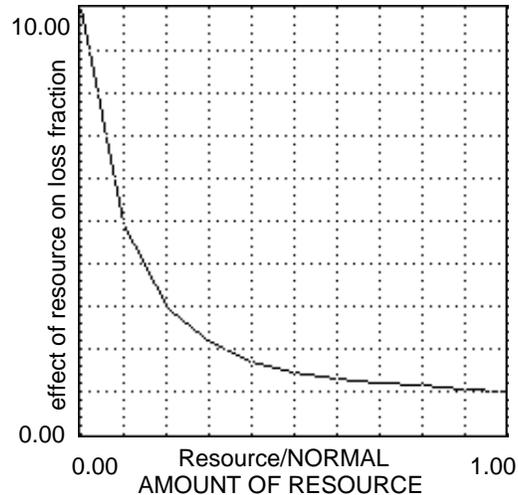
Figure 11: The "effect of resource on loss fraction" table function

The slope of the function is negative because of the negative relationship between the "loss fraction" and the fraction of "Resource" remaining, which is the ratio of "Resource" to "NORMAL AMOUNT OF RESOURCE." The higher the fraction of the "Resource" remaining, the closer the "loss fraction" to the "NORMAL LOSS FRACTION," so the table function outputs a value close to 1. As the ratio decreases, the "loss fraction" becomes more significant, so the table function outputs a value larger than 1.

The "effect of resource on loss fraction" table function has a negative slope that decreases in magnitude. While the fraction of the "Resource" remaining is still high, the depletion of the "Resource" will not have a strong impact on the "loss fraction." For example, in the table function in Figure 11, the "effect of resource on loss fraction" is lower than 2 when the fraction of "Resource" remaining is greater than 0.4. As the "Resource" declines further, however, its effect on the "loss fraction" becomes stronger, so the curve is steeper as the ratio decreases.

The table function ranges from 0 to 1 for the fraction of the "Resource" remaining. When the "Resource" is equal to (or even greater than) the "NORMAL AMOUNT OF RESOURCE," the "loss fraction" is equal to the "NORMAL LOSS FRACTION." The table function then outputs a value of 1.[10] The minimum value of the "effect of resource on loss fraction" is therefore 1. As the remaining fraction of "Resource" decreases, the "loss fraction" becomes higher than the "NORMAL LOSS FRACTION," so the table function outputs values greater than 1.

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[10] Assuming that an overabundance of the "Resource" will not reduce the "loss fraction" below the value of the "NORMAL LOSS FRACTION."

The maximum value of the "effect of resource on loss fraction" can significantly affect the behavior of the model. In the generic structure, the maximum value is set to 10. Thus, the "loss fraction" becomes ten times the "NORMAL LOSS FRACTION" when the "Resource" is completely depleted. If the value of the "COMPOUNDING FRACTION" is less than ten times the "NORMAL LOSS FRACTION," then the "loss fraction" will eventually become greater than the "COMPOUNDING FRACTION." When the "loss fraction" is greater than the "COMPOUNDING FRACTION," the "outflow" will be greater than the "inflow," and the "Stock" will decline.

As soon as the table function outputs a value that makes the "loss fraction" greater than the "COMPOUNDING FRACTION," the "Stock" will start to decline. With the values used to produce the simulation of the generic structure ("COMPOUNDING FRACTION" = 0.25, "NORMAL LOSS FRACTION" = 0.1), the "outflow" reaches the "inflow" when the table function outputs the value of 2.5. Because the table function ranges up to 10, the "outflow" will significantly exceed the "inflow" when the "Resource" is almost depleted, and thus the "Stock" collapses.

5. SUMMARY OF IMPORTANT CHARACTERISTICS

- **Structure**

The structure generating overshoot and collapse is a second-order negative feedback loop. The "Stock" feeds on the "Resource" whose depletion eventually causes the "Stock" to collapse. The "loss fraction" that determines the "outflow" from the "Stock" depends on the remaining amount of "Resource." The "Resource" does not have to be nonrenewable but should at least be slow to regenerate.

- **Behavior**

Initially, the "Stock" undergoes exponential growth. Because of the accumulating "Stock," the "Resource" is depleted. The negative feedback from the "Resource" to the "loss fraction" slows the growth of the "Stock," until the "outflow" becomes greater than the "inflow," which causes a reversal of the behavior: the negative feedback loop of the "outflow" becomes very strong and the "Stock" collapses.

6. USING INSIGHTS GAINED FROM THE GENERIC STRUCTURE

Generic structures are useful because they help a modeler to transfer knowledge among different systems. The following exercises will emphasize the concept of transferability of structure.

6.1 Exercise 1: Health Deterioration

This exercise discusses the effects of too much work on a person's health. Bryant, a graduate student, just started working on a new project. At the beginning, he is excited and enthusiastic about the project, so he starts spending more time working on the project than the usual eight hours a day. As he spends more and more time on the project, he starts getting tired and becomes ill—his health deteriorates. Eventually, by the time Bryant works more than 15 hours a day, his health becomes so weak that he has to quickly start cutting back on the number of hours he works, until he stops working completely.

Using the generic structure from section 3, build a model showing how the number of hours Bryant spends on the project exhausts his health. Additional variables or parameters may need to be added to the generic structure.

First choose the variables corresponding to the "Stock" and the "Resource." For Bryant, the number of hours he spends on the project every day depletes his health. The level of the resource of health can be measured on a scale either from 0 to 1, or from 0 to 100, representing the percentage of Bryant's health remaining. Assume that initially Bryant is perfectly healthy. Also assume that, as Bryant's health declines, the fraction of his health that deteriorates for every additional hour he works becomes smaller. Note that the time horizon of the model (approximately a month) is too short for Bryant's health to recover significantly.

6.2 Solution to Exercise 1

Figure 16 shows a possible health deterioration model. Documented equations are in section 7.3 of the Appendix.
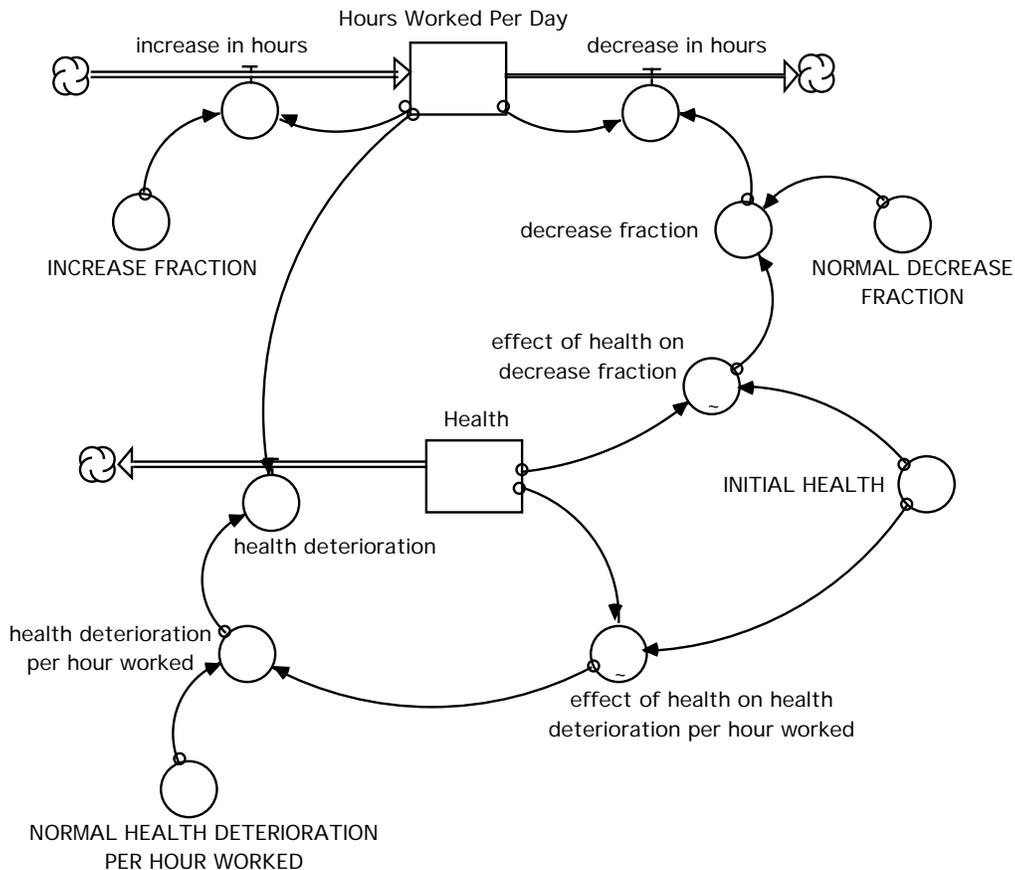

Figure 12: Model of Bryant's health

One can build a model of Bryant's health that has the same structure as the generic structure presented in section 3. The stock of "Hours Worked Per Day" has an inflow of "increase in hours," determined by the number of "Hours Worked Per Day" and a constant "INCREASE FRACTION." The "increase in hours" positive feedback loop shows that the more Bryant works on the project, the more he likes it, causing him to work even more. The number of "Hours Worked Per Day" decreases through a "decrease in hours," which is the product of the "Hours Worked Per Day" and a "decrease fraction." The "decrease fraction" depends on a "NORMAL DECREASE FRACTION" and an "effect of health on decrease fraction" multiplier. The multiplier reflects a negative relationship between "Health" and the "decrease fraction": as the ratio of "Health" to "INITIAL HEALTH" decreases, the "decrease fraction" increases. While

his "Health" is still relatively strong, the "effect of health on decrease fraction" will be quite small. As his "Health" declines further, the multiplier outputs a higher value, until it reaches the value of 100, which occurs when Bryant's "Health" has dropped 70 percent. Such a high value for the multiplier emphasizes the fact that Bryant would start cutting back on his hours quite drastically if he started to become seriously ill.

Bryant's "Health" becomes worse as he works more hours every day. His "health deterioration" is the product of the number of "Hours Worked Per Day" and a fraction called "health deterioration per hour worked." The fraction is determined by a "NORMAL HEALTH DETERIORATION PER HOUR WORKED" and a multiplier called "effect of health on health deterioration per hour worked." The multiplier shows that as the ratio of Bryant's "Health" to his "INITIAL HEALTH" decreases, "health deterioration per hour worked" becomes smaller. Over a one-month time horizon, Bryant's "Health" will not be able to regenerate significantly, so "Health" does not have an inflow.

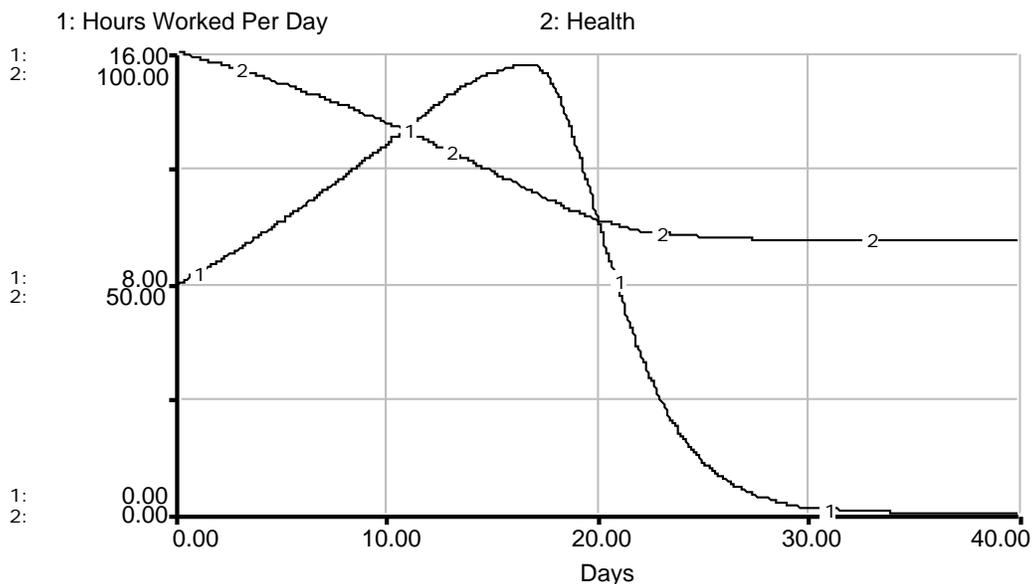Figure 17 shows the behavior of the model.

Figure 13: Behavior of the health deterioration model

As Bryant starts spending more hours working on the project every day, his "Health" starts deteriorating slowly. After approximately 16 days of intensive work, when he works more that 15 hours every day, Bryant realizes the bad effects of too much work on his "Health," so he starts working less and less. Eventually, after day 30, Bryant decides not to do any work at all, so his "Health" stops deteriorating.

6.3 Exercise 2: Mayan Population

The second exercise examines a possible cause of the decline of the Mayan empire. Please read carefully the following excerpts from an article entitled "Study Depicts Mayan Decline."

> One of the great mysteries of human history has been the sudden collapse of one of the main centers of Mayan civilization in Central America at a time when it was apparently at a peak of culture, architecture and population around A.D. 800. ... Just before the final cataclysm, the new research suggests, the population in one area ranged from about 200 to 500 persons per square kilometer. ... Within two to four Mayan generations, which probably meant less than 100 years, the population dropped back to what it had been almost 2,000 years before—20 or less per square kilometer. ... [New] research ... showed there was an exponential growth in Mayan population during at least 1,700 years... Human numbers doubled every 408 years... [The population] grew at a steadily increasing pace, but, for many centuries, the growth was too slow for any single generation to see what was happening. ... Gradually-intensified agriculture seems to have done severe cumulative damage to an originally verdant environment. To this was added the impact of increases in human dwellings and other major architectural works on the land. Essential nutrients washed, slid and were moved downhill to be lost in the lakes, diminishing the fertility of agricultural land. ... [The new research gives] documented estimates for the growth pattern of Mayan population and correlate[s] this growth with the damage to the environment that went with it. [11]

Based on the generic structure from section 3, build a model showing the collapse of the Mayan population after its environment is destroyed. Use the numerical values provided in the article to calculate approximate values for the model parameters. The model does not have to reproduce the generic structure exactly; additional converters or table functions may be needed to represent the system more realistically. Here are some additional hints for building the model:

First, determine which variables correspond to the "Stock" and "Resource" of the generic structure. The "Stock" will most likely be the Mayan population, while the "Resource" could be an aggregate variable representing the state of the environment in which the population lives ranging from 0 to 100 percent of the environment capacity. Assume that initially the environment is undamaged. The time horizon of the simulation should be at least 2000 years, as specified in the article, and should include A.D. 800.

Notice that it is not the actual level of the Mayan population that damages the environment, but rather the density of the population. Therefore, the model should

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[11] From Harold M. Schmeck, Jr. "Study Depicts Mayan Decline." *New York Times*, October 23, 1979.

contain a converter representing density, the ratio of the population to a constant land area. The size of land area is somewhat arbitrary; it should, however, be set such that the initial density is approximately 20 people per square kilometer, as indicated in the article. The density should then be compared to a normal value of density, to determine the effect of population density on the fraction of the environment that is damaged. The effect will in turn multiply a normal damage fraction to determine the actual fractional damage to the environment.

In addition, the model should contain an inflow into the resource of the environment because the environment is renewable over a time horizon of 2000 years. Depending on the assumptions of the modeler, the time over which the environment regenerates may be either constant or may vary as a function of the environment.

6.4 Solution to Exercise 2

Figure 18 shows a possible model of the Mayan population based on the description from section 6.3. Documented equations are in section 7.4 of the Appendix.
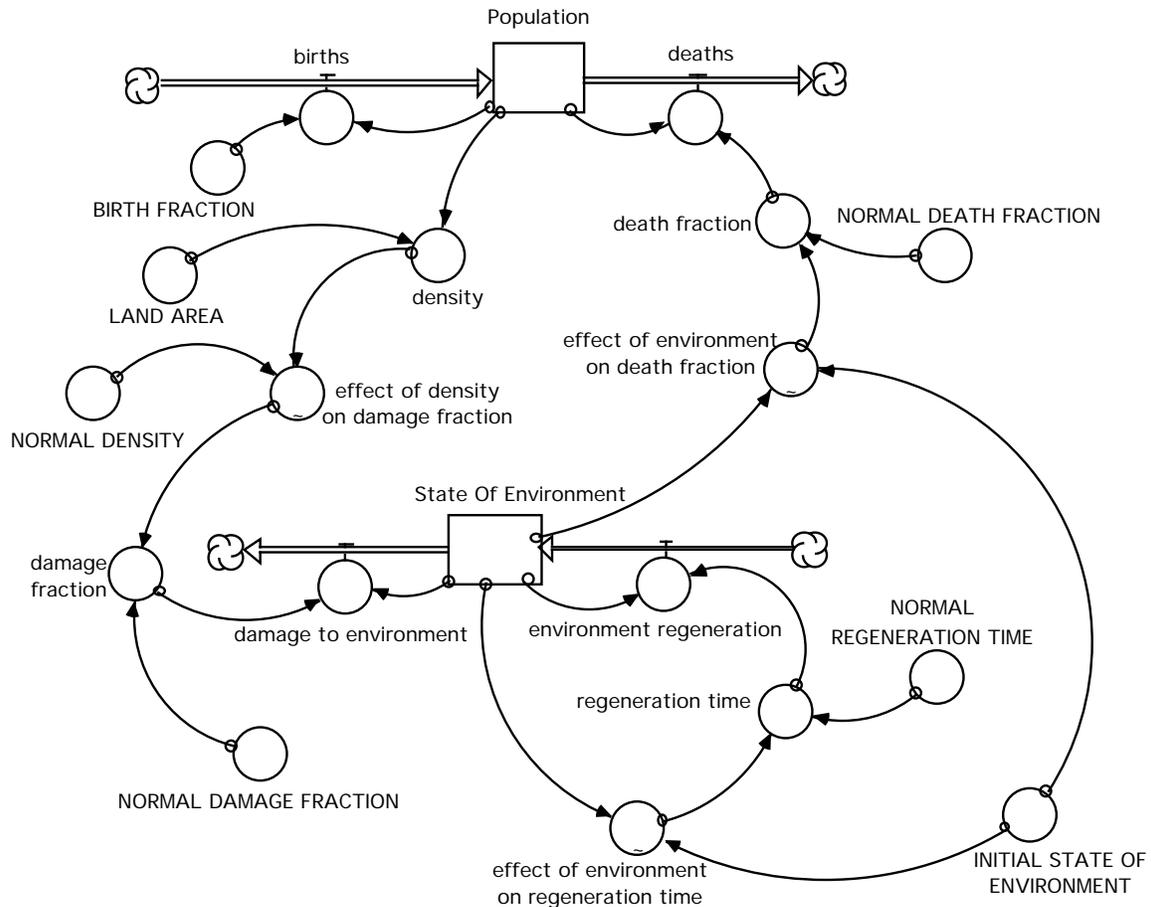

Figure 14: Model of the Mayan population

The two stocks in the model, corresponding to "Stock" and "Resource" of the generic structure, are "Population" and "State of Environment," respectively.

"Population" increases by "births," the product of the "Population" and a constant "BIRTH FRACTION." The outflow from "Population" is "deaths," determined by the level of "Population" and a "death fraction." The "death fraction" is the product of a "NORMAL DEATH FRACTION" and a multiplier called "effect of environment on death fraction." The multiplier captures a negative relationship between the "State of Environment" and the "death fraction": as the "State of Environment" declines, the "death fraction" rises.

The "State of Environment" resource is depleted through an outflow of "damage to environment." The "damage to environment" is the "damage fraction" of the "State of

Environment."   The "damage fraction" is the product of a "NORMAL DAMAGE FRACTION" and a multiplier called "effect of density on damage fraction."   The multiplier shows that as the "density" increases, the "damage fraction" becomes larger. The "effect of density on damage fraction" is a function of the ratio of the actual "density" to a "NORMAL DENSITY." The "density" is the ratio of "Population" to a constant "LAND AREA."

Because environment is a renewable resource, "State of Environment" increases through an inflow of "environment regeneration." The "environment regeneration" is determined by the "State of Environment" and a "regeneration time." Assuming that the "regeneration time" varies with "State of Environment," "regeneration time" is the product of a "NORMAL REGENERATION TIME" and an "effect of environment on regeneration time" multiplier. The multiplier shows that as the ratio of "State of Environment" to the "INITIAL STATE OF ENVIRONMENT" decreases, it takes longer for the environment to regenerate itself, so the "regeneration time" increases.

Before running the model to see whether it demonstrates the expected behavior of overshoot and collapse of the Mayan population, the modeler should determine an appropriate time horizon for the model and decide on the time unit to be used in the simulation. As the article indicates, the Mayan population collapsed around A.D. 800 after growing exponentially for at least 1700 years. With such a long time horizon over which the behavior occurred, running the simulation in units of years would be very slow. Thus, it might be more convenient to choose "Century" as a time unit, and to run the simulation from the 10th century B.C. to the 10th century A.D.

The modeler must also specify values for constant parameters in the model, as well as determine the functional relationships captured by the table functions. To find the value of the "BIRTH FRACTION" and "NORMAL DEATH FRACTION," remember that the article mentioned a population doubling time of 408 years, or 4.08 centuries, corresponding to a net growth fraction of approximately 0.18 per century.[12] The net growth fraction is the difference between the "BIRTH FRACTION" and the "NORMAL DEATH FRACTION."   Thus, the modeler could pick the value of 2 per century for "BIRTH FRACTION," and 2 – 0.18, or 1.82 per century, for "NORMAL DEATH FRACTION," corresponding to an average lifetime of about 55 years.[13]

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[12] Remember that the doubling time of exponential growth is approximately 0.7/compounding fraction. Thus the compounding fraction is 0.7/doubling time. For the Mayan population, the net growth fraction is 0.7 / 4.08, which is approximately 0.18.

[13] The average lifetime is calculated as 1/"NORMAL DEATH FRACTION."

The value of "LAND AREA" can be chosen by the modeler, but should have a specific relation to the initial value of the "Population" to give the historically observed initial value for "density" as described in the article, 20 people per square kilometer. For example, with a "LAND AREA" of 10,000 square kilometers, the initial value of "Population" would be 200,000 people.

Considering that the "State of Environment" ranges on a scale from 0 to 100, it seems reasonable to choose the value of the "NORMAL DAMAGE FRACTION" to be, for example, 1 per century. Also, a reasonable value for "NORMAL REGENERATION TIME" could be 1 century.[14]

The "effect of environment on death fraction" table function should have the same general shape as the "effect of resource on loss fraction" described in section 4. The multiplier takes on values close to 1 while the ratio of "State of Environment" to "INITIAL STATE OF ENVIRONMENT" is relatively high, and the multiplier only starts increasing significantly when the ratio decreases to approximately 0.25. When the ratio reaches 0, the multiplier has the value of 5.

The "effect of density on damage fraction" table function represents the relationship between the "damage fraction" and the ratio of "density" to "NORMAL DENSITY." According to the article, the "density" of the Mayan population in some areas reached up to 500 persons per square kilometer, leading to a value of 25 for the ratio (assuming that "NORMAL DENSITY" is the initial density of the population, 20 persons per square kilometer). To have a more robust model, let the ratio range from 0 up to, say, 35. As the ratio increases, the "effect of density on damage fraction" also increases. The table function, however, gives relatively small values when the ratio is less than 20. Then, even a small increase in the ratio leads to a large increase in the multiplier, so the slope of the table function will be large. The slope, however, flattens out as the ratio exceeds the value of 30. The "effect of density on damage fraction" eventually reaches the value of 50 as the ratio of "density" to "NORMAL DENSITY" reaches 35.

Finally, the "effect of environment on regeneration time" multiplier shows the relationship between the environment "regeneration time" and the ratio of "State of Environment" to "INITIAL STATE OF ENVIRONMENT." The multiplier stays close

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[14] The exact value of the parameters will usually not have a significant effect on the overall behavior mode of the system. See Lucia Breierova, 1996. *An Introduction to Sensitivity Analysis* (D-4526), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, September 6, 40 pp.

to 1 for values of the ratio larger than 0.5, and then starts increasing faster up to the value of 2 when the ratio becomes 0.
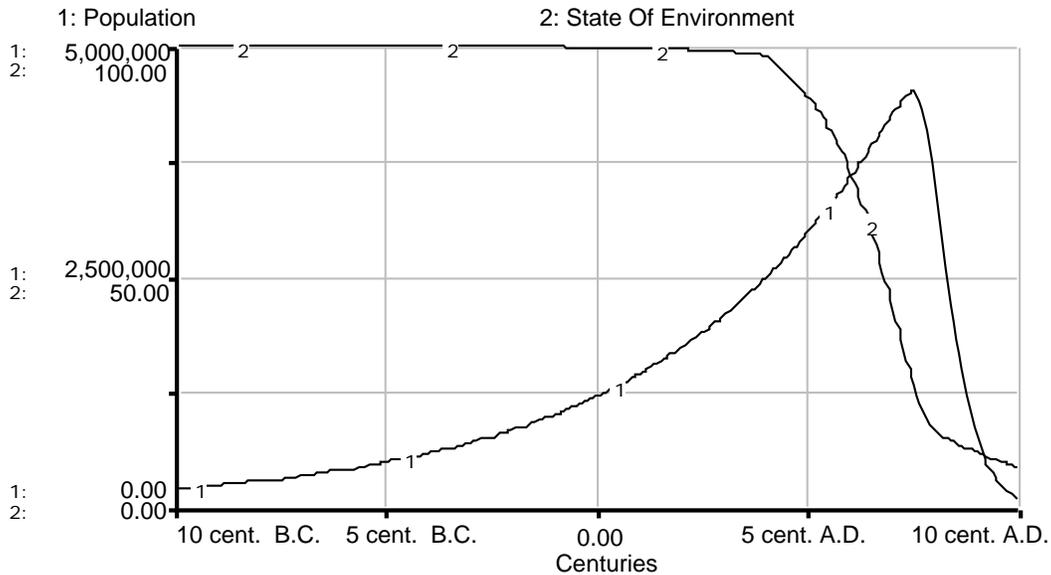
Figure 19 shows the behavior of the model.

Figure 15: Behavior of the Mayan population model

As expected, the model exhibits the overshoot and collapse behavior mode. The Mayan population grows exponentially from the 10th century B.C. to approximately the 7th century A.D., when it peaks sharply and quickly collapses within about one century.

The decline of the "State of Environment" is almost unnoticeable until around the 4th century A.D. Then, as the "Population" rises, the "density" also increases, so the "effect of density on damage fraction" increases, causing a faster decline of the "State of Environment." As the "State of Environment" declines, the "effect of environment on death fraction" multiplier rises sharply. Hence, the "death fraction" eventually exceeds the "BIRTH FRACTION," causing a quick collapse of the "Population."

Therefore, the model closely replicates the behavior described in the article. As long as the reader's model exhibits overshoot and collapse, it is not necessary to try to replicate the described behavior precisely. A closer fit to the historically observed behavior may be achieved by changing some parameter values or modifying the table functions.

7. APPENDIX: MODEL DOCUMENTATION

7.1 Petroleum Extraction Model

☐ **Petroleum_Reserves(t)** = Petroleum_Reserves(t - dt) + (- extraction) * dt
      INIT Petroleum_Reserves = 10^9
      DOCUMENT: The estimated number of barrels of petroleum under the field is
      initially a billion, or 10^9.
      Units: Barrels

      � OUTFLOWS:
      extraction = Wells * extraction_per_well
      DOCUMENT: Number of barrels of petroleum extracted each year.
      Units: Barrels/Year

☐ **Wells(t)** = Wells(t - dt) + (drilling_wells - closing_wells) * dt
      INIT Wells = 1
      DOCUMENT: Number of wells on the field extracting petroleum.
      Units: Wells

      ☐masculine INFLOWS:
      drilling_wells = Wells * drilling_fraction
      DOCUMENT: Number of wells drilled each year.
      Units: Wells/Year

      ☐masculine OUTFLOWS:
      closing_wells = Wells * closing_fraction
      DOCUMENT: Number of wells closing each year.
      Units: Wells/Year

○ **closing_fraction** = NORMAL_CLOSING_FRACTION * effect_of_extraction_on_
      closing_fraction
      DOCUMENT: The fraction of wells closing each year.
      Units: 1/Year

○ **drilling_fraction** = NORMAL_DRILLING_FRACTION * effect_of_reserves_on_
      drilling_fraction
      DOCUMENT: The fraction of wells drilled each year.
      Units: 1/Year

○ **extraction_per_well** = MAXIMUM_EXTRACTION_PER_WELL * effect_of_
      reserves_on_extraction_per_well
      DOCUMENT: The amount of petroleum extracted per well each year.
      Units: (Barrels/Well)/Year

○ **INITIAL_PETROLEUM_RESERVES** = 10^9
      DOCUMENT: The initial amount of petroleum in the field is estimated to be a
      billion, or 10^9.
      Units: Barrels

○ **MAXIMUM_EXTRACTION_PER_WELL** = 1000000
        DOCUMENT: The amount of petroleum that a well can extract each year at full
        capacity when there is ample petroleum left.
        Units: (Barrels/Well)/Year

○ **NORMAL_CLOSING_FRACTION** = 0.05
        DOCUMENT: The normal fraction of wells that close each year.
        Units: 1/Year

○ **NORMAL_DRILLING_FRACTION** = 0.25
        DOCUMENT: The normal fraction of wells drilled each year.
        Units: 1/Year

⊘ **effect_of_extraction_on_closing_fraction** =
        GRAPH(extraction_per_well/MAXIMUM_EXTRACTION_PER_WELL)
        (0.00, 20.0), (0.1, 19.9), (0.2, 19.8), (0.3, 18.8), (0.4, 11.1), (0.5, 4.60), (0.6, 2.30),
        (0.7, 1.40), (0.8, 1.05), (0.9, 1.00), (1, 1.00)
        DOCUMENT: The effect of the remaining percentage of petroleum reserves on
        the closing fraction.
        Units: dimensionless

⊘ **effect_of_reserves_on_drilling_fraction** =
        GRAPH(Petroleum_Reserves/INITIAL_PETROLEUM_RESERVES)
        (0.00, 0.00), (0.1, 0.00), (0.2, 0.00), (0.3, 0.00), (0.4, 0.41), (0.5, 0.695), (0.6,
        0.84), (0.7, 0.935), (0.8, 0.98), (0.9, 0.995), (1, 1.00)
        DOCUMENT: The effect of the remaining percentage of petroleum reserves on
        the drilling fraction.
        Units: dimensionless

⊘ **effect_of_reserves_on_extraction_per_well** =
        GRAPH(Petroleum_Reserves/INITIAL_PETROLEUM_RESERVES)
        (0.00, 0.00), (0.05, 0.00), (0.1, 0.00), (0.15, 0.00), (0.2, 0.00), (0.25, 0.00), (0.3,
        0.00), (0.35, 0.00), (0.4, 0.00), (0.45, 0.00), (0.5, 0.00), (0.55, 0.00), (0.6, 0.005),
        (0.65, 0.045), (0.7, 0.15), (0.75, 0.37), (0.8, 0.725), (0.85, 0.92), (0.9, 0.985),
        (0.95, 0.995), (1.00, 1.00)
        DOCUMENT: The effect of the remaining percentage of petroleum reserves on
        the extraction per well.
        Units: dimensionless

7.2 Deer Population Model

☐ **Deer(t)** = Deer(t - dt) + (births - deaths) * dt
        INIT Deer = 100
        DOCUMENT: The population of deer.
        Units: Deer

        ♄ INFLOWS:
        births = Deer * BIRTH_FRACTION
        DOCUMENT: Number of deer born every year.
        Units: Deer/Year

☃ OUTFLOWS:
deaths = Deer * death_fraction
DOCUMENT: Number of deer dying every year.
Units: Deer/Year

☐ **Vegetation(t)** = Vegetation(t - dt) + (growth - consumption) * dt
INIT Vegetation = 10000
DOCUMENT: Amount of vegetation present in the area.
Units: Unit of vegetation

☃ INFLOWS:
growth = Vegetation * GROWTH_PER_UNIT_OF_VEGETATION
DOCUMENT: Rate of regeneration of the vegetation.
Units: Units of vegetation/Year

☃ OUTFLOWS:
consumption = Deer * consumption_per_deer
DOCUMENT: Rate of consumption of the vegetation.
Units: Units of vegetation/Year

○ **BIRTH_FRACTION** = 0.5
DOCUMENT: Number of deer born per deer every year.
Units: 1/Year

○ **consumption_per_deer** = NORMAL_CONSUMPTION_PER_DEER * effect_of_
vegetation_on_consumption_per_deer
DOCUMENT: Amount of vegetation consumed per deer per year.
Units: (Units of vegetation/Deer)/Year

○ **death_fraction** = NORMAL_DEATH_FRACTION * effect_of_vegetation_on_
death_fraction
DOCUMENT: Fraction of deer dying each year.
Units: 1/Year

○ **GROWTH_PER_UNIT_OF_VEGETATION** = 0.1
DOCUMENT: Units of vegetation regenerated per unit of vegetation every year.
Every year, the amount of vegetation in the area grows by approximately 10
percent.
Units: 1/Year

○ **NORMAL_AMOUNT_OF_VEGETATION** = 10000
DOCUMENT: The normal amount of vegetation in the area.
Units: Unit of vegetation

○ **NORMAL_CONSUMPTION_PER_DEER** = 1
DOCUMENT: Number of units of vegetation that a deer consumes per year if
there is enough vegetation available.
Units: (Units of vegetation/Deer)/Year

○ **NORMAL_DEATH_FRACTION** = 0.1
DOCUMENT: Fraction of deer dying per deer every year when there is enough
vegetation available.
Units: 1/Year

⊘ **effect_of_vegetation_on_consumption_per_deer** =
　　　GRAPH(Vegetation/NORMAL_AMOUNT_OF_VEGETATION)
　　　(0.00, 0.00), (0.1, 0.305), (0.2, 0.545), (0.3, 0.72), (0.4, 0.835), (0.5, 0.905), (0.6,
　　　0.945), (0.7, 0.97), (0.8, 0.985), (0.9, 1.00), (1, 1.00)
　　　DOCUMENT: Effect of the availability of vegetation on consumption per deer.
　　　Units: dimensionless

⊘ **effect_of_vegetation_on_death_fraction** =
　　　GRAPH(Vegetation/NORMAL_AMOUNT_OF_VEGETATION)
　　　(0.00, 10.0), (0.1, 7.15), (0.2, 5.05), (0.3, 3.15), (0.4, 2.15), (0.5, 1.60), (0.6, 1.35),
　　　(0.7, 1.15), (0.8, 1.05), (0.9, 1.00), (1, 1.00)
　　　DOCUMENT: Effect of the availability of vegetation on the deer death fraction.
　　　Units: dimensionless

7.3 Health Deterioration Model

☐ **Health(t)** = Health(t - dt) + (- health_deterioration) * dt
　　　INIT Health = 100
　　　DOCUMENT: State of Bryant's health. A perfect health is defined to have the
　　　value of 100, as in 100%.
　　　Units: units of health.

　　　☞ OUTFLOWS:
　　　health_deterioration = Hours_Worked_Per_Day * health_deterioration_per_hour_
　　　worked
　　　DOCUMENT: Bryant's health gets worse as he works more.
　　　Units: units of health/day

☐ **Hours_Worked_Per_Day(t)** = Hours_Worked_Per_Day(t - dt) + (increase_in_hours -
　　　decrease_in_hours) * dt
　　　INIT Hours_Worked_Per_Day = 8
　　　DOCUMENT: Number of hours that Bryant works per day.
　　　Units: hours per day

　　　☞ INFLOWS:
　　　increase_in_hours = Hours_Worked_Per_Day * INCREASE_FRACTION
　　　DOCUMENT: The number of hours by which Bryant daily increases the number
　　　of hours he works per day.
　　　Units: hours per day/day

　　　☞ OUTFLOWS:
　　　decrease_in_hours = Hours_Worked_Per_Day * decrease_fraction
　　　DOCUMENT: The number of hours by which Bryant daily decreases the number
　　　of hours he works per day.
　　　Units: hours per day/day

○ **decrease_fraction** = NORMAL_DECREASE_FRACTION * effect_of_health_on_
　　　decrease_fraction
　　　DOCUMENT: The fraction of an hour by which Bryant decreases the number of
　　　hours he works per day.
　　　Units: 1/day

○ **health_deterioration_per_hour_worked** =
NORMAL_HEALTH_DETERIORATION_PER_HOUR_WORKED * effect_
of_health_on_health_deterioration_per_hour_worked
DOCUMENT: The fraction by which Bryant's health deteriorates each day for
every hour he works.
Units: (units of health/hours per day)/day

○ **INCREASE_FRACTION** = 0.1
DOCUMENT: The fraction by which Bryant increases the number of hours he
works each day. As Bryant spends more hours working on the project, he likes it
more and becomes addicted to his work, so he spends even more hours working.
Units: 1/day

○ **INITIAL_HEALTH** = 100
DOCUMENT: The initial state of Bryant's health. Assume Bryant is initially
perfectly healthy, so his health is at 100%.
Units: units of health

○ **NORMAL_DECREASE_FRACTION** = 0.05
DOCUMENT: The fraction by which Bryant decreases the number of hours he
works each day while his health is perfect.
Units: 1/week

○ **NORMAL_HEALTH_DETERIORATION_PER_HOUR_WORKED** = 0.15
DOCUMENT: The fraction by which Bryant's health deteriorates with every
hour he works.
Units: (units of health/hours per day)/day

⊘ **effect_of_health_on_decrease_fraction** = GRAPH(Health/INITIAL_HEALTH)
(0.00, 100), (0.1, 100), (0.2, 100), (0.3, 100), (0.4, 50.0), (0.5, 26.0), (0.6, 10.0),
(0.7, 2.00), (0.8, 1.20), (0.9, 1.05), (1, 1.00)
DOCUMENT: The effect of Bryant's health on the "decrease fraction." As his
health deteriorates, Bryant realizes that his has to stop working so many hours
every day, so the "decrease fraction" grows.
Units: dimensionless

⊘ **effect_of_health_on_health_deterioration_per_hour_worked** =
GRAPH(Health/INITIAL_HEALTH)
(0.00, 0.00), (0.1, 0.435), (0.2, 0.645), (0.3, 0.79), (0.4, 0.87), (0.5, 0.915), (0.6,
0.945), (0.7, 0.97), (0.8, 0.985), (0.9, 0.995), (1, 1.00)
DOCUMENT: The effect of Bryant's health on the "health deterioration per
hour." As Bryant's health gets worse, the fraction of his health that deteriorates
for every hour he works becomes smaller.
Units: dimensionless

7.4 Mayan Population Model

□ **Population(t)** = Population(t - dt) + (births - deaths) * dt
INIT Population = 200000
DOCUMENT: The size of the Mayan population.
Units: People

☞ INFLOWS:
births = Population * BIRTH_FRACTION
DOCUMENT: Number of people born per century.
Units: People/Century

☞ OUTFLOWS:
deaths = Population * death_fraction
DOCUMENT: Number of people dying per century.
Units: People/Century

☐ **State_Of_Environment(t)** = State_Of_Environment(t - dt) +
(environment_regeneration - damage_to_environment) * dt
INIT State_Of_Environment = INITIAL_STATE_OF_ENVIRONMENT
DOCUMENT: The state of the environment in which the Mayan population
lives. The initial value is 100, as in 100%.
Units: Environment units

☞ INFLOWS:
environment_regeneration = State_Of_Environment/regeneration_time
DOCUMENT: Regeneration of the environment per century.
Units: Environment units/Century

☞ OUTFLOWS:
damage_to_environment = State_Of_Environment * damage_fraction
DOCUMENT: Environment damage per century.
Units: Environment units/Century

○ **BIRTH_FRACTION** = 2
DOCUMENT: Number of people born per person per century.
Units: 1/Century

○ **damage_fraction** = NORMAL_DAMAGE_FRACTION * effect_of_density_on_
damage_fraction
DOCUMENT: Fraction of the environment that is damaged per century.
Units: 1/Century

○ **death_fraction** = NORMAL_DEATH_FRACTION * effect_of_environment_on_
death_fraction
DOCUMENT: Number of people dying per person per century.
Units: 1/Century

○ **density** = Population/LAND_AREA
DOCUMENT: Density of the Mayan population.
Units: People/square kilometer

○ **INITIAL_STATE_OF_ENVIRONMENT** = 100
DOCUMENT: Initial state of the environment.
Units: Environment units

○ **LAND_AREA** = 10000
DOCUMENT: Area on which the Mayan population lived.
Units: square kilometers

○ **NORMAL_DAMAGE_FRACTION** = 1
　　　DOCUMENT: Fraction of the environment that is damaged per century when
　　　density is at its initial value.
　　　Units: 1/Century

○ **NORMAL_DEATH_FRACTION** = 1.82
　　　DOCUMENT: Number of people dying per person per century when the
　　　environment has no negative impact on the lifetime.
　　　Units: 1/Century

○ **NORMAL_DENSITY** = 20
　　　DOCUMENT: Density of the Mayan population that does not have any
　　　damaging effect on the environment.
　　　Units: People/square kilometer

○ **NORMAL_REGENERATION_TIME** = 1
　　　DOCUMENT: Time necessary for the environment to regenerate itself when no
　　　substantial damage has been done yet.
　　　Units: Century

○ **regeneration_time** = NORMAL_REGENERATION_TIME * effect_of_
　　　environment_on_regeneration_time
　　　DOCUMENT: Time necessary for the environment to regenerate itself.
　　　Units: Century

⊘ **effect_of_density_on_damage_fraction** = GRAPH(density/NORMAL_DENSITY)
　　　(0.00, 1.00), (1.00, 1.00), (2.00, 1.00), (3.00, 1.00), (4.00, 1.00), (5.00, 1.00),
　　　(6.00, 1.00), (7.00, 1.00), (8.00, 1.00), (9.00, 1.00), (10.0, 1.00), (11.0, 1.00),
　　　(12.0, 1.00), (13.0, 1.10), (14.0, 1.10), (15.0, 1.10), (16.0, 1.20), (17.0, 1.20),
　　　(18.0, 1.35), (19.0, 1.35), (20.0, 1.60), (21.0, 1.60), (22.0, 1.90), (23.0, 1.90),
　　　(24.0, 2.30), (25.0, 2.80), (26.0, 3.40), (27.0, 4.25), (28.0, 6.25), (29.0, 26.8),
　　　DOCUMENT: Effect of the density as compared to the normal density on the
　　　damage fraction.
　　　Units: dimensionless

⊘ **effect_of_environment_on_death_fraction** =
　　　GRAPH(State_Of_Environment/INITIAL_STATE_OF_ENVIRONMENT)
　　　(0.00, 5.00), (0.1, 2.33), (0.2, 1.33), (0.3, 1.03), (0.4, 1.02), (0.5, 1.00), (0.6, 1.00),
　　　(0.7, 1.00), (0.8, 1.00), (0.9, 1.00), (1, 1.00)
　　　DOCUMENT: Effect of the environment as compared to the initial state of the
　　　environment on the death fraction.
　　　Units: dimensionless

⊘ **effect_of_environment_on_regeneration_time** =
　　　GRAPH(State_Of_Environment/INITIAL_STATE_OF_ENVIRONMENT)
　　　(0.00, 2.00), (0.1, 1.50), (0.2, 1.32), (0.3, 1.18), (0.4, 1.09), (0.5, 1.04), (0.6, 1.01),
　　　(0.7, 1.00), (0.8, 1.00), (0.9, 1.00), (1, 1.00)
　　　DOCUMENT: Effect of the environment as compared to the initial state of the
　　　environment on the time necessary for the environment to regenerate itself.
　　　Units: dimensionless