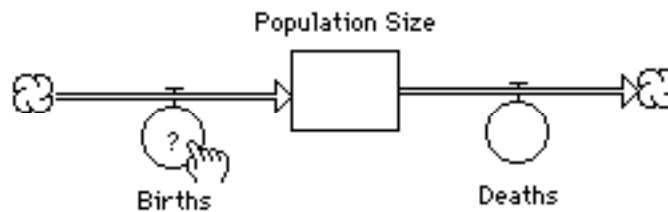


Modeling Exercises

Section 1



Prepared for
MIT System Dynamics Education Project
Under the Supervision of
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by
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Vensim Examples added October 2001

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The exercises in this paper require a Macintosh¹ Computer with 2 Mb of RAM if running System 6.0.4 and at least 4 Mb of RAM if running System 7 or higher. Also required is the STELLA² II software package. If you do not have STELLA, it can be obtained from High Performance Systems:

High Performance Systems, Inc.
45 Lyme Road
Hanover, NH 03755
(800) 332-1202

Abstract

This paper is the first in a series of papers designed to utilize and enhance the modeling capabilities of the reader. This series of papers is part of a project called Road Maps. Road Maps is a self study guide to lead the reader through the process of learning system dynamics. Road Maps is currently being worked on by the System Dynamics in Education Project at MIT under Jay Forrester, the founder of system dynamics.

The beginning of this paper outlines the importance and the process of model building. This paper also contains three separate modeling exercises. Each exercise begins with a brief description of the system after which the reader constructs a system dynamics model of the system. The reader then runs the model under varying conditions and answers some questions. This paper is not intended as an introduction to system dynamics or model building. The reader is expected to have prior experience with both. Solutions to all the exercises are included at the end. The modeling exercises are presented approximately in order of difficulty, but need not be done in this sequence.

¹Macintosh is a registered trademark of Apple Computer, Inc.

²STELLA is a registered trademark of High Performance Systems, Inc..

Introduction

Modeling is a crucial element of system dynamics. We use mental models every day of our lives to make decisions without addressing the model being used. Creating and simulating models using STELLA or other modeling software forces us to solidify our mental models and allows us to observe the behavior of complex systems where mental models are inadequate. One drawback of using a computer to simulate systems is that the computer will always do exactly what you tell it to do. Your computer won't say "Hey wait a minute, you can't have a negative population," or "You can't make the number of sick people greater than the total population." You must be the one to generate and verify a valid model.

To make system dynamics modeling as useful as possible, a modeler must acquire many skills. Below is a list of some important skills:

- **Have a general idea of how the system should behave to test the accuracy of the computer simulation.**
- **Generate a valid model of the system to be analyzed.**
- **Enter the model correctly into the computer.**
- **Make the model clear so that others can understand its structure.**

The exercises you encountered in Road Maps I-IV have prepared you for the task of creating your own models. This paper will allow you to exercise some of the skills you have already learned and enable you to acquire new skills as well. Future parts of Road Maps will contain more sections of the Modeling Exercises series (this is the first section) enabling you to further increase your modeling skills.

How to Build Models

Building a model can be a very rewarding, yet challenging process. There is no set way to build a model. Each model that you build will be different, and likewise the steps you follow in building your models will vary. In some cases, you may be able to jump directly into STELLA and start mapping. In other cases the modeling process may take a considerable amount of time.

Below is a rough guideline for how to go about the process of model building. It is not intended as a "cookbook" for model building but rather as a way to get you started.

1. Define the purpose. What you want to do with your model is entirely up to you. However, you must define the purpose for your model before you can begin to build it. Do you want to explore different management policies for a park? Perhaps you merely wish to teach your third graders about population dynamics. You must have a goal in mind to decide how complex to make your model. This purpose will also guide you in your decisions regarding how much certainty you require in your numbers, where to set your model boundary and many other aspects of your model.

2. Define the system you wish to model. You will have to decide exactly what is "inside" and "outside" your system before you begin modeling. You can't include the whole world in your model, but neither can you make everything exogenous³. You must find a happy medium.

3. Identify key variables in the system. What factors have the most effect on your system? Be careful in naming your model variables. Use names that can be associated with a real quantity such as fish, cows or account balance. Model elements can also represent soft variables, but be sure to use something that can be scaled, like happiness. Don't use vague names like mental attitude or lifestyle.

4. Describe the behavior of the key variables. How do the key variables behave in real life? You may not know an exact answer to this, but you should at least define some constraints. (i.e., population can't be negative or infinite) This is an important step because it helps you evaluate and improve your model later on.

³Exogenous inputs are factors that are not enclosed by the system boundary. They affect the system, but are not affected by the elements within the system. Exogenous inputs are either constant, or dependent on time only.

5. Identify the stocks and flows in your system. What are the levels of your system? What flows affect them?

6. Map the stocks and flows on your STELLA diagram pad. Begin building the model in STELLA with the stocks and flows.

7. Define the flows. Recall that the flows are defined only by the stocks in the system and constants. There may be several intermediate steps, but each flow must be dependent only on the stocks and the constants. Model these relationships in your STELLA model.

8. Include quantitative information. Once you have built a preliminary flow diagram for the model in STELLA you should enter the equations for the model elements. Many relationships are simple and intuitive. Sometimes, the relationship between two variables is too complicated for a simple equation. If so, you will have to define a graph to represent the relationship.

9. Run the model. Now is the time to test your model. Did it behave as expected? Is the behavior possible? You may need to return to one of the previous steps at this point and correct any problems you come across.

10. Evaluate the model. Your model may not have produced a reasonable behavior. Look at each relationship in the model to see if you have made a mistake. Here is a list of common mistakes:

- The solution interval (DT) is too large.
- A flow that should be bi-directional is defined as a one way flow⁴.
- A stock is incorrectly defined as non-negative.
- Incorrect flow direction. (An outflow is modeled as an inflow or vice versa.)

⁴It is generally good practice to define all flows as bi-directional and allow all stocks to become negative. You should select the bi-directional flow option for all your flows and de-select the non-negative option for all your stocks. This allows the modeler to see the behavior of the system without any unnatural constraints. If a stock in your model is inappropriately becoming negative, examine the causes of this behavior from a structural point of view and correct the model rather than clicking the non-negative box. This principle also applies to constraints on flow direction.

- **Incorrect Equations.** (Don't be too quick to guess the equation for a relationship. Dividing instead of multiplying (or a similar mistake) is easy to do and will make the model behave incorrectly.)

11. Improve the Model. Try to find causes of any anomalous behavior, then modify the model so it produces an appropriate behavior. Once you get an appropriate behavior, try a few different scenarios to test your model under varying conditions. You may also wish to add additional structure to the model.

Now, return to the first step and see if your model accomplished what you wanted it to. You may find that once you have a working model it doesn't accomplish for you what you had intended. In that case, go back and change the model. There is no set point at which your model is "done" or "perfect". It can only be "good for what I wanted it for."

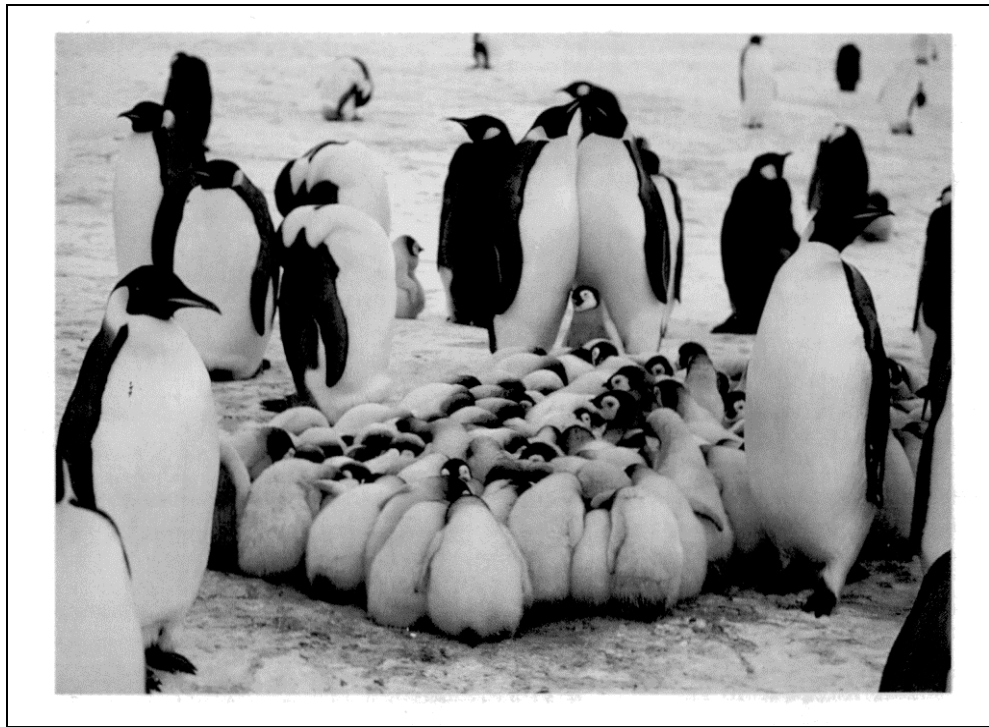
NOTE: The book, An Introduction to Systems Thinking, that came with your STELLA software provides a more in-depth look at the process of model building. You may want to read the section entitled "The Modeling Process" before proceeding with exercises that follow.

Building Models

This paper contains three exercises. Each begins with a written description. All of the information necessary to build a model may not be in the written description, so you will have to make some decisions for parameter values and other variable relationships. Suggested answers are included in the Appendix.

- Read the Description.
- Use the steps outlined previously to guide you in building a STELLA model.
- Follow the directions and answer the questions in the exploration that follows.

Model 1: An Ecological System⁵



This model description is taken from an ecology textbook. A causal-loop diagram was made for this system in Exercise 1 of Study Notes in System Dynamics⁶ from Road Maps Part II. The exercises included after the system description will help you in formulating and testing your model.

⁵Photograph © Doug Allen/Oxford Scientific Films

⁶Goodman, Michael R.(1974) *Study Notes in System Dynamics*, Portland, OR: Productivity Press, pp. 137-147.

Description:

...At a critical time in the life history of a given population, a physical factor such as light or a nutrient may be significant as a regulatory agent; at another time, parasitism, predation, or competition, or even some other physical factor may become the operative factor. As complex and as variable as the niche of any species is, it is unlikely that the regulation comes about by any single agency. However, there does appear to be considerable and mounting evidence, both empirical and theoretical, to suggest that populations are self-regulating through automatic feedback mechanisms. Various mechanisms and interactions appear to operate both in providing the information and in the manner of responding to it, and with the exceptional case of a catastrophe, the stimulus to do so appears to depend on the density of the population. The end effect is one of avoiding destruction of a population's own environment and thereby avoiding its own extinction.⁷

You should be able to build a simple, one level model based on the above description. Use the steps outlined in the previous section to guide you through the process. Remember that you have been provided with a general guideline, not a cookbook. It is not necessary to follow the steps exactly.

You may wish to refer to the causal-loop diagram you created for the exercise in Road Maps Part II. In building your model, you may wish to generate a graphical function to show the relationship between the density of the population and the death rate. After you have built your model, proceed to the exploration section below:



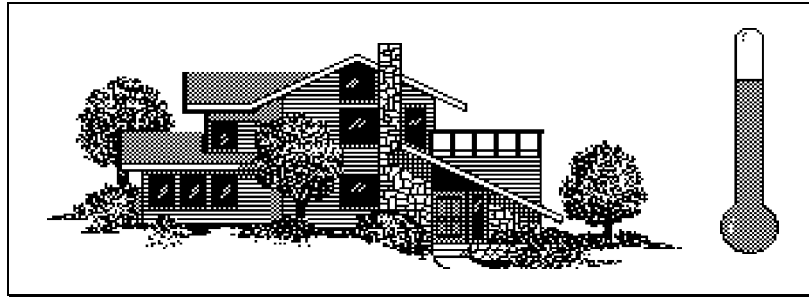
Exploration:

1. This system can be modeled with only one level. What physical quantity does the level represent? What flows affect the level?
2. Describe the state of equilibrium for your model. What is the size of the level? What are the flow rates? Does this seem like a plausible equilibrium? (Note: If birth rate = death rate = 0, the model would be in equilibrium, but it would not be very helpful.)
3. Simulate a catastrophe by adding another outflow to the population. Use the PULSE function in STELLA to cause the catastrophe to kill half the population (i.e. the size of the pulse will be half the equilibrium population) Is the system "self-regulating through automatic feedback" as described in the passage?
4. Add another flow taking away from the population stock labeled *Parasites*. Set the catastrophe flow equal to 0. What is the effect of a constant flow out of the population? What is the new equilibrium?

⁷Kormondy, E.J. (1969) *Concepts of Ecology*, Englewood Cliffs: Prentice-Hall, p.110.

Could you have predicted the new equilibrium? Compare this behavior to the one-time catastrophe response.

Model 2: A Heating/Cooling System



This is a description taken from Systems 1: An Introduction to Systems Thinking⁸. This book was used in Part I of Road Maps.

Description Part A:

One of the most common mechanical feedback systems is the heating system found in most homes and buildings. Once a temperature has been set on the thermostat, the system will try to keep the temperature in the house as close to that level as possible. If the temperature drops below that level, the thermostat responds by turning the furnace on. The furnace produces heat, which warms the air back up. Eventually the temperature rises above the desired level and the thermostat shuts the furnace off. Then, if it is colder outside than inside, the house will cool down again until the thermostat turns the furnace on, and the whole cycle is repeated.⁹

You should include the effects of both the environment and the furnace on the room temperature. While the furnace is either on or off, the effect of the environment is somewhat more complicated. You may wish to refer back to the cooling cup of coffee model you built in Road Maps Part III¹⁰. A similar goal-gap mechanism is at work here.

⁸Kauffman, Draper L. J. (1980) *Systems 1: An Introduction to Systems Thinking*, Minneapolis, MN: S. A. Carlton, Publishers.

⁹ibid., Page 6

¹⁰Nancy Roberts, David F. Andersen, Ralph M. Deal, Michael S. Garet, William A. Shaffer (1983) *Introduction to Computer Simulation: A System Dynamics Modeling Approach*. pp. 267-272 (Chapter 15).

Hints:

- Both the furnace and the outside environment affect the state of the house by adding and removing energy in the form of *heat*.
- Changes in the house's *temperature* are proportional to the change in the amount of *heat*¹¹ in the house.
- The rate at which *heat* is transferred from the environment to the house is directly proportional to the difference in temperature between the house and the environment.
- The furnace does not react instantly to changes in room temperature. You need to add a delay between the actual house temperature and the temperature registered by the thermostat.

Follow the model building guideline provided to you earlier to build a STELLA model of this system. After building the model, proceed with the following exercises.



Exploration Part A:

1. What is the level in this system?
2. What rate(s) affect the level?
3. How are the rates determined?
4. Run your STELLA model. What happens if the outside temperature is lower than the thermostat setting? What if it's higher?
5. What happens if it is very, very cold outside? Does this make sense?

¹¹Since all we care about for this system is *changes* in energy, we can set zero reference anywhere we want. For example, you could set the zero energy point to correspond to a house temperature of 75 degrees.

Now let's add an air conditioning system to take care of the situation where the outside temperature is higher than the inside temperature.

Description Part B:

“What about hot days? A heating system can’t do anything to keep a house cool, but at least the thermostat can shut the furnace off and keep it from making the house even hotter. Of course, we could add an air conditioner to the system and connect it to the thermostat. Now if we set the heating system for 70 degrees and the cooling systems form 78 degrees, the thermostat will turn the furnace on if the temperature drops below 70 degrees and the air conditioner on if it rises above 78 degrees. This changes a one-directional feedback system which keeps us from getting cold into a two-directional system which will keep the house comfortable in any weather.¹²”



Exploration Part B:

6. Now, run the model again with the external temperature higher than the room temperature. Is the problem solved? What happens if the heater setting is higher than the air conditioner setting?

¹²Kauffman, Draper L. J. (1980) *Systems I: An Introduction to Systems Thinking*, Minneapolis, MN: S. A. Carlton, Publishers. Page 6.

Model 3: A Predator Prey System¹³

Description:



Interesting and only partly understood density variations are those which are not related to seasonal or obvious annual changes, but which involve regular oscillations or cycles of abundance with peaks and depressions every few years, often occurring with such regularity that population size may be predicted in advance. The best known cases concern mammals, birds, insects, fish and seed production in plants in northern environments. Among the mammals, the best-studied examples exhibit either a 9 to 10 year or a 3 to 4 year periodicity... Peaks of abundance often are followed by "crashes," or rapid declines...¹⁴

You should be able to model this phenomenon by connecting two simple population models and making them interdependent. Your two populations will be similar to the population model you built in the first exercise. In this case, you should make the death fraction of the predators dependent on the prey density. The number of prey killed per predator should also depend on the prey density. You may want to use the Define Graph option in STELLA to make graphical functions for these relationships.



Exploration:

1. Did your model produce the characteristic oscillations? What is the relationship between the predator and the prey oscillations?
2. What are the periods of oscillation for your two populations? Are they the same? Can they be different?

¹³Fox photograph © Leonard Lee Rue III/Animals Animals

¹⁴Odum Eugene P. (1971) *Fundamentals of Ecology*, Philadelphia: W.B. Saunders Co., pp. 190-192.

3. Can you make one or both populations in your model go to infinity? (This may have happened in the early stages of your model.) Why does this happen? Obviously this won't happen in real life; what is your model missing?
4. Add a PULSE function to the predator death rate and kill half the predators a few years into the simulation. What are the short-term (1-2 cycles) effects on the predator and prey populations? What are the long-term (more than 5 cycles) effects on the two populations?
5. Assume the predator population represented insects and the prey population represented a crop. What would be the result of spraying insecticides on your crop? If you looked only at the short-term results, what might be your reaction to this?

Solutions

This section contains one possible model solution for each of the descriptions in Modeling Exercises: Section 1. I wish to stress that the models and equations outlined here represent only one of many possible solutions. There is more than one right answer for all modeling exercises. Your model may have more complexity, but it should be similar. If your model displays plausible behavior and you can explain each relation in the model, then the model is probably useful. You will be learning more about testing whether or not your model is adequate in future sections of Road Maps.

Model 1: An Ecological System

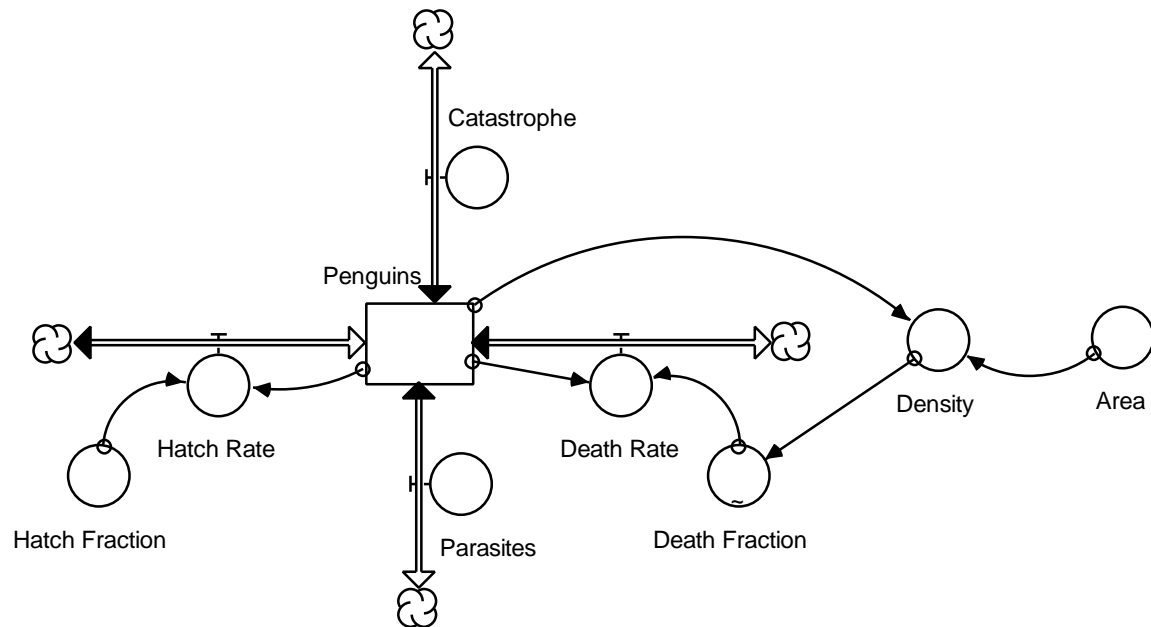
For the first modeling exercise, I have included my progress through the steps outlined in the How to Build Models section included earlier in this paper. For this simple system, your results will probably be similar.

- 1. Define the purpose:** My purpose for building this model is to simulate the behavior described in the reading passage. I also wish to learn about population dynamics and why this behavior occurs.
- 2. Define the system:** I wish to model the population. I will use penguins in this case.
- 3. Identify key variables:** The key variables mentioned in the passage are: Density, the population, and external regulatory agents. I will also include the hatch rate and the death rate.
- 4. Describe the behavior of the key variables:** I want the population to center on some standard value. It should change up or down, depending on what external events affect the population, but when these events end, the population should return to the standard value.
- 5. Identify the stocks and flows:** The one stock in my model is the population of penguins. It is increased by the hatch rate flow and decreased by the death rate flow.
- 6. Map the stocks and flows on your STELLA diagram pad.** See the model in Figure 1.
- 7. Define the flows:** The hatch rate is equal to the size of the penguin population multiplied by a hatch fraction. The death rate is similar, defined by the population times a death fraction. To include the regulatory effects of density, I made the death fraction dependent on density. The hatch fraction is constant.
- 8. Include quantitative information:** Since the purpose of this model does not require much accuracy in the numerical data of the model, I did not research the reproductive characteristics of penguins. Instead, I simply chose an initial population of 800 penguins in an area of 10 acres. I let the hatch fraction be equal to .2 and the death fraction varied from .08 for an extremely small population to .294 for crowded conditions. (The model equations that follow contain more detailed information)

- 9. Run the model:** The results of running the model are included in the exploration answers below.
- 10. Evaluate the model:** The model displayed behavior consistent with the description upon which it was based. Running the scenarios suggested in the exploration helped to test the robustness of the model.
- 11. Improve and expand upon the model:** The model served its purpose so far. From here, you could add the effects of a limited food supply, or show the effect of density on the hatch fraction.

Solutions to Exploration Questions:

1. This model is centered around a level that represents the total population being analyzed. I decided to examine a population of penguins for my model. The population level is increased by the hatch rate and decreased by the death rate. For this model, I made the hatch fraction constant at 0.2. The death rate is dependent on the density of the population. The values of the parameters in the system (area, hatch fraction, initial population, etc.) are somewhat arbitrary. Your values will be different, but you should have a similar structure.

Model Diagram:**Figure 1: STELLA Diagram of the Population System**

This population system is a simple, one-level model with a constant hatch fraction and a death fraction dependent on density.

To represent the dependency of the death fraction on density, I created the following graphical function:

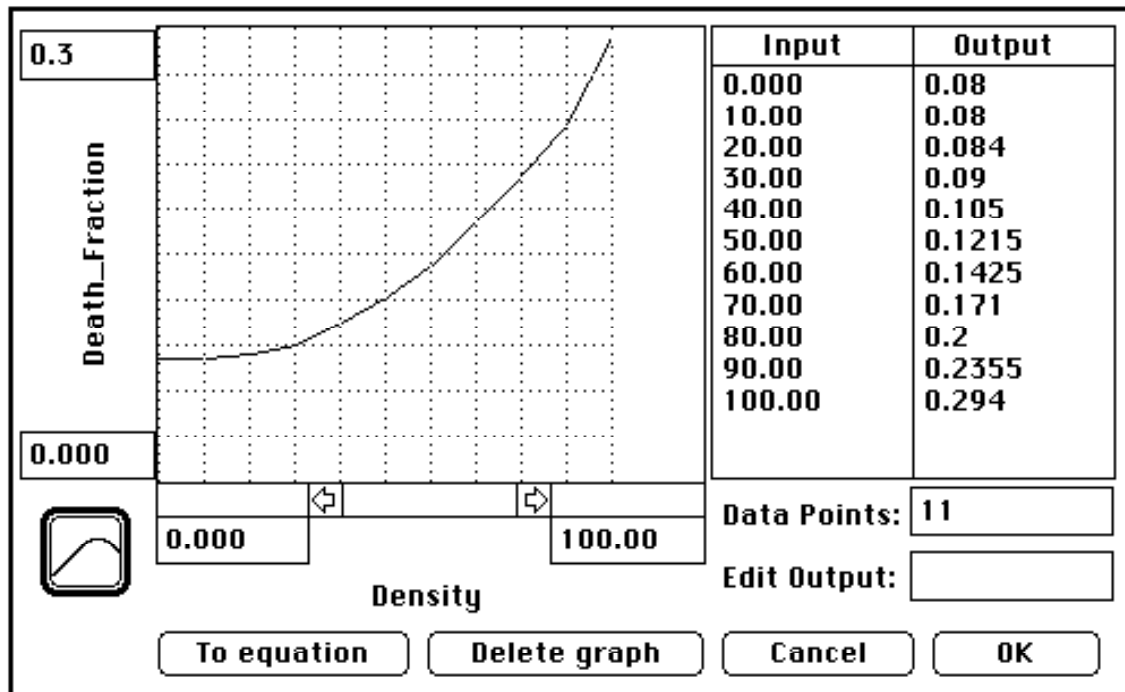


Figure 2: Death Fraction graphical function

This graph determines the dependency of the death fraction on the density. When density is very low, the death fraction is constant at 0.08, less than the hatch fraction. As the density increases, the death fraction also increases.

2. Equilibrium will occur when the hatch rate and the death rate are equal. For my model, this occurred when the population was equal to 800 penguins. The density at this point was 80 penguins per acre, which gives us a death fraction of 0.2 (see Figure 2) This makes sense because the hatch fraction is a constant at 0.2.

3. To simulate a catastrophe, I use the PULSE function in STELLA as follows:

$$\text{Catastrophe} = \text{PULSE}(400,5,0)$$

This caused a decrease in the population from 800 to 400 after 5 years. The density dropped down to 40 penguins per acre, causing the death fraction to fall below the hatch fraction and the population slowly rose back up to its equilibrium value of 800.

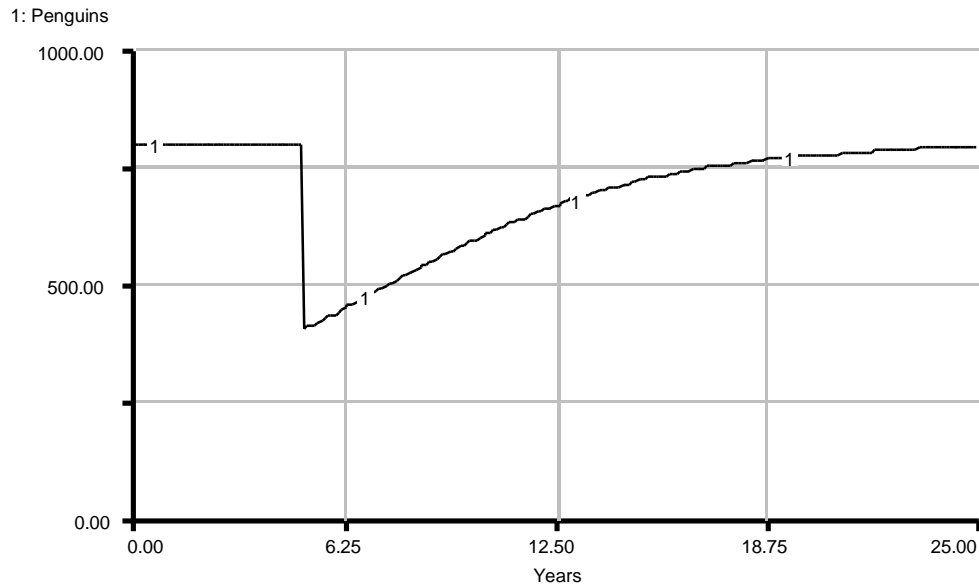


Figure 3: Graph of population response to a one time catastrophe

4. The response to the constant outflow was very different. I added a constant outflow labeled Parasites and set it to 20 penguins/year. The population dropped to a new equilibrium at approximately 700 penguins. Notice that the new equilibrium is lowered significantly by a loss of only 20 extra penguins per year. The reason for this is that the population must drop down to a level where the hatch rate is larger than the death rate by 20 penguins per year. The new equilibrium will vary depending on the parameters in your model. If both your hatch and death fractions are high, the drop will be less substantial. It will also be less if the dependency of your death fraction on density is steeper.

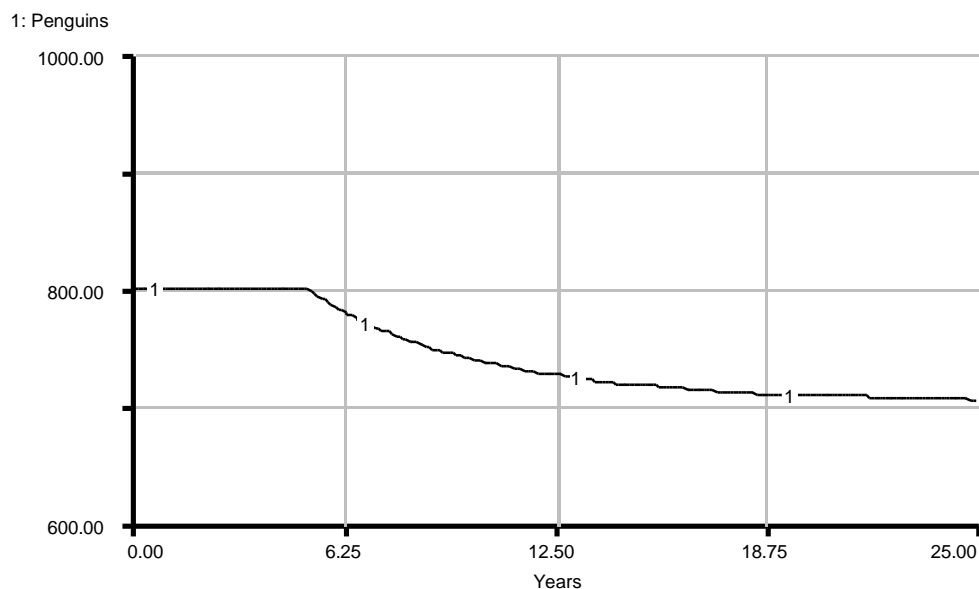


Figure 4: Graph of population response to a constant outflow.

Model Equations:

Penguins(t) = Penguins(t - dt) + (Hatch_Rate - Death_Rate - Catastrophe - Parasites) * dt
 INIT Penguins = 800

DOCUMENT: This stock represents the number of penguins in the population. It is increased by hatching and decreased by deaths.

UNITS: penguins

INFLOWS:

Hatch_Rate = Penguins * Hatch_Fraction

DOCUMENT: This is the hatch rate for the population. It is determined by multiplying the population by a hatch fraction.

UNITS: penguins/year

OUTFLOWS:

Death_Rate = Penguins * Death_Fraction

DOCUMENT: This is the death rate of the population. It is determined by multiplying the population by a death fraction.

UNITS: penguins/year

Catastrophe = PULSE(400,5,0)

DOCUMENT: This flow was used to explore the effects of a sudden drop in the population.

UNITS: penguins/year

Parasites = 20

DOCUMENT: This flow is used to represent a constant outflow from the population.

UNITS: penguins/year

Area = 10

DOCUMENT: This is the total land area occupied by the population.

UNITS: acres

Density = Penguins/Area

DOCUMENT: This is the population density. It affects the death fraction of the population.

UNITS: penguins/acre

Hatch_Fraction = .2

DOCUMENT: This represents the fraction of the population that reproduces each year. If the hatch fraction is 0.2, then the hatch rate will be 20% of the population.

UNITS: 1/years

Death_Fraction = GRAPH(Density)

(0.00, 0.08), (10.0, 0.08), (20.0, 0.085), (30.0, 0.09), (40.0, 0.1), (50.0, 0.12), (60.0, 0.14), (70.0, 0.17), (80.0, 0.2), (90.0, 0.24), (100, 0.3)

DOCUMENT: The death fraction is dependent on the density of the population. The higher the density, the larger the fraction of the population that dies each year.

UNITS: 1/years

Model 2: A Heating/Cooling System

Model Diagram:

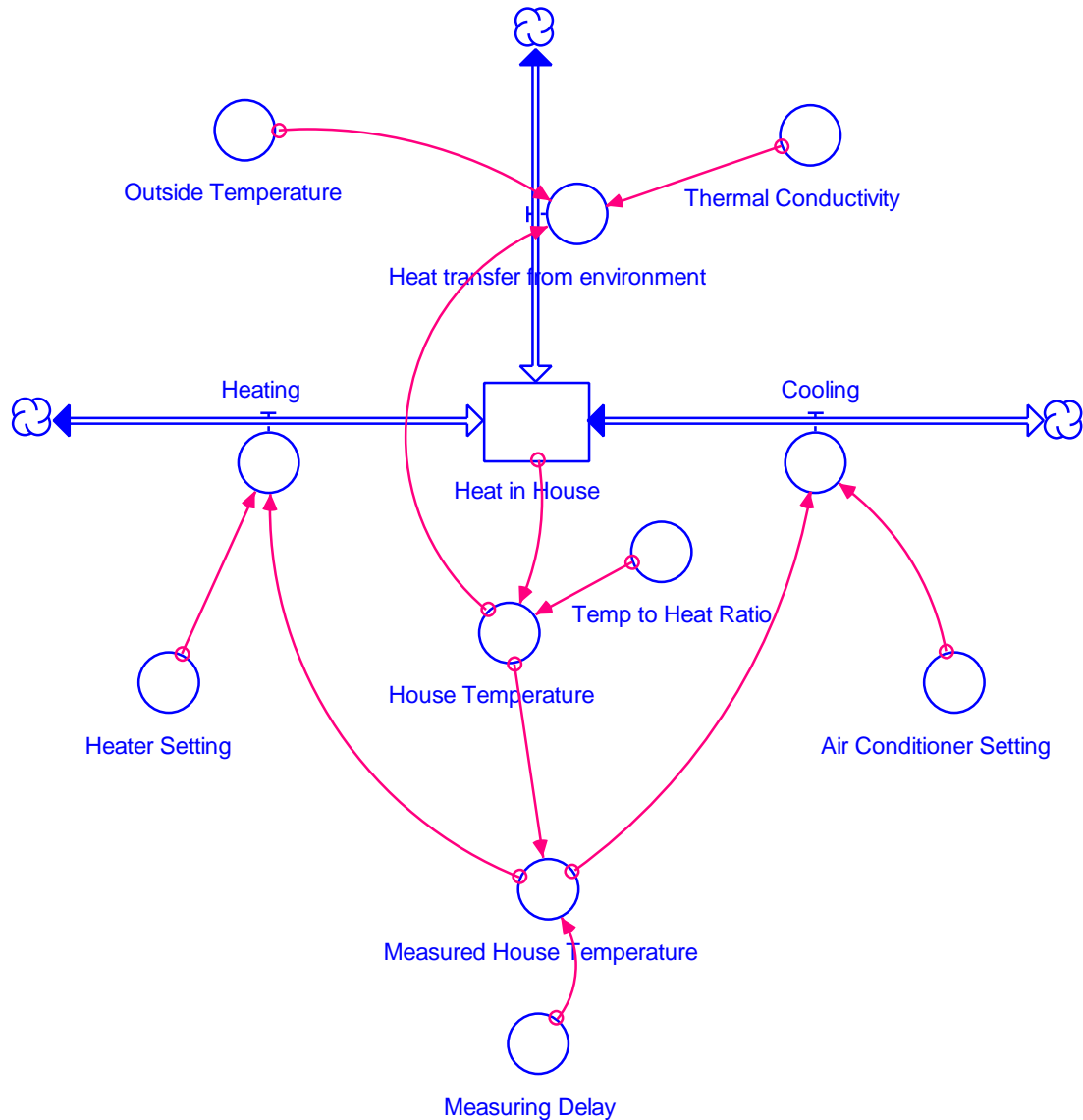


Figure 5: A Heating/Cooling System Model

The heat in the house is affected by the external environment as well as the heating and cooling systems. The heat transfer from the environment is dependent on the difference in temperature between the house and the environment. The heating and cooling systems are either on or off dependent on the measured house temperature and their respective settings.

1. The level in this model is the amount of heat in the house. The heat is then used to calculate the temperature using the relation given in the hints.

Temperature ? Heat in house

I used this to define the house temperature. I assumed that the heat in the house was equal to zero when the temperature was 75 degrees Fahrenheit.

$$\text{House Temperature} ? \frac{\text{Heat In House}}{\text{Temp to Heat Ratio}} ? 75$$

2. The house temperature is affected by the heat transferred from the environment, and the heating system.
3. The heating system will add heat to the house at a rate of 100,000 joules per hour whenever the measured temperature in the house is lower than the heater setting temperature. I used the following relation to define the energy transferred from the environment to the house:

Energy Transfer ? (External Temperature ? House Temperature)

So, the heat transfer from the environment was defined as follows:

(Outside Temperature ? House Temperature) ? Thermal Conductivity

The Thermal Conductivity accounts for the ability of the whole house to transfer heat to and from the environment. In effect, the lower the value, the better insulated the house.

4. If the outside temperature is lower than the thermostat setting, the house temperature will cycle around the heater setting value. The cycling occurs because of the delays involved with measuring and changing the house temperature. If the outside temperature is higher than the heater setting, then the house will gradually heat up to the outside temperature. Heating will remain at 0 because the measured house temperature is above the heater setting.

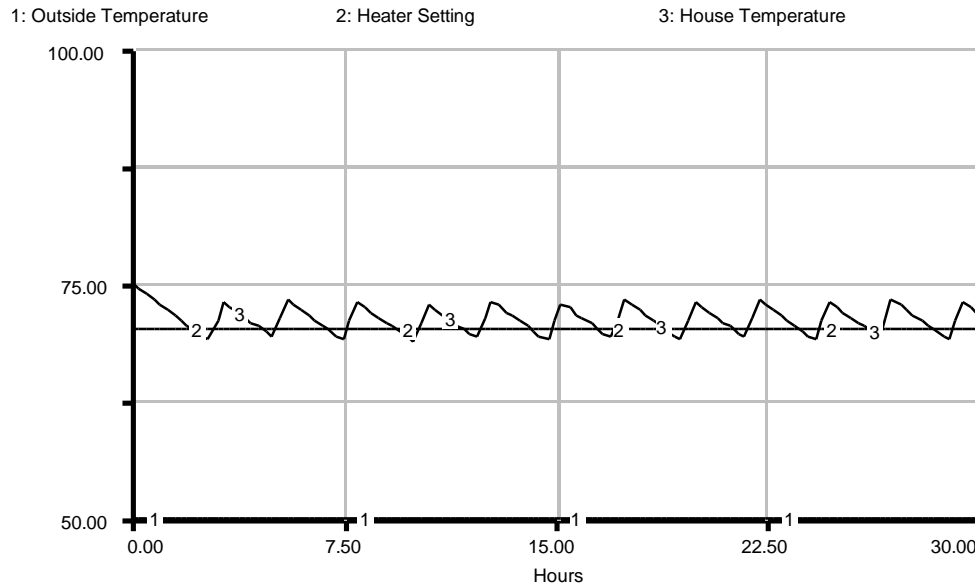


Figure 6: Graph of house temperature with heating system only.

5. The heating system will be unable to maintain the temperature of the house if heat is being removed from the house faster than the heating system can add it back in. In my model this occurs when the external temperature is more than 100 degrees below the heater setting temperature. This value is dependent on the power of the heating system as well as the thermal conductivity of the house.

The cooling system is very similar to the heating system. It will remove heat from the house at a rate of 100,000 joules per hour whenever the measured temperature in the house is higher than the air setting.

6. Now, the system will cycle about the air setting temperature if the outside temperature is higher than the measured house temperature. (See Figure 7) You should notice that the cycling period is longer. This occurs because the outside temperature is closer to the room temperature so heat transfer with the environment is slower. If the outside temperature is between the two settings, the house temperature will converge on it and remain at that temperature.

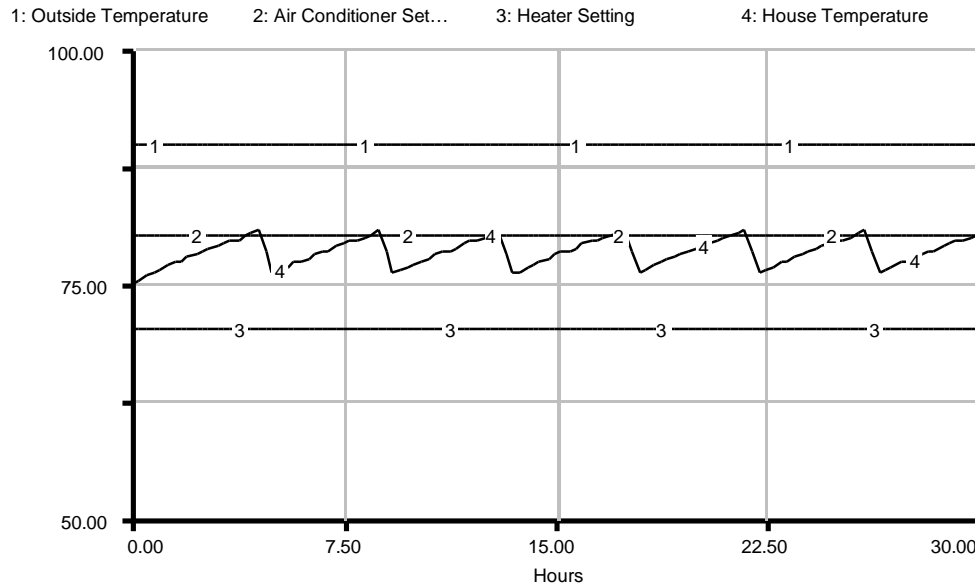


Figure 7: Graph of house temperature with both heating and air conditioning.

In this run, the outside temperature is higher than the air conditioner setting and the temperature cycles around the air conditioner setting temperature.

Setting the heater setting higher than the air conditioner setting gave some interesting results. When the outside temperature was between the two settings, the house temperature converged on it as before except that both the heater and the air conditioner were on. (Somewhat expensive) When the outside temperature was higher than both settings, the house temperature rose slowly to the heater setting because only heat transfer from the environment was affecting it. (heating and cooling balanced out) Then, when the temperature rose slightly above the heater setting the heater shut off and the temperature rapidly dropped until the heating system turned back on after a delay. The response to an outside temperature below both settings was similar. There was a cycle of slow decreases until the air setting temperature was reached, followed by rapid increases as the cooling system turned off.

Model Equations:

$$\text{Heat_in_House}(t) = \text{Heat_in_House}(t - dt) + (\text{Heating} + \text{Heat_transfer_from_environment} - \text{Cooling}) * dt$$

INIT Heat_in_House = 0

DOCUMENT: This is the amount of heat in the house. It is affected by the heating and air conditioning systems as well as the heat transferred from the environment. The initial amount of heat in the house is set to zero as a reference point. This amount of heat is assumed to correspond to a temperature of 75 degrees Fahrenheit.

Units: Joules

INFLOWS:

Heating = IF Measured_House_Temperature < Heater_Setting THEN 100000 ELSE 0

DOCUMENT: This is the heat transferred to the house by the heating system. If the measured house temperature is lower than the Heater_Setting, then Heating = +100,000 Joules per hour. If the house temperature is higher, then Heating = 0

Units: Joules/hour

Heat_transfer_from_environment = (Outside_Temperature-House_Temperature)*Thermal_Conductivity

DOCUMENT: This is the amount of heat transferred into the house from the environment. It is proportional to the temperature difference between the house and the environment.

Units: Joules/hour

OUTFLOWS:

Cooling = IF Measured_House_Temperature > Air_Conditioner_Setting THEN 100000 ELSE 0

DOCUMENT: This is the heat removed from the room by the air conditioning system. If the measured house temperature is higher than the Air_Setting, then Cooling = +100,000 joules per hour. If the house temperature is lower, then Cooling = 0

Units: Joules/hour

Air_Conditioner_Setting = 80

DOCUMENT: This is the thermostat setting for the cooling system.

Units: Degrees

Heater_Setting = 70

DOCUMENT: This is the thermostat setting for the heater system.

Units: Degrees

House_Temperature = (Heat_in_House/Temp_to_Heat_Ratio)+75

DOCUMENT: This is the temperature in the house. Changes in the temperature are proportional to changes in the house's energy. The change in temperature is calculated by multiplying the energy by the temp to heat ratio. The actual temperature is calculated by adding 75 (the reference temperature corresponding to zero energy).

Units: degrees

Measured_House_Temperature = SMTH1(House_Temperature,Measuring_Delay)

DOCUMENT: This is the room temperature as perceived by the thermostat. It is calculated by smoothing the actual room temperature with a delay.

Units: degrees

Measuring_Delay = .2

DOCUMENT: This is the delay involved in measuring the room temperature.

Units: hours

Outside_Temperature = 50

DOCUMENT: This is the temperature of the environment external to the house. The house temperature would eventually reach this temperature without a heating/cooling system.

Units: Degrees

Temp_to_Heat_Ratio = 10000

DOCUMENT: This is the ratio between temperature and energy in the house. This number tells how many joules of energy are required to heat the house by 1 degree.

UNITS: Joules/Degree

Thermal_Conductivity = 1000

DOCUMENT: This factor represents how easily energy is transferred from the outside environment to the house.

UNITS: Joules/hour/degree

Model 3: A Predator/Prey System

This is the model I came up with to simulate the predator/prey system.

Model Diagram:

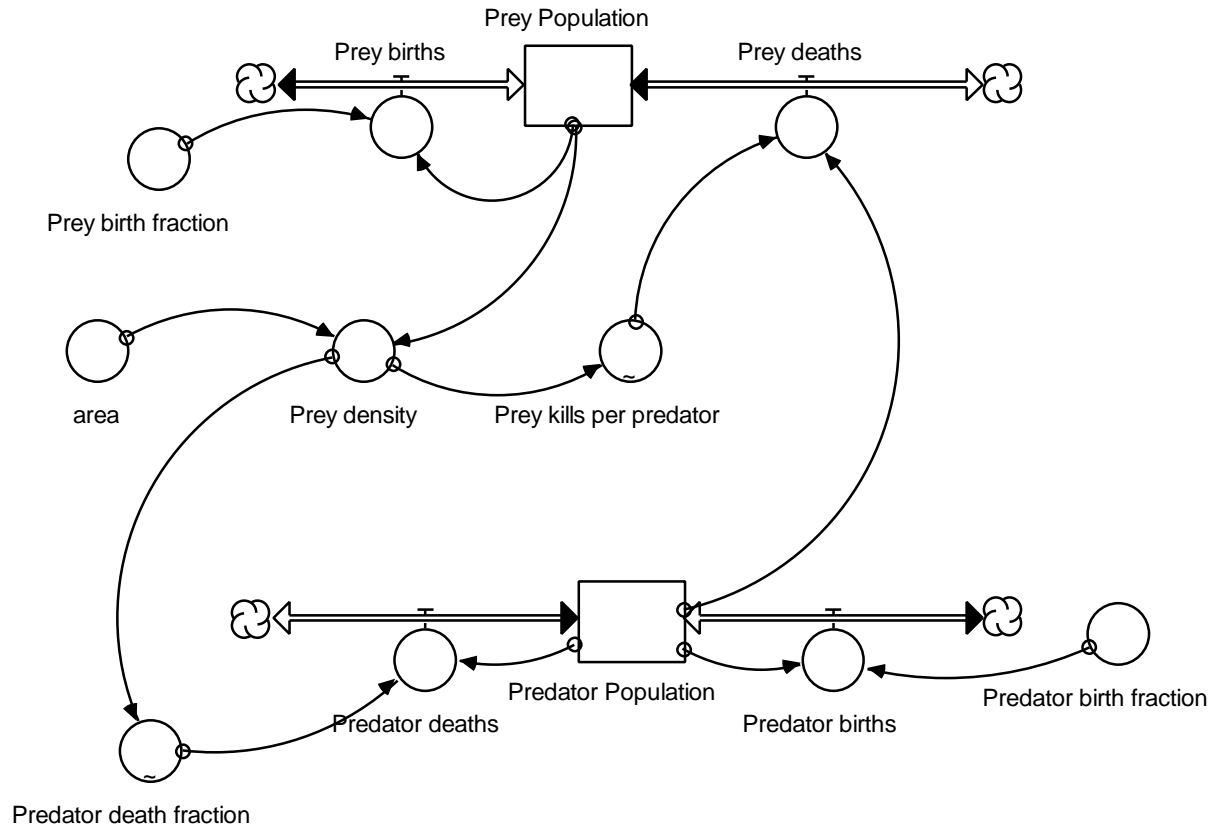


Figure 4: A Predator/Prey System Model

Two population subsystems interact to produce oscillation. The predator deaths and the prey kills per predator vary with the prey density. Both predator and prey birth fractions are constant.

I used the following graphical functions to define the dependencies of prey kills per predator and predator death fraction on prey density.

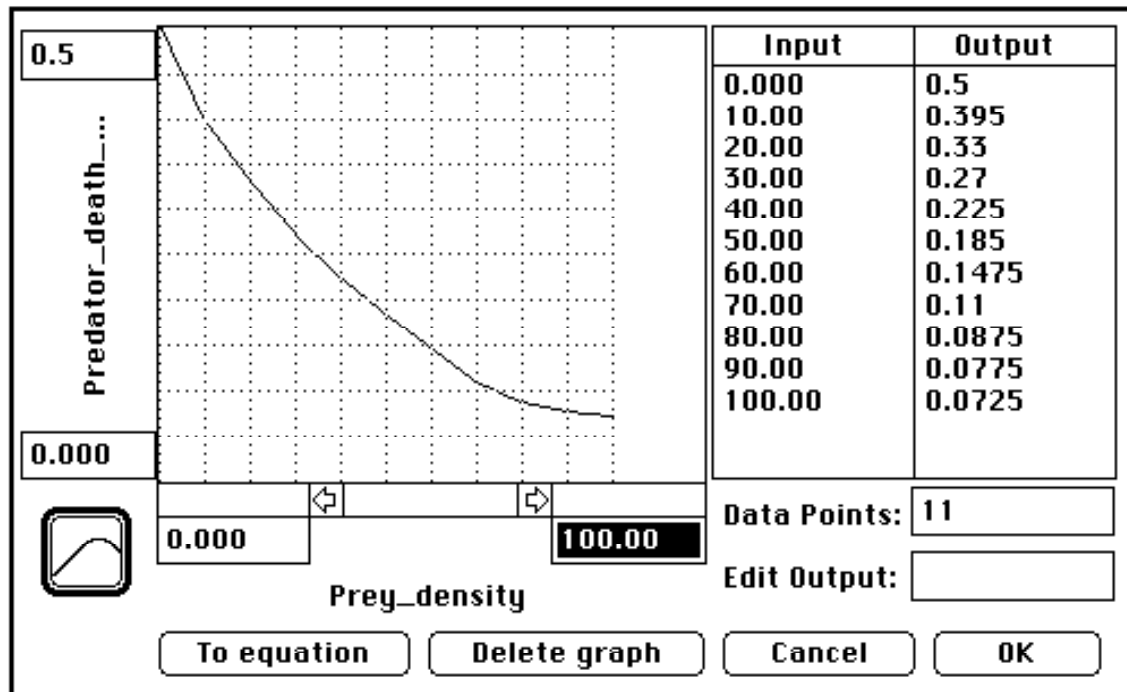


Figure 5: Predator Death Fraction

The predator death fraction varies with prey density. When there are no prey, the predators die very rapidly. As the prey density increases, the number of predators that starve decreases until the death fraction levels off at .0725.

This graph shows that the predator death fraction varies widely with the prey density. When the number of prey in the area is small, the predators are unable to catch very many and their survival rate drops. When there are no prey at all, the predators are dying twice as fast as they are being born. As the density of the prey increases, the death fraction of the predators decreases. Eventually, the predators reach a point where they are no longer at risk of dying of starvation, but rather from other factors. This is represented by the leveling-off of the predator death fraction graph when prey density is high.

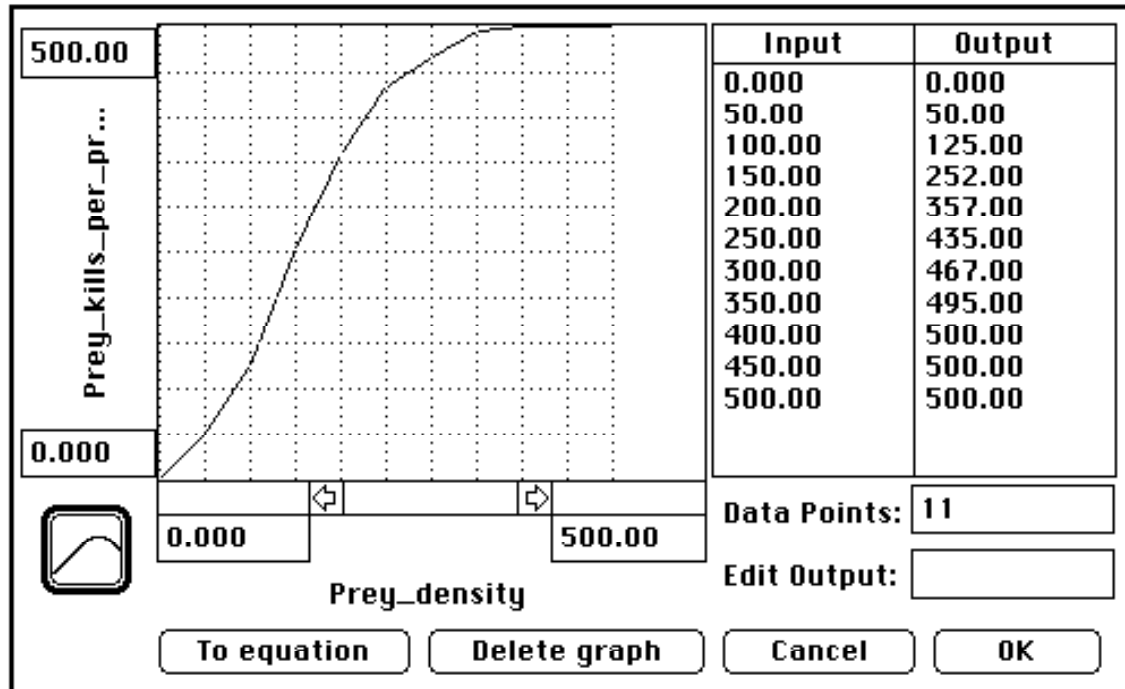


Figure 6: Prey kills per predator

When there are no prey, the predators don't catch any. As the prey density rises, so does the catch per predator until the predators are satiated at 500 prey per year.

This graph shows that prey kills per predator varies with the prey density. Naturally, when there are no prey, the density is 0 and prey kills per predator is also 0. When the prey density begins to rise, prey kills per predator rises quickly, and then levels off. This leveling off represents the point at which the predators are satiated and do not wish to eat any more prey regardless of how abundant they are.

1. You should notice that the prey population is decreasing the fastest when the predator population is highest and increasing the most when the predator population is lowest.
2. Both populations in my model oscillated with a period of about 14 years. Both species must oscillate with the same period because they are interdependent. The behavior of the prey is directly dependent on the predators and vice-versa.

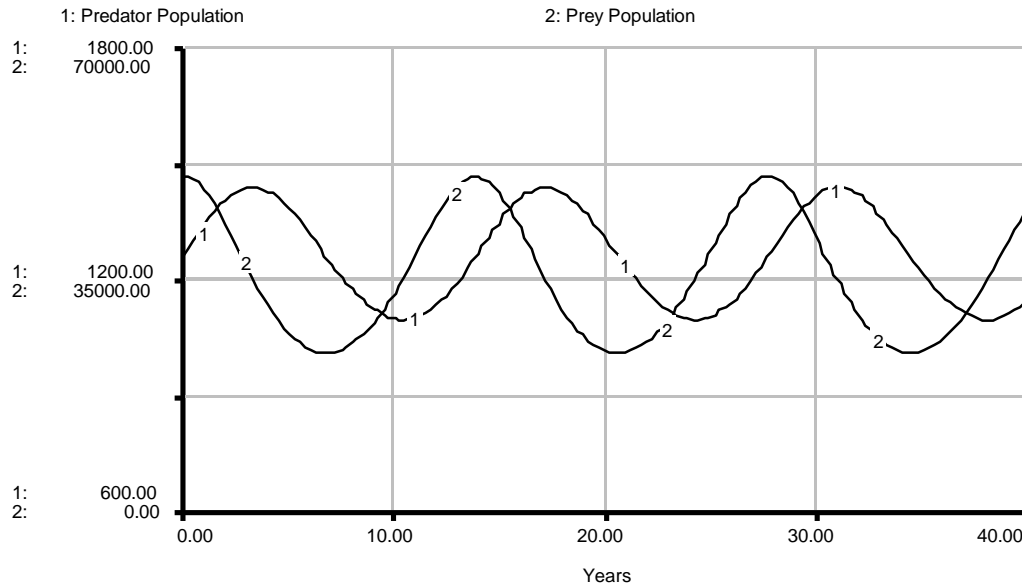


Figure 7: Graph of predator-prey system behavior

This graph shows a response of the predator prey system. The period is about 14 years. Notice how the predator population lags behind the prey population. (Please note that the scales for the two populations are different.)

3. In my model, the system became unstable if I started the prey population above 580,000. Both populations would increase exponentially without bound. Beginning with a large prey population meant that the predators were catching as many prey as possible, but this was inadequate given the tremendous birth rate of the prey. Because the prey birth fraction is larger than the predator birth fraction, the predators will never catch up. In real life this would never happen. Other limiting factors like food and water would restrict the prey population until the predators caught up.
4. I used the following function to kill half of my predators in year 15:

$$\text{PULSE}(\text{Predator_Population}/2, 15, 0)$$

This resulted in an instant decrease in the predator population followed by a large increase in the prey population. In the next cycle, the predator population greatly overshot its previous maximum and the prey population fell below its previous minimum value. After a few more cycles, both populations returned to their previous oscillation with the same period and amplitude.

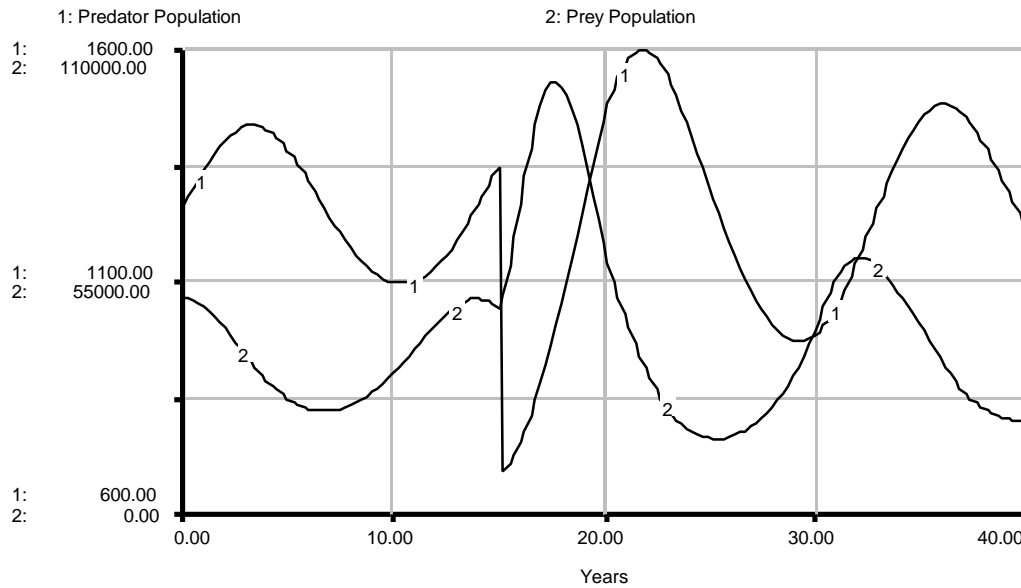


Figure 8: Graph of predator-prey system behavior

In this run, half of the predator population was eliminated in year 15. Just following the shock, the prey population rose quickly. However, the next few cycles had a much greater amplitude than before the shock. Eventually, the system returns to the original oscillation behavior.

5. This model shows that spraying crops with insecticide can have some unwanted consequences. Initially, spraying has the effect decreasing the insect population and increasing crop yield. But, a few years later, the insect population is higher than ever and crop yield is worse than it has ever been. The immediate reaction to this effect might be to spray with an even larger amount of insecticide. Using the model, you can see that this will result in even greater extremes than the first time. The farmer can end up in a vicious cycle of larger and larger sprayings of insecticide and greater and greater fluctuations in his crop yield and the size of the insect population.

Model Equations:

Predator_Population(t) = Predator_Population(t - dt) + (Predator_births - Predator_deaths) * dt
 INIT Predator_Population = 1250

DOCUMENT: Initially, there are 1250 predators.

Units: Predators

INFLOWS:

Predator_births = Predator_Population*Predator_birth_fraction

DOCUMENT: A compounding process is used to depict predator births. The births flow is defined as the product of predator population and their birth fraction.

Units: Predators/year

OUTFLOWS:

Predator_deaths = Predator_Population*Predator_death_fraction+PULSE(Predator_Population/2,15,1000)

DOCUMENT: This is a draining process. Some fraction of the Predator population will die each year. The PULSE function is used to jolt the system out of its initial steady-state.

Units: Predators/year

Prey_Population(t) = Prey_Population(t - dt) + (Prey_births - Prey_deaths) * dt
 INIT Prey_Population = 50000

DOCUMENT: Initially, there are 50,000 prey in the ecosystem.

Units: Prey

INFLOWS:

Prey_births = Prey_Population*Prey_birth_fraction

DOCUMENT: A compounding process is used to depict prey births. The births flow is defined as the product of Prey population and their birth fraction.

Units: Prey/year

OUTFLOWS:

Prey_deaths = Predator_Population*Prey_kills_per_predator

DOCUMENT: Each predator has a productivity, given by prey kills per predator.

Units: Prey/year

area = 1000

DOCUMENT: The area of the ecosystem is defined as 1000 acres.

Units: acres

Predator_birth_fraction = .25

DOCUMENT: On average, each Predator will generate .25 offspring per year.

Units: 1/years

Prey_birth_fraction = 1.25

DOCUMENT: Each prey in the ecosystem is assumed to produce 1.25 offspring (on average) per year.

Units: 1/years

Prey_density = Prey_Population/area

DOCUMENT: This equation calculates the average density of prey in the ecosystem.

Units: Prey/acre

Predator_death_fraction = GRAPH(Prey_density)

(0.00, 0.5), (10.0, 0.395), (20.0, 0.33), (30.0, 0.27), (40.0, 0.225), (50.0, 0.185), (60.0, 0.147), (70.0, 0.11), (80.0, 0.0875), (90.0, 0.0775), (100, 0.0725)

DOCUMENT: Predator mortality -- the fraction of the predator population that dies each year, is assumed to depend on prey density. With fewer prey, a greater fraction of the predator population will starve.

Units: 1/years

Prey_kills_per_predator = GRAPH(Prey_density)

(0.00, 0.00), (50.0, 50.0), (100, 125), (150, 252), (200, 357), (250, 435), (300, 467), (350, 495), (400, 500), (450, 500), (500, 500)

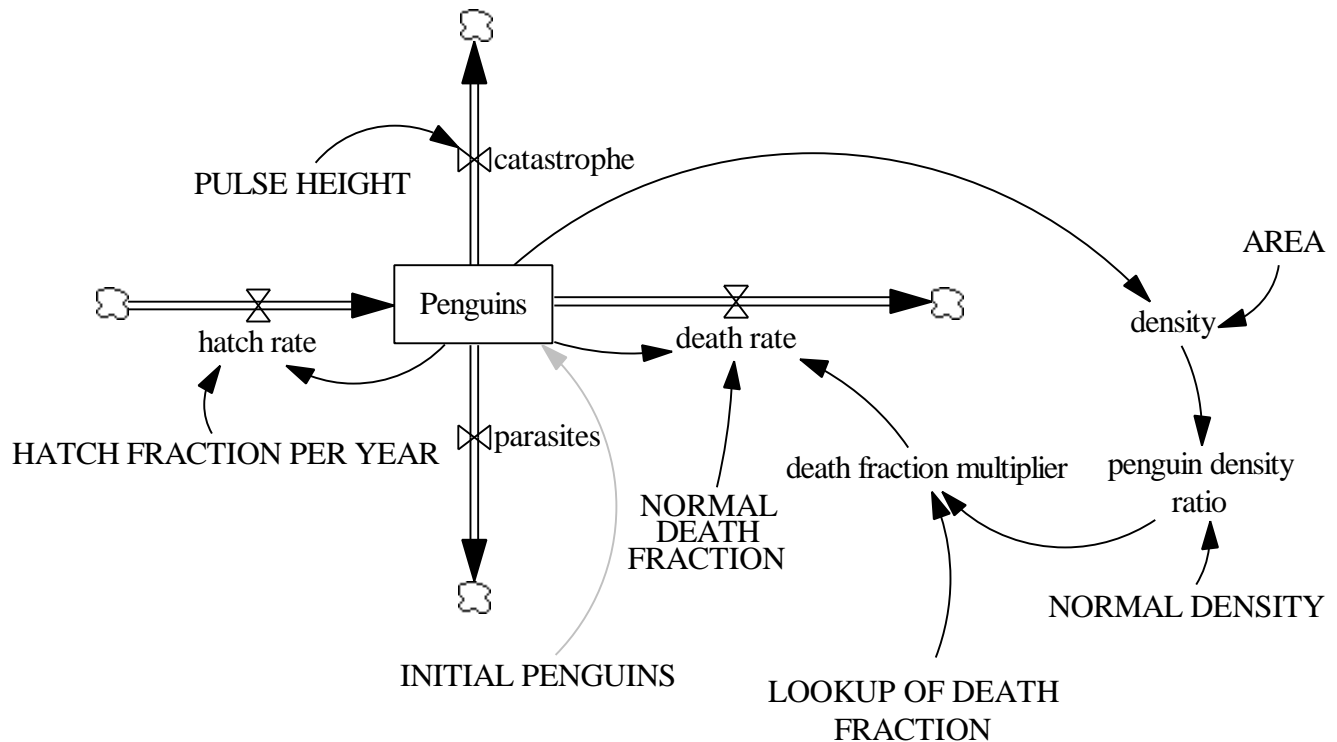
DOCUMENT: The number of prey killed per predator (per year) increases with the density of prey in the ecosystem. Note that when density is 0 (there are no prey), kills per predator per year must be 0.

Units: Prey/year/Predator

Vensim Examples: Modeling Exercises Section 1

Lei Lei and Nathaniel Choge
October 2001

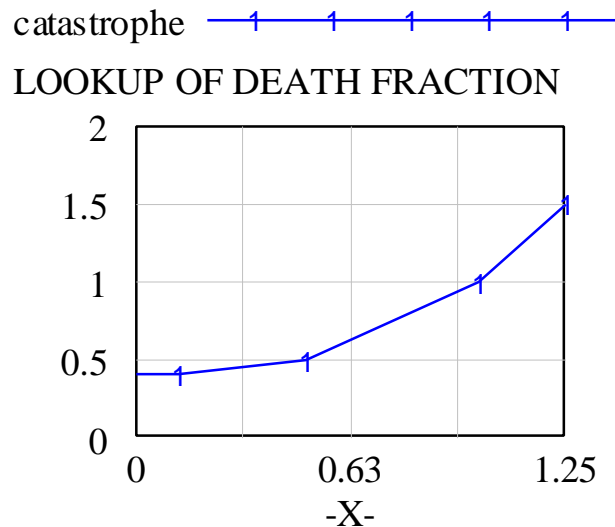
Model 1: An Ecological System



Documentation for Ecological System:

- (01) AREA = 10
Units: acres
This is the total land area occupied by the population.
- (02) catastrophe = PULSE HEIGHT * PULSE(5,1)
Units: penguins/year
This flow was used to explore the effects of a sudden drop in the population.
- (03) death fraction multiplier = LOOKUP OF DEATH FRACTION(penguin density ratio)
Units: dmnl
This is the equation that gives the death fraction is a function of the penguin density ratio.

- (04) $\text{death rate} = \text{Penguins} * \text{death fraction multiplier} * \text{NORMAL DEATH FRACTION}$
 Units: penguins/year
 This is the death rate of the population. It is determined by multiplying the population by a death fraction, times a death rate multiplier which serves to produce the correct dimensions for the flow.
- (05) $\text{density} = \text{Penguins} / \text{AREA}$
 Units: penguins/acres
 This is the population density. It affects the death fraction of the population.
- (06) $\text{FINAL TIME} = 25$
 Units: year
 The final time for the simulation.
- (07) $\text{HATCH FRACTION PER YEAR} = 0.2$
 Units: 1/year
 This represents the fraction of the population that reproduces each year. If the hatch fraction is 0.2, then the hatch rate will be 20% of the population. To obtain 0.2 we assume that half of the penguins are female and that each female produces an offspring every 2.5 years.
- (08) $\text{hatch rate} = \text{Penguins} * \text{HATCH FRACTION PER YEAR}$
 Units: penguins/year
 This is the hatch rate for the population. It is determined by multiplying the population by a hatch fraction.
- (09) $\text{INITIAL PENGUINS} = 800$
 Units: penguins
 The system starts with 800 penguins.
- (10) $\text{INITIAL TIME} = 0$
 Units: year
 The initial time for the simulation.
- (11) $\text{LOOKUP OF DEATH FRACTION}([(0,0)-(1.25,1.5)], (0,0.4), (0.125,0.4), (0.375,0.45), (0.5,0.5), (1,1), (1.25,1.5))$
 Units: dmnl
 The death fraction is dependent on the density of the population. When the penguin density ratio is 1, the density is equal to the normal density and the death fraction is 0.2. The higher the penguin density ratio, the larger the fraction of the population that dies each year.



- (12) NORMAL DEATH FRACTION = 0.2
 Units: penguins/year
 This multiplier produces the correct dimensions for death rate. Since "Penguins" has units of Penguins and "death fraction" is dimensionless we need to divide by year to produce the correct dimensions for "death rate."
- (13) NORMAL DENSITY = 80
 Units: penguins/acres
 This variable is introduced in order to make the model dimensionless. We assume that the initial population divided by the area represents the normal density of penguins.
- (14) parasites = STEP(20 , 5)
 Units: penguins/year
 This flow is used to represent a constant outflow from the population.
- (15) penguin density ratio = density/NORMAL DENSITY
 Units: dmnl
 This ratio will be used as a dimensionless input to the lookup table.
- (16) Penguins= INTEG (+hatch rate-catastrophe-death rate-parasites, INITIAL PENGUINS)
 Units: penguins
 This stock represents the number of penguins in the population. It is increased by hatching and decreased by deaths.
- (17) PULSE HEIGHT = 400
 Units: penguins/year
 This is the number of penguins removed after 5 years due to catastrophe.
- (18) SAVEPER = TIME STEP
 Units: year

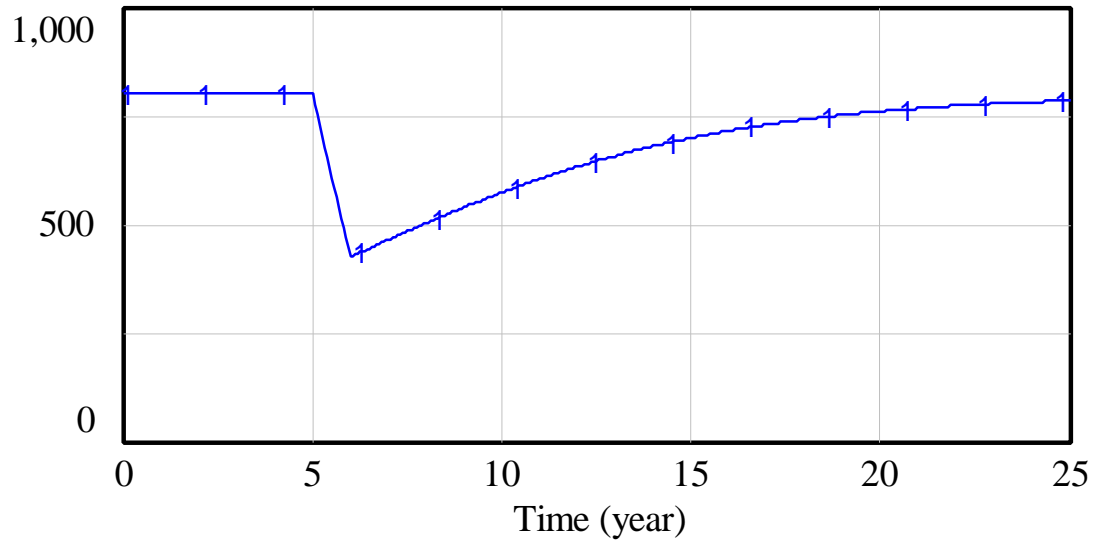
The frequency with which output is stored.

(19) TIME STEP = 0.0625

Units: year

The time step for the simulation.

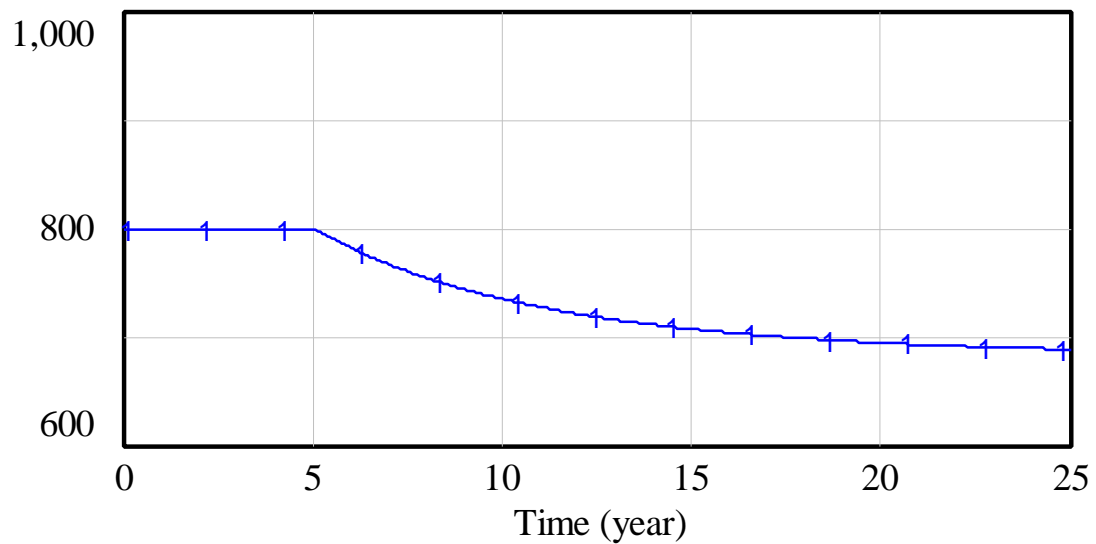
Graph for Penguins



Penguins : catastrophe ————— penguins

Figure 10: Vensim Equivalent of Figure 3: Simulation of response to one-time Catastrophe

Graph for Penguins



Penguins : parasites ————— penguins

11: Vensim Equivalent of Figure 4: Simulation of response to constant outflow

Model 2: Heating/Cooling System

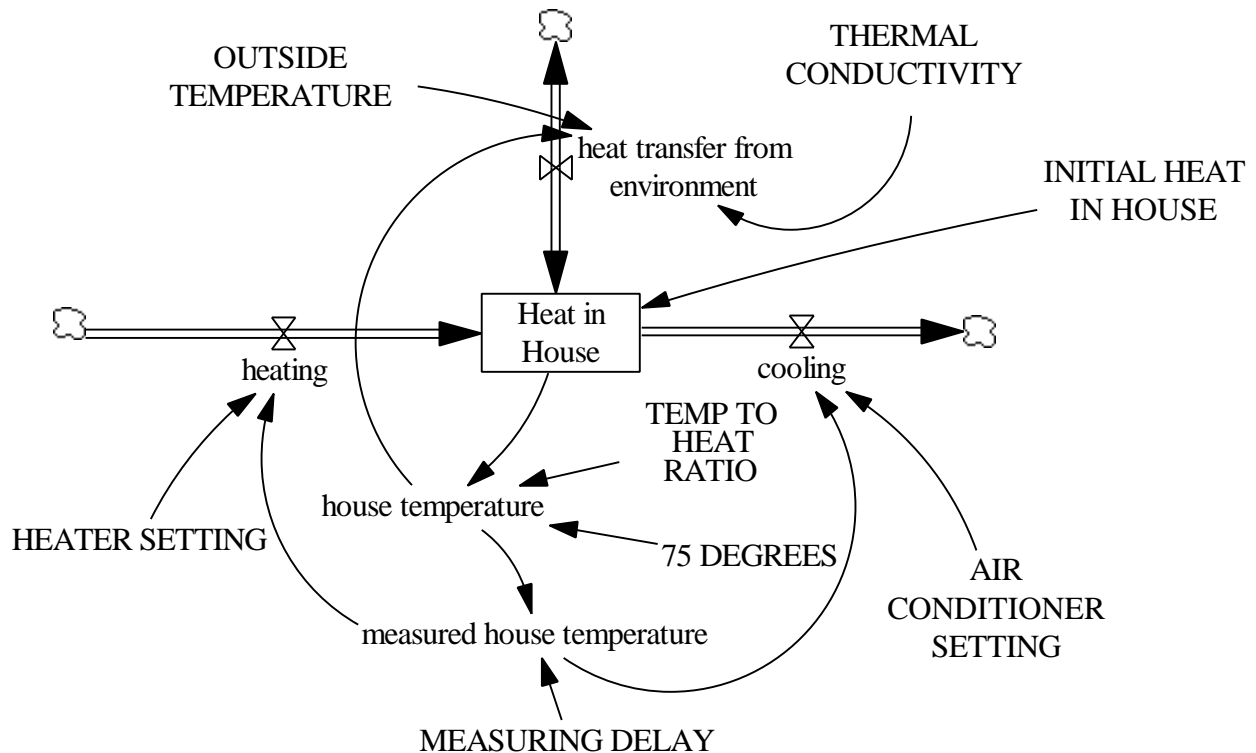


Figure 12: Vensim Equivalent of Figure 5: Heating/Cooling System Model

NOTE: Differences between the Stella and Vensim versions of the Heating/Cooling System Model are described below the model documentation.

Documentation for Heating/Cooling System Model

- (01) "75 DEGREES" = 75
Units: degrees
This is the initial temperature of the house. It is the temperature that determines the reference heat in house for our simulation. The reference is the heat that we take as already present in the house as the base for our simulation.
- (02) AIR CONDITIONER SETTING = 80
Units: degrees
This is the thermostat setting for the cooling system.
- (03) cooling = IF THEN ELSE(measured house temperature>AIR CONDITIONER SETTING, 100000 , 0)
Units: btu/hour

This is the heat removed from the room by the air conditioning system. If the measured house temperature is higher than the air setting, then cooling = 100,000 joules per hour. If the house temperature is lower, then cooling = 0

- (04) FINAL TIME = 30
Units: hour
The final time for the simulation.

- (05) Heat in House= INTEG (+heating-cooling+heat transfer from environment, INITIAL HEAT IN HOUSE)
Units: btu
This is the amount of heat in the house relative to the heat in the house at 75 degrees. It is affected by the heating and air conditioning systems as well as the heat transferred from the environment. The initial amount of heat in the house is set to the heat at 75 degrees by setting "INITIAL HEAT IN HOUSE" to zero.

- (06) heat transfer from environment = (OUTSIDE TEMPERATURE-house temperature)*THERMAL CONDUCTIVITY
Units: btu/hour

- (07) HEATER SETTING = 70
Units: degrees
This is the thermostat setting for the heater system.

- (08) heating = IF THEN ELSE(measured house temperature<HEATER SETTING,100000 , 0)
Units: btu/hour
This is the heat transferred to the house by the heating system. If the measured house temperature is lower than the heater setting, then heating = 100,000 btu per hour. If the house temperature is higher, then heating = 0.

- (09) house temperature = (Heat in House/TEMP TO HEAT RATIO)+"75 DEGREES"
Units: degrees
This is the temperature in the house. Changes in the temperature are proportional to changes in the house's energy. The change in temperature is calculated by multiplying the energy by the temp to heat ratio. The actual temperature is calculated by adding 75 degrees (the initial reference temperature).

- (10) INITIAL HEAT IN HOUSE = 0
Units: btu

- (11) INITIAL TIME = 0
Units: hour
The initial time for the simulation.

- (12) measured house temperature = SMOOTHI(house temperature, MEASURING DELAY, 75)
Units: degrees
This is the room temperature as perceived by the thermostat. It is calculated by smoothing the actual room temperature with a delay.
- (13) MEASURING DELAY = 0.2
Units: hour
This is the delay involved in measuring the room temperature.
- (14) OUTSIDE TEMPERATURE = 90
Units: degrees
This is the temperature of the environment external to the house. The house temperature would eventually reach this temperature without a heating/cooling system.
- (15) SAVEPER = TIME STEP
Units: hour
The frequency with which output is stored.
- (16) TEMP TO HEAT RATIO = 10000
Units: btu/degrees
This is the ratio between temperature and energy in the house. This number depends on the size and insulative properties of the house. It tells how many btus of energy are required to heat the house by 1 degree.
- (17) THERMAL CONDUCTIVITY = 1000
Units: btu/hour/degrees
This factor represents how easily energy is transferred from the outside environment to the house.
- (18) TIME STEP = 0.0625
Units: hour
The time step for the simulation.

Differences between the Vensim and Stella Heating/ Cooling System Models:

5/8/2001

1. The units for heat have been changed from joules to BTU to use the common terms of the air conditioning field.
2. All units for temperature have been set to degrees Fahrenheit to get rid of the dimensional inconsistencies in the original paper and assumption was made that heat in the house was equal to zero when the temperature was 75 degrees Fahrenheit which is physically incorrect. In the Vensim version we take the heat in the house at 75 degrees Fahrenheit as the reference heat for our simulation. i.e. Heat in House is the amount of heat in the house relative to the heat in the house at 75 degrees and not the absolute amount of heat.

Please note outside temperature is set to 50 degrees for this run only.

Graph with Heating System Only

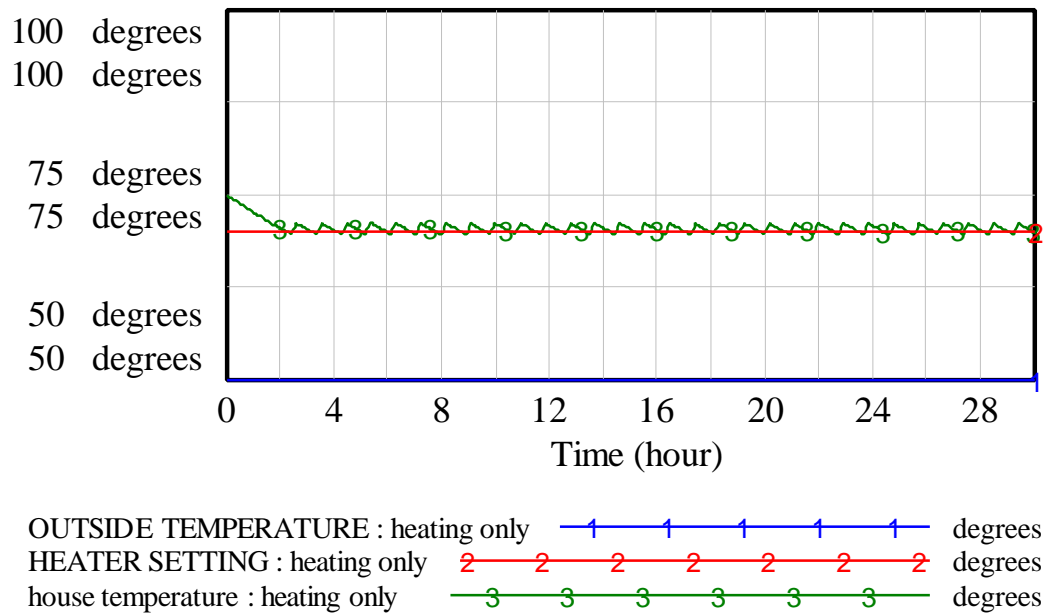


Figure 13: Vensim Equivalent of Figure 6: Simulation with heating system only

Please note outside temperature is set to 90 degrees for this run only.

Graph with Heating and Cooling

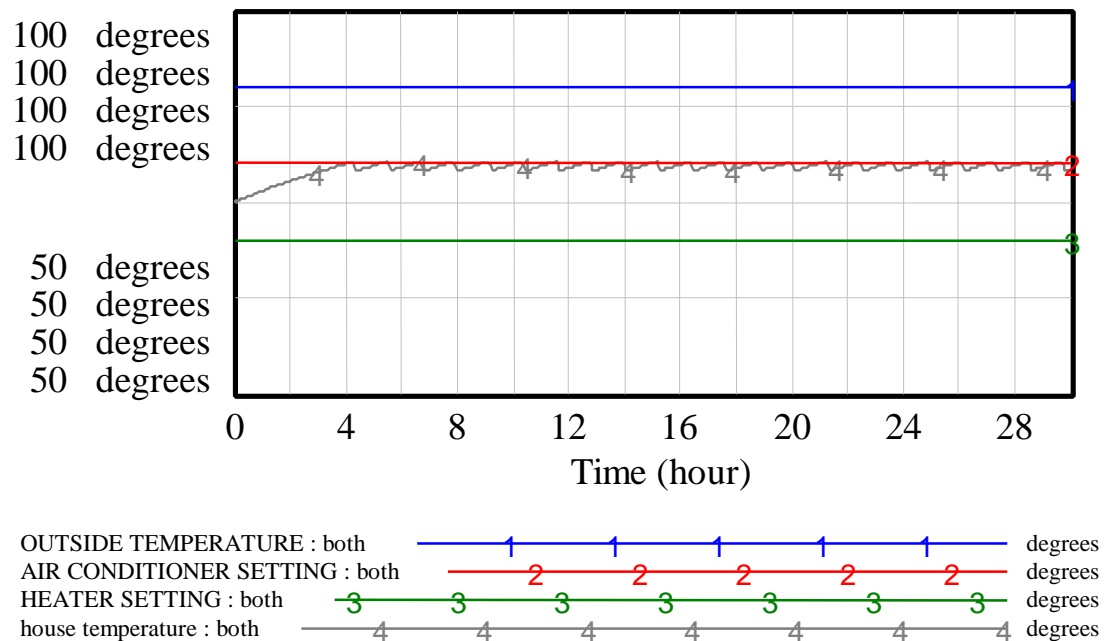
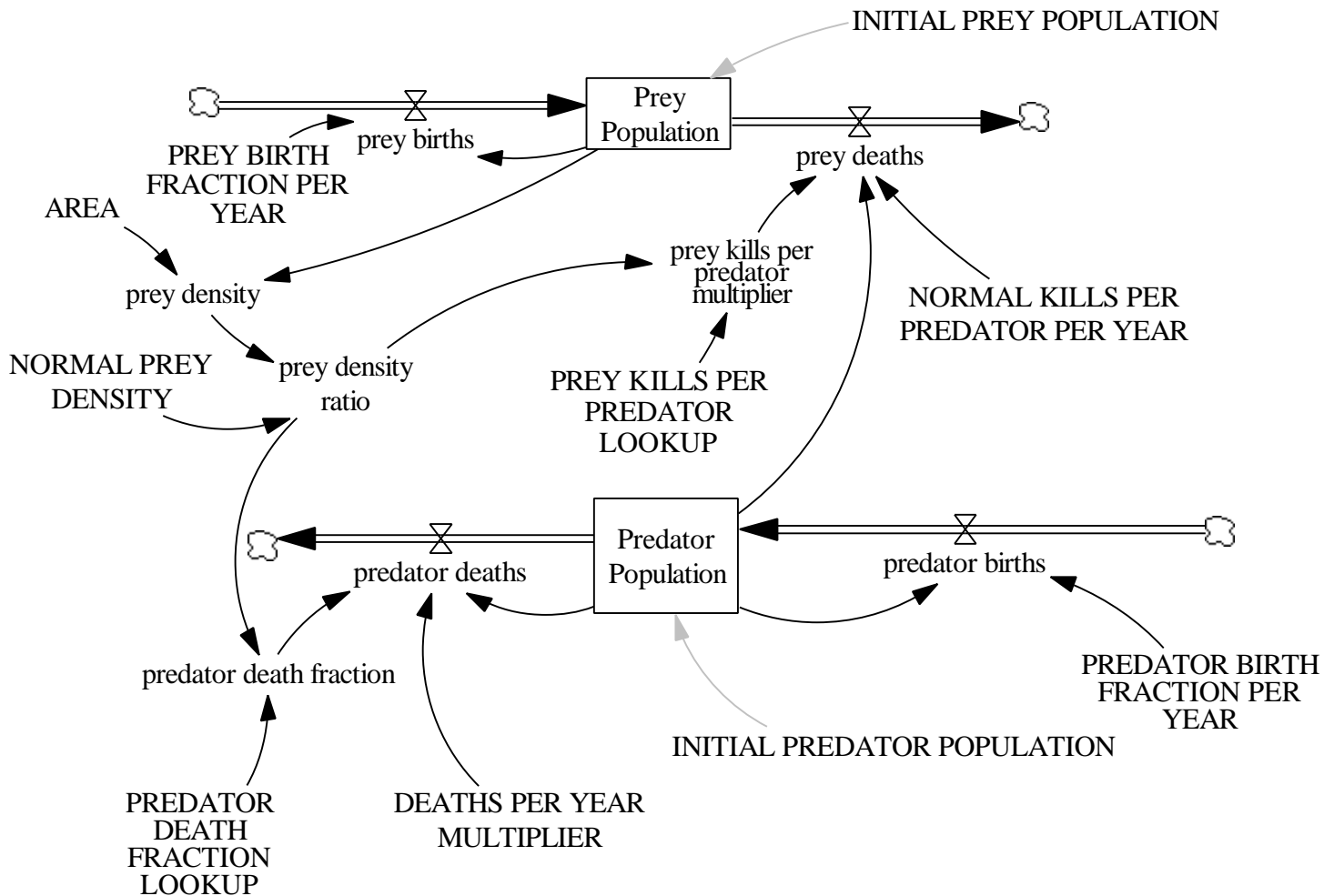


Figure 14: Vensim Equivalent of Figure 7: Simulation with Heating and Cooling Systems.

Model 3: A Predator/Prey System



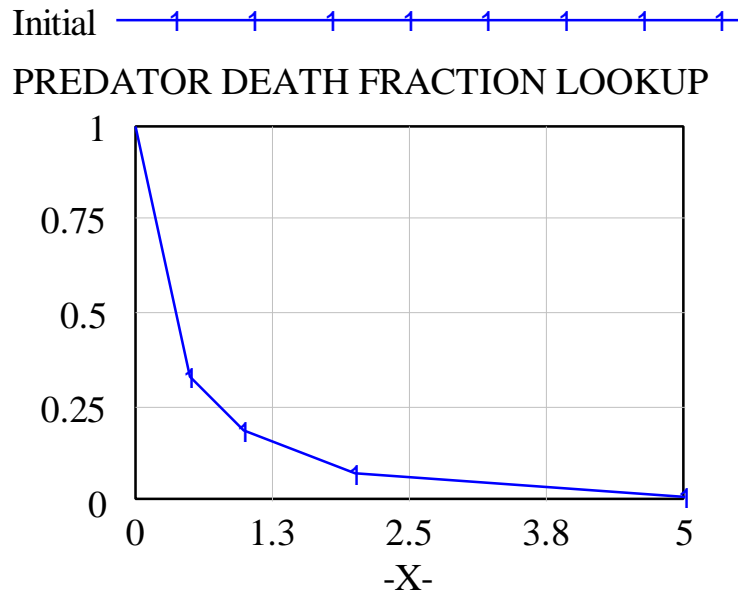
Documentation for Predator/Prey Model:

- (01) AREA = 1000
Units: acres
The area of the ecosystem is defined as 1000 acres.
- (02) DEATHS PER YEAR MULTIPLIER = 1
Units: 1/year
This multiplier produces the correct dimensions for predator deaths. Since "Predator Polulation" has units of Predators and "predator death fraction" is dimensionless we need to divide by year to ptoduce the correct dimensions for "predator deaths."

- (03) FINAL TIME = 40
Units: year
The final time for the simulation.
- (04) INITIAL PREDATOR POPULATION = 1250
Units: predators
Initially there are 1250 prey in the ecosystem.
- (05) INITIAL PREY POPULATION = 50000
Units: prey
Initially, there are 50,000 prey in the ecosystem.
- (06) INITIAL TIME = 0
Units: year
The initial time for the simulation.
- (07) NORMAL KILLS PER PREDATOR PER YEAR = 50
Units: prey/(predator*year)
At the normal prey density, each predator is able to kill 50 prey in the year. This represents the level of kills at which the predator death fraction is not affected by scarcity of prey.
- (08) NORMAL PREY DENSITY = 50
Units: prey/acres
The normal density of prey per acre.
- (09) PREDATOR BIRTH FRACTION PER YEAR = 0.25
Units: 1/year
On average, each Predator will generate .25 offspring per year. This is obtained by assuming that half the predators are female and that each female has a cub every two years.
- (10) predator births = Predator Population*PREDATOR BIRTH FRACTION PER YEAR
Units: predators/year
A compounding process is used to depict predator births. The births flow is defined as the product of predator population and their birth fraction.
- (11) predator death fraction = PREDATOR DEATH FRACTION LOOKUP (prey density ratio)
Units: dmnl
This equation relates predator death fraction as a function of relative prey density.
- (12) PREDATOR DEATH FRACTION LOOKUP([(0,0)-(5,1)],(0,1),(0.5,0.33),(1,0.185),(2,0.07),(5,0.01))

Units: dmnl

Predator mortality--the fraction of the predator population that dies each year-- is assumed to depend on prey density. With fewer prey, a greater fraction of the predator population will starve. At normal "prey density ratio" the death fraction is equal to 0.185, which we take as the normal death fraction of predators per year. The .0185 is obtained by giving each predator a life expectancy of about five and a half years. At zero prey density, the death fraction is 1, since the predator cannot survive with no food. When prey density is five times the normal, the death fraction of predators is only 0.01



- (13) $\text{predator deaths} = \text{Predator Population} * \text{predator death fraction} * \text{DEATHS PER YEAR MULTIPLIER} + \text{Predator Population} / 2 * \text{PULSE}(15, 1)$

Units: predators/year

This is a draining process. Some fraction of the Predator population will die each year. A death fraction of .5 would mean that half of the predator death population dies each year. The PULSE function is added in the second model run to jolt the system out of its initial steady-state and show the effect of a sudden decrease of predators to the ecosystem.

- (14) $\text{Predator Population} = \text{INTEG}(\text{predator births} - \text{predator deaths}, \text{INITIAL PREDATOR POPULATION})$

Units: predators

Initially, there are 1,250 predators.

- (15) $\text{PREY BIRTH FRACTION PER YEAR} = 1.25$

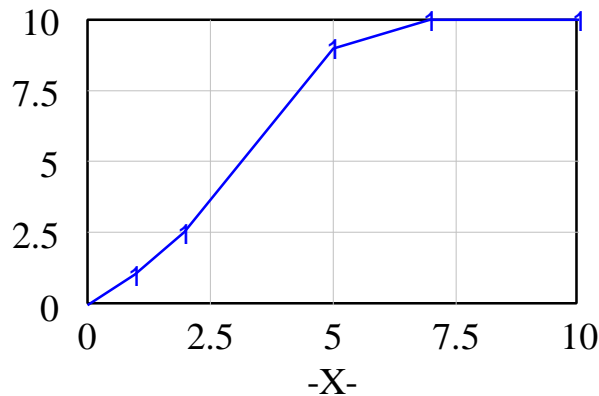
Units: 1/year

Each prey in the ecosystem is assumed to produce 1.25 offspring (on average) per year. This is obtained by assuming half the prey are female and that the females on average have 2.5 offspring per year.

- (16) $\text{prey births} = \text{Prey Population} * \text{PREY BIRTH FRACTION PER YEAR}$
 Units: prey/year
 A compounding process is used to depict prey births. The births flow is defined as the product of Prey Population and their birth fraction.
- (17) $\text{prey deaths} = \text{Predator Population} * \text{NORMAL KILLS PER PREDATOR PER YEAR} * \text{prey kills per predator multiplier}$
 Units: prey/year
 Each predator kills a number of prey per year, which depends on the "normal kills per predator" times a multiplier affected by the relative scarcity or abundance of prey.
- (18) $\text{prey density} = \text{Prey Population} / \text{AREA}$
 Units: prey/acres
 This equation calculates the average density of prey in the ecosystem.
- (19) $\text{prey density ratio} = \text{prey density} / \text{NORMAL PREY DENSITY}$
 Units: dmn1
 Ratio of prey density to NORMAL PREY DENSITY of the area. This ratio is used to provide a dimensionless input to the lookup table function for "prey kills per predator lookup."
- (20) $\text{PREY KILLS PER PREDATOR LOOKUP}([(0,0)-10,10], (0,0), (1,1), (2,2.5), (5,9), (7,10), (10,10))$
 Units: dmn1
 The number of prey killed per predator per year increases with the density of prey in the ecosystem. Note that when density is 0 (there are no prey), kills per predator must be 0. The number of prey killed per predator increases as the "prey density ratio" increases, however when the ratio reaches values above 7, the prey kills per predator multiplier stays at 10 because we assume the predators cannot kill more than 500 prey each per year.

Initial ———— 1 ———— 1 ———— 1 ———— 1 ———— 1 ———— 1 ————

PREY KILLS PER PREDATOR LOOKUP



- (21) prey kills per predator multiplier = PREY KILLS PER PREDATOR LOOKUP
(prey density ratio)
Units: dmnl
This equation says that prey kills per predator is a function of the prey density.
- (22) Prey Population= INTEG (prey births-prey deaths,INITIAL PREY POPULATION)
Units: prey
Initially, there are 50,000 prey in the ecosystem. Predator population increases with births and decreases with deaths. The formulation assumes no other outflows or inflows.
- (23) SAVEPER = TIME STEP
Units: year
The frequency with which output is stored.
- (24) TIME STEP = 0.0625
Units: year
The time step for the simulation.

Graph of Predator and Prey Population Without a Pulse

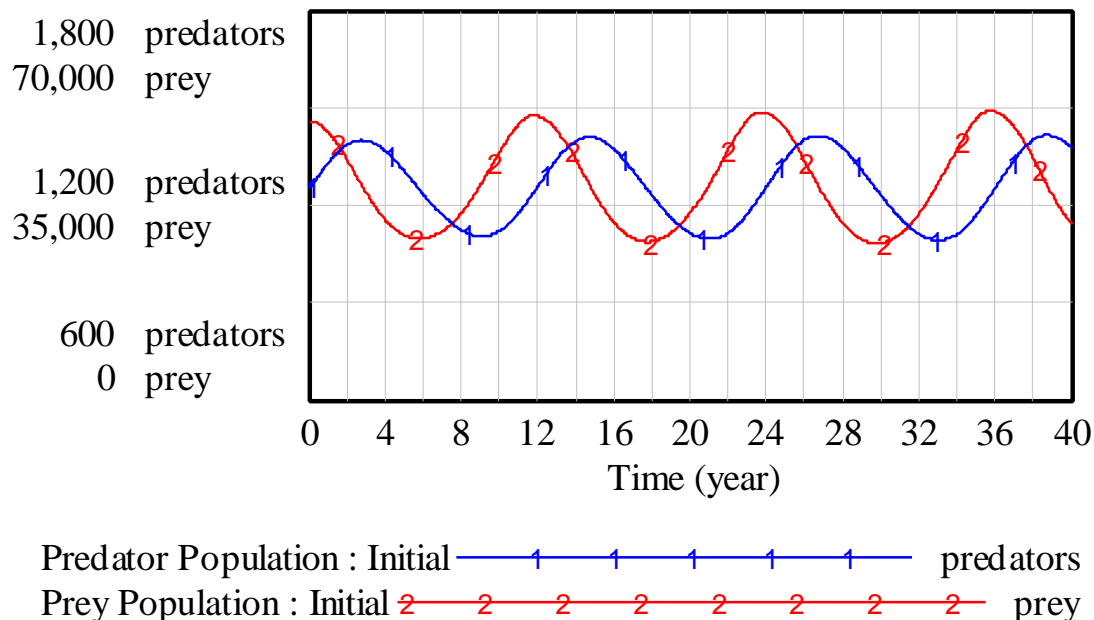
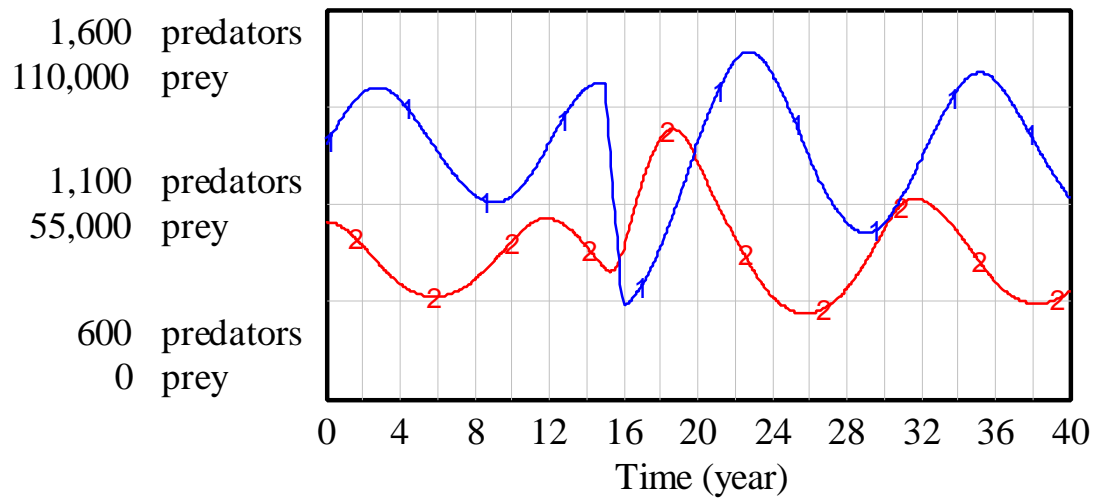


Figure 16: Vensim Equivalent of Figure 7: Simulation without a Pulse Function

Graph of Prey and Predator Populations with Pulse



Predator Population : pulse ——— 1 ——— 1 ——— 1 ——— 1 ——— 1 ——— predators
 Prey Population : pulse ——— 2 ——— 2 ——— 2 ——— 2 ——— 2 ——— 2 ——— prey

Vensim Equivalent of Figure 8: Simulation with a Pulse