Graphical Integration Exercises

Part Three: Combined Flows

Prepared for the
MIT System Dynamics in Education Project
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1. Abstract

This paper is the third in a series of papers on developing skills in graphical integration. It will study the use of graphical integration to estimate the behavior of simple systems with constant and linearly increasing or decreasing inflows and outflows. This paper assumes you can estimate behaviors of models with constant, step, and variable net flows. Also, you should be familiar with the terms "slope" and "area," and you should know how to calculate them for simple behaviors.[1]

2. Introduction

Let us consider a simple system, that of a bathtub with a constant inflow through a faucet. We know that if we turn on the faucet to fill a tub, go outside for five minutes and come back to find the tub one-fourth full, then it will take five more minutes to fill up halfway. It will then take five more minutes to be three-fourths full, and a total of twenty minutes for the tub to be completely full of water.

What we just did, essentially, is graphical integration without explicitly drawing graphs. We were able to perform this integration because the system in question was very simple. However, many systems are not as simple as the bathtub system. Thus, it becomes difficult to predict how they will behave. Practice in using graphical integration will make it easier to use it to better estimate the behavior of many systems.

Although we have access to sophisticated computer programs that simulate models of great complexity, it is important that we can intuitively estimate and understand what we see on the graph pad before and after running a simulation. This helps to improve confidence in the model. The series of graphical integration papers will enable us to do that. In this third paper of the series, we will graphically integrate systems with separate inflows and outflows.

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[1] If you need to develop these skills, please refer to *Graphical Integration Exercises Part Two: Ramp Functions* (D-4571), by Kevin Agatstein & Lucia Breierova, System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, Jan., 1996.

3. Graphical Integration with Independent Inflows and Outflows

The first paper in the series, *Graphical Integration Exercises Part One: Exogenous Rates*,[2] studied graphical integration of constant flows and step functions. In the second paper in the series, *Graphical Integration Exercises Part Two: Ramp Functions*,[3] we saw how to graphically integrate linearly increasing and decreasing flow ramp functions. In both of these papers we dealt only with a single net flow. We will now turn our attention to graphical integration with both inflows and outflows to a single stock.

An example of such a system with different inflows and outflows is a ship whose hull has punctured a leak. Water rushes into the hull while the sailors furiously work at the pumps to expel the water. This is modeled in Figure 1:

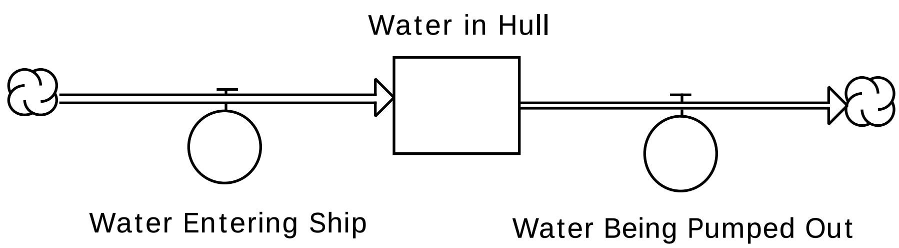

Figure 1: Model of Ships Hull.

(Note that the Inflow and Outflow are separate)

In these situations ship captains intuitively graphically integrate (without drawing the actual graphs) to determine whether or not to "abandon ship." In this paper we will quantitatively graphically integrate similar systems with separate inflows and outflows. Even though we are dealing with separate inflows and outflows, it does not mean that the material we discussed in the first two papers in the *Graphical Integration Exercises* series does not apply. In fact, as you shall soon see, graphically integrating systems with separate inflows and outflows is nearly identical to other types of graphical integration. The only difference is the first step of calculating a "net flow." Let's learn how to do that now.

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[2] *Graphical Integration Exercises Part One: Exogenous Rates* (D-4547), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, Dec 1., 1995.

[3] *Graphical Integration Exercises Part Two: Ramp Functions* (D-4571).

3.1 Calculating a Net Flow

For graphically integrating systems with separate inflows and outflows, the first step is to calculate this single net flow. Then the problem becomes identical to the ones discussed in the previous two papers in the *Graphical Integration Exercises* series.

To calculate the net flow into a system all we need to do is subtract the outflow from the inflow. If this net flow is positive, then the stock will fill; if it is negative, the stock will empty. The net flow is written mathematically for any time interval as:

Net Flow = Change in Value of Stock **= Inflow - Outflow**

This means that the new value of the stock can be expressed as:

New Value of Stock = Initial Value of Stock + (Inflow - Outflow) * time.

Don't worry if you are confused by these formulas. Once we do the following example the concept should become much clearer.

Let us consider a system where oil is being added to an initially empty storage tank at a constant rate of 15 gallons per minute and at the same time is being pumped out of the tank at a constant rate of 5 gallons per minute. Can we estimate the behavior of the stock?

Graphically, the two rates that affect the system are shown in Figure 2.

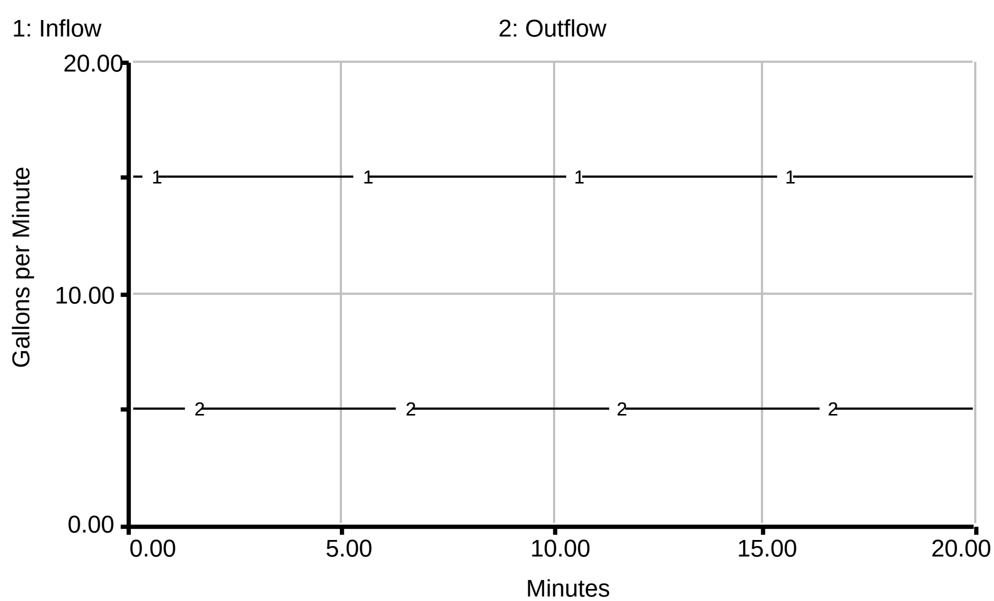


Figure 2: Flows Into and Out of the Oil Storage Tank.

The first step in estimating the behavior of this system is to determine the net flow. The net flow is simply the difference between inflow and outflow. In this system:

Initially:
Net Flow at Time 0 = Inflow at Time 0 - Outflow at Time 0
Net Flow at Time 0 = 15 gallons per minute - 5 gallons per minute
= 10 gallons per minute

At 5 min:
Net Flow at 5 min = Inflow at 5 min- Outflow at 5 min
Net Flow at 5 min = 15 gallons per minute - 5 gallons per minute
= 10 gallons per minute

At 10 min:
Net Flow at 10 min= Inflow at 10 min - Outflow at 10 min
Net Flow at 10 min= 15 gallons per minute - 5 gallons per minute
= 10 gallons per minute

The same holds true for every point in time, since both the inflow and outflow are constant. This means that we can rewrite the system with a single flow, the net flow, as opposed to the separate inflow and outflow drawn above. This is done in Figure 3.

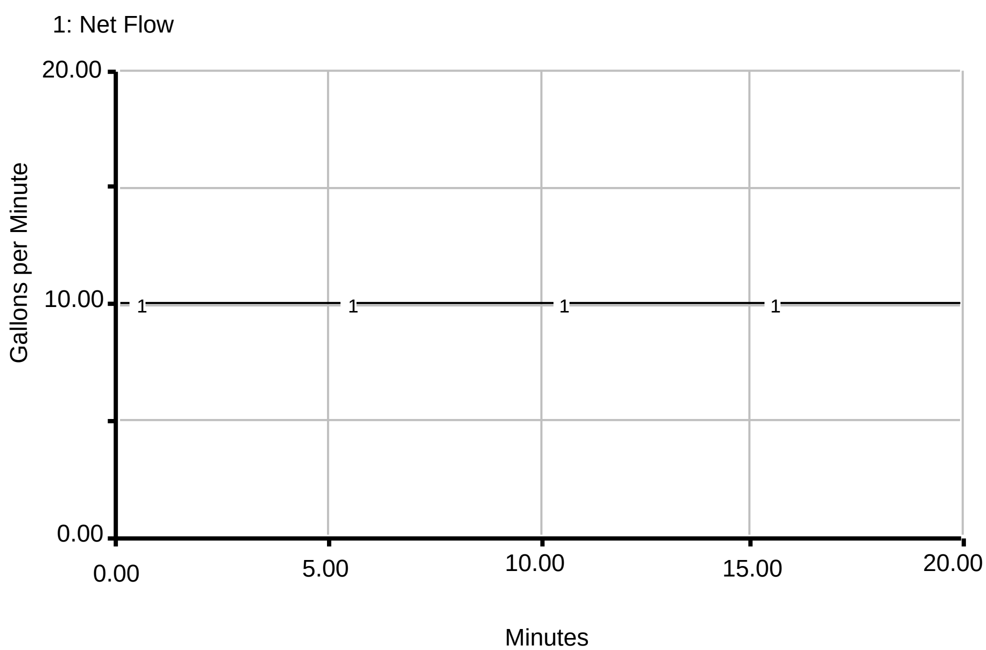


Figure 3: Net Flow for Oil Tank System

According to this graph at any given point in time there is a net addition of 10 gallons of oil per minute to the tank. This is logical since we are always adding 15 gallons per minute to the tank, and subtracting 5 gallons per minute.

Now that we have calculated the net flow, estimating the behavior of the system should be easy. We learned in *Graphical Integration Exercises Part 1: Exogenous Rates* how to graphically integrate a constant flow. For review, we will do it again here.

We know that constant net flows generate linear stock behaviors. Also, we know that for any given interval the area under the net flow graph is the change is the value of the stock for that interval. Thus from Time 0 to 5 minutes:

Change in value of Stock from 0 to 5 min = Area under net flow graph from 0 to 5 min
Change in value of Stock from 0 to 5 min = 10 gallons per minute * 5 minutes
Change in value of Stock from 0 to 5 min = 50 gallons

Now we can calculate the value of the stock at 5 minutes using:

Value of Stock at 5 min = Value of Stock at Time 0 + Change in value of Stock from 0 to 5 min
Value of Stock at 5 min = 0 gallons + 50 gallons = 50 gallons

Repeating this calculation from 5 minutes to 10 minutes, we see that:

Change in value of Stock from 5 to 10 min= Area under net flow graph from 5 to 10 min
Change in value of Stock from 5 to 10 min= 10 gallons per minute * 5 minutes
Change in value of Stock from 5 to 10 min= 50 gallons

Thus, the value of the stock at 10 minutes can be calculated as follows:

Value of Stock at 10 min = Value of Stock at 5 min + Change in value of Stock from 5 to 10 min
Value of Stock at 10 min = 50 gallons + 50 gallons = 100 gallons

The same calculation can be repeated for the remainder of the net flow graph. After doing this, we get the behavior shown in Figure 4.

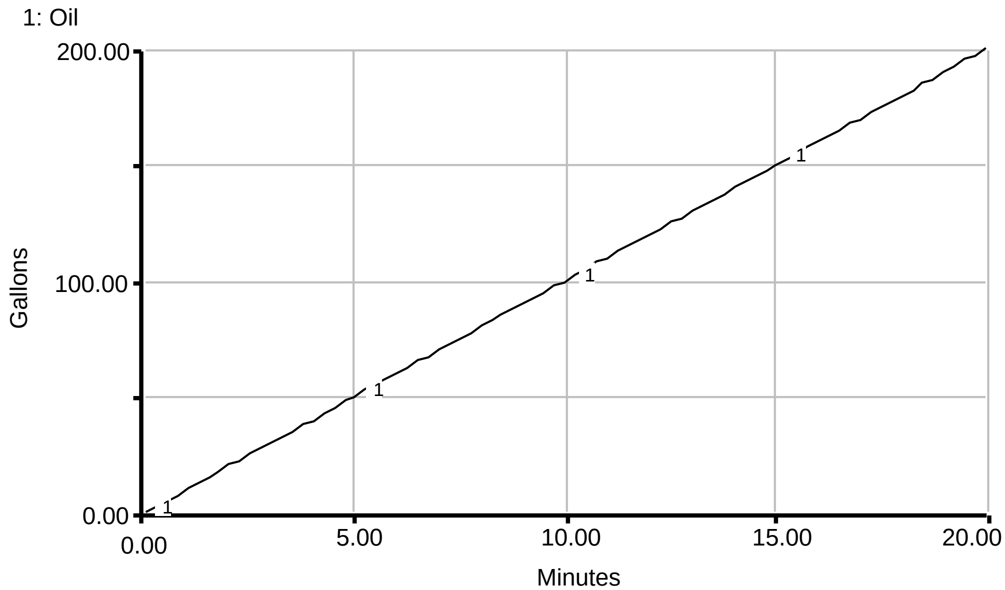


Figure 4: Behavior of the Oil Storage Tank with an Inflow of 15 Gallons per Minute and n Outflow of 5 Gallons per Minute

It might be a good idea to reflect back at this point on what the "net flow" is and how it relates to what we have studied in the two previous graphical integration papers. In these papers we always calculated the area between the flow graph and the line representing flow = 0. What we were really doing was treating the net flow as the inflow, and assuming there was no outflow. In other words, we treated the outflow as constant at zero units per time. We then found the area between the inflow and the outflow curves. In the next example we will apply this concept to graphically integrate a similar system, but without going through the trouble of calculating the net flow.

3.2 Combining Constant Flows

Let us look at another example of determining net flows when the inflow and outflow are constant. Let's imagine the same oil tank, this time initially with 25 gallons of oil in it. To this tank we are constantly adding 75 gallons per minute, and draining 50 gallons per minute. This can be represented graphically, as shown in Figure 5:

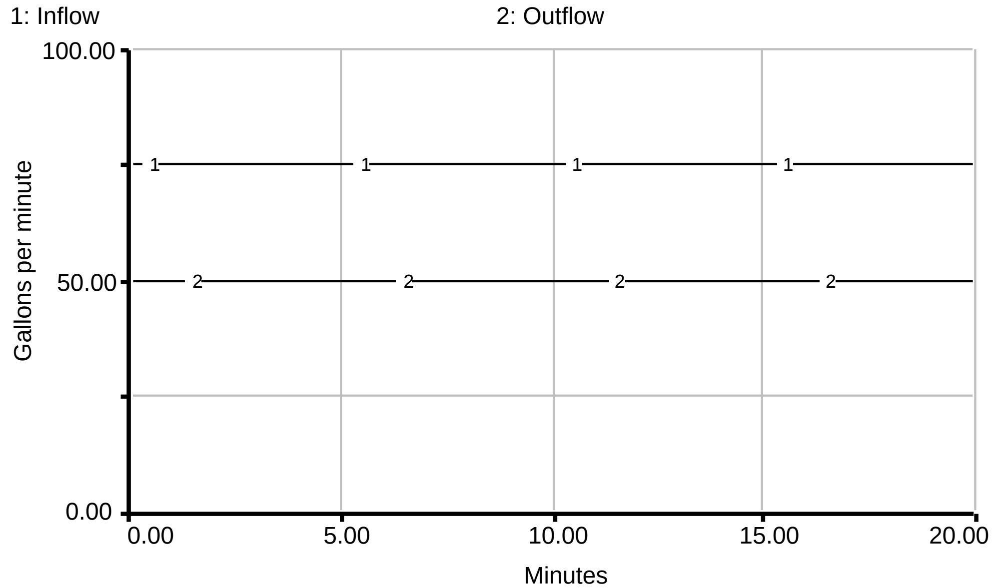


Figure 5: Inflow and Outflow to the Oil Storage Tank

To estimate the behavior of this system we could do exactly what we did in the previous example: draw the net flow graph and integrate it the same way we did in the previous two *Graphical Integration Exercises* papers. Briefly, the net flow graph would be a flat line at:

Net Flow = 75 gallons per minute - 50 gallons per minute = 25 gallons per minute.

Thus, the graph of the oil in the tank would be:

25 gallons at time 0

25 gallons + (25 gallons per minute * 5 minutes) = 150 gallons at time 5

150 gallons + (25 gallons per minute * 5 minutes) = 275 gallons at time 10,

and so on and so on.

For this example though, let us apply the fact that the change in the value of the stock is the area between the inflow and outflow curves. Thus, for Time 0 to 5 minutes:

Change in value of Stock = area between inflow and outflow graphs

Change in value of Stock = area of rectangle with a width of 5 minutes and a height of (75 gallons per minute - 50 gallons per minute)

Change in value of Stock = 5 minutes * 25 gallons per minute = 125 gallons

This is shown in Figure 6:

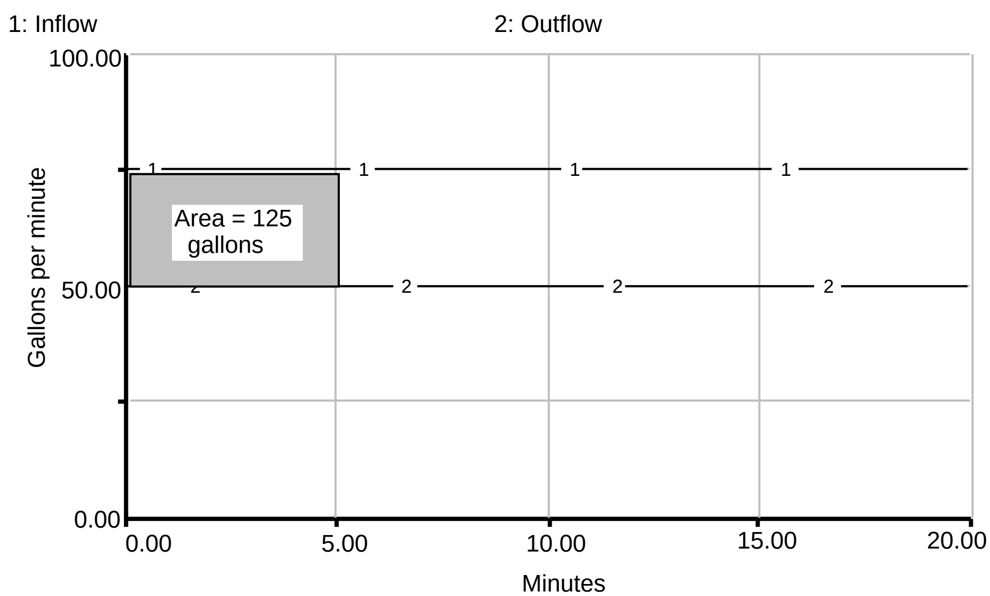


Figure 6: Graph of Flows
The area between the inflow and outflow for the first 5 minutes is shown.

Thus, we know that the value of the stock (which is the oil in the tank) at 5 minutes is:

Stock at 5 min = Initial Value of Stock + Change in value of Stock for 0 to 5 min
Stock at 5 min = 25 gallons + 125 gallons = 150 gallons.

For the time interval of 5 minutes to 10 minutes, we can repeat the calculation.

Change in value of Stock = area between inflow and outflow
Change in value of Stock = rectangle with a length of (10-5) minutes and a height of (75-50) gallons per minute
Change in value of Stock = 125 gallons

Stock at 10 min= Initial value of Stock + Change in value of Stock
= Stock at 5 min + 125 gallons
= 150 gallons + 125 gallons
= 275 gallons

Doing this calculation for time 10 minutes to 15 minutes will yield a value of the stock of 400 gallons at 15 minutes and 525 gallons at 20 minutes.

This behavior is shown graphically in Figure 7:

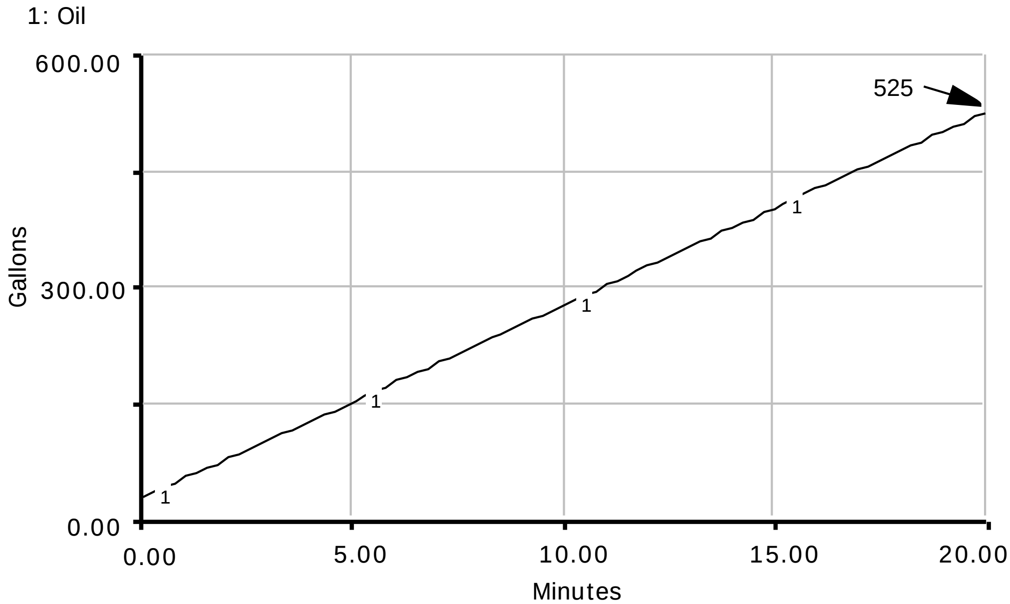


Figure 7: Behavior of the Oil Volume in the Storage Tank with Constant Inflow and Outflow

3.3 Variable Inflow with a Constant Outflow

Now that we have graphically integrated systems with constant inflows and outflows, let us look at a system that is a little more interesting. Let us consider an initially empty oil storage tank with 10 gallons per minute constantly being pumped out. Oil is added to the system initially at a rate of 20 gallons per minute which increases linearly with a slope of +1 gallon per minute per minute. Therefore at 20 minutes oil is being added at a rate of 40 gallons per minute.

These two flows are shown in Figure 8:

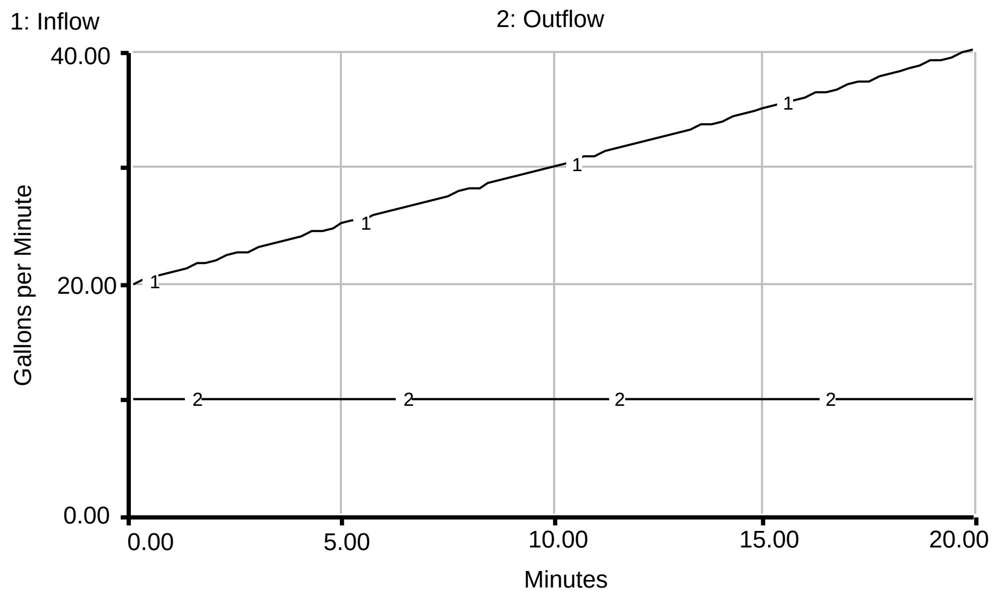


Figure 8: Graph of Inflow and Outflow for Storage Tank.

Let us solve this system by determining the net flow. (However, we could calculate the area between the Inflow and Outflow graph, like we did in the previous example. Either method is fine. You should pick the one you find easier for any given example.)

Initially:

Net flow at 0 min = Inflow at 0 min - Outflow at 0 min
= 20 gallons per min - 10 gallons per min
= 10 gallons per min

At 5 minutes:

Net flow at 5 min = Inflow at 5 min - Outflow at 5 min
= 25 gallons per min - 10 gallons per min
= 15 gallons per min

At 10 minutes:

Net flow at 10 min = Inflow at 10 min - Outflow at 10 min
= 30 gallons per min - 10 gallons per min
= 20 gallons per min

At 20 minutes:

Net flow at 20 min = Inflow at 20 min - Outflow at 20 min
= 40 gallons per min - 10 gallons per min
= 30 gallons per min

The net flow can then be drawn as in Figure 9:[4]

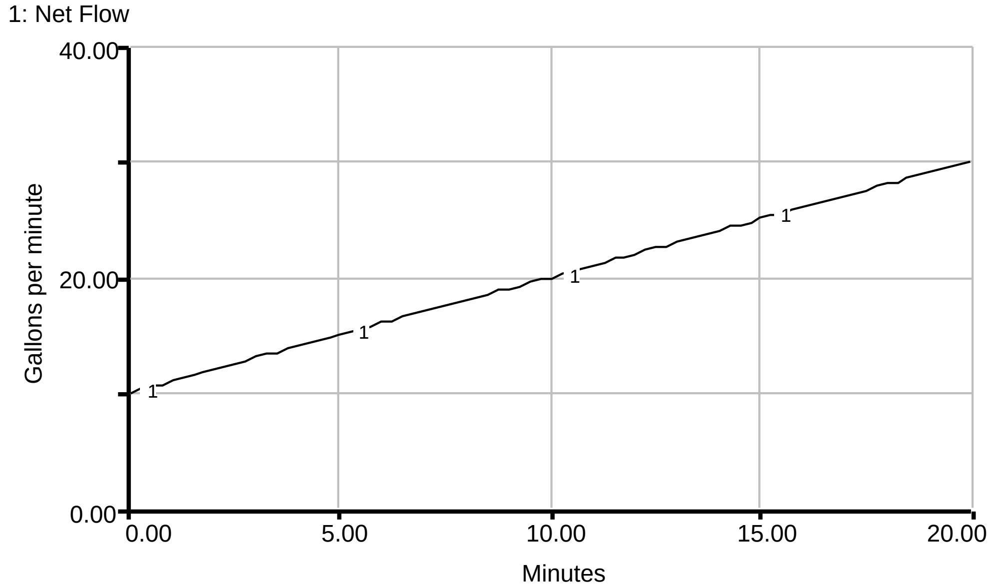


Figure 9: The Net Flow Into The Storage Tank.

In *Graphical Integration Exercises Part Two: Ramp Functions*, we went into great detail on how to integrate linearly increasing flows. We learned in that paper that the stock will display parabolic growth. We can estimate the behavior of the stock by calculating the area under the net flow graph to the line flow = 0, and adding that value to the initial value of the stock. For review purposes, let us carefully determine the behavior of the stock.

For 0 to 5 minutes:

Change in value of Stock = area under net flow graph
Change in value of Stock = area of rectangle plus area of triangle

[4] It is important to notice that the net flow is a straight line. Any time you subtract one line from another, you will get another line.

Figure 10 shows these two areas.

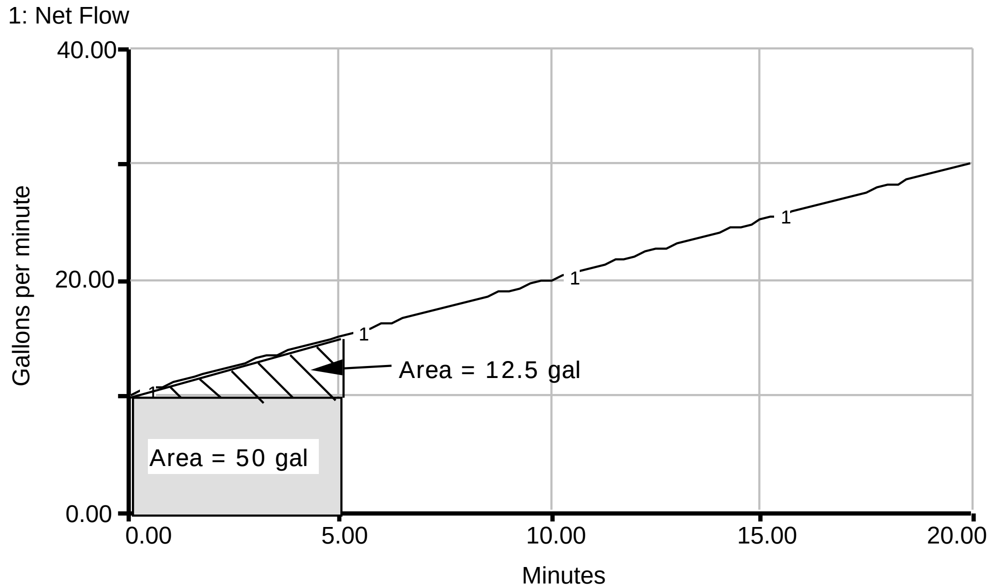


Figure 10: Calculation of Area Under Net Flow Graph

Change in value of Stock = (5 min * 10 gal per min) + (0.5 * 5 min * 5 gal per min)
= 50 gal + 12.5 gal = 62.5 gal

Stock at 5 min = Stock at 0 min + change in value of Stock from 0 to 5 min
Stock at 5 min = 0 gal + 62.5 gal = 62.5 gallons

To calculate the value of the stock at 10 minutes, we can perform a similar process.

Change in value of stock from 5 to 10 min = area under net flow curve from 5 to 10 min

Area under curve = area of triangle + area of rectangle.

These areas are shown in Figure 11.

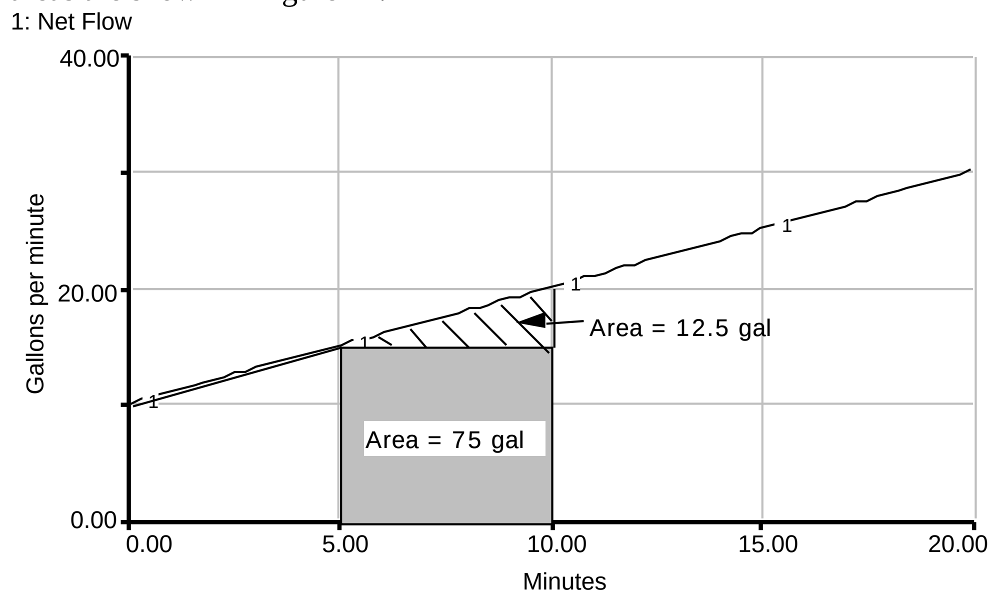


Figure 11: Calculation of Area Under Net Flow Graph

Thus, the area under the net flow graph from 5 to 10 minutes is:
Area = 75 gallons + 12.5 gallons = 87.5 gallons

The stock at 10 minutes is:
Stock at 10 min = Stock at 5 min + Change in value of Stock from 5 min to 10 min
= 62.5 gallons + 87.5 gallons = 150 gallons.

We could continue this exact same type of calculation for the rest of the net flow curve. Figure 12 shows the final result of such a calculation, the parabolic behavior of the stock (which is the amount of oil in the storage tank for this example.)

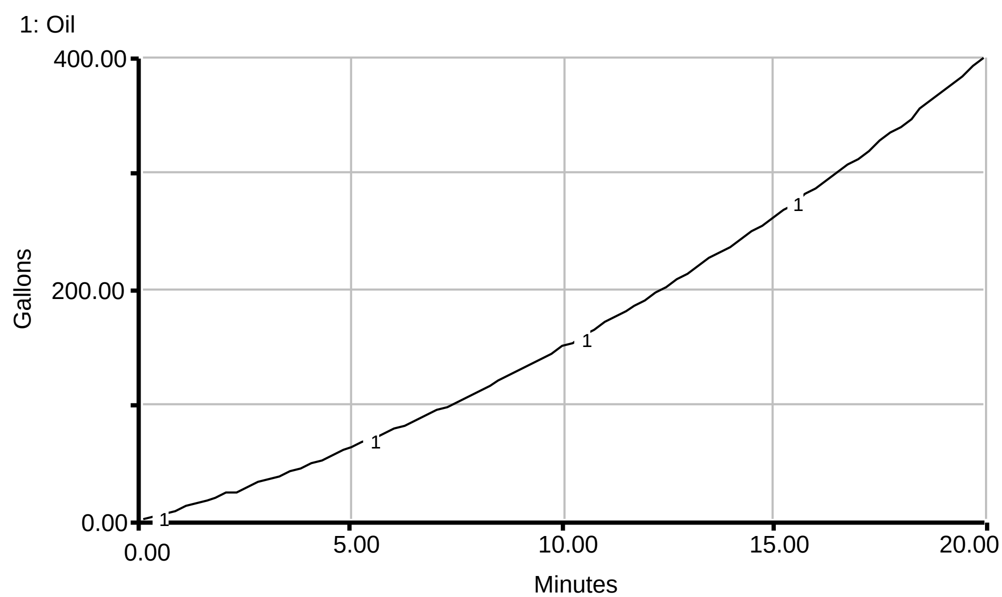


Figure 12: Behavior of the Oil Storage Tank System.

3.4 Variable Inflow and Outflow

For our fourth and final example, let us look at a system whose inflow and outflow are both changing over time. In this system the outflow, which is the oil being pumped out of our storage tank, is linearly decreasing with a slope of –2 from 40 gallons per minute to zero in 20 minutes. The inflow is constant at 20 gallons per minute for time 0 to 10 minutes and then it ramps up with a slope of +1 until 20 minutes. The oil tank is initially filled with 100 gallons of oil.

The inflow and outflow are shown in Figure 13.

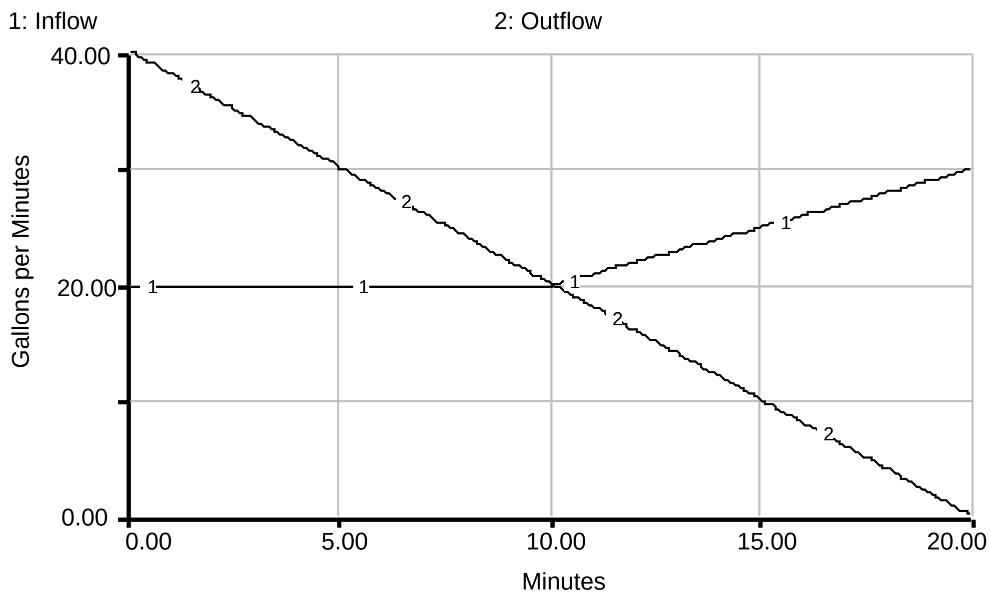


Figure 13: Graph of Variable Inflows and Outflows.

For this example we will use the net flow determination method. We could of course use the method of finding the area between the inflow and outflow graphs and get the correct answer. However, finding those areas looks a little tricky, so let's find the net flow. Remember:

Net Flow = Inflow - Outflow.

The net flow at 0 minutes is:

Net flow at 0 min = Inflow at 0 min- Outflow at 0 min
= 20 gallons per min - 40 gallons per min
= –20 gallons per minute[5]

[5] Note that the net flow is negative. All this means is that the stock (the storage tank) is being emptied, not filled.

The net flow at 10 minutes is:

Net flow at 10 min = Inflow at 10 min - Outflow at 10 min
= 20 gallons per min - 20 gallons per min
= 0 gallons per minute

We could have done this net flow calculation for all points between 0 and 10 minutes but it is easier to simply note that any time you subtract two lines you get another line. Thus, by calculating the net flow at 0 and 10 minutes the flow can be represented as in Figure 14.

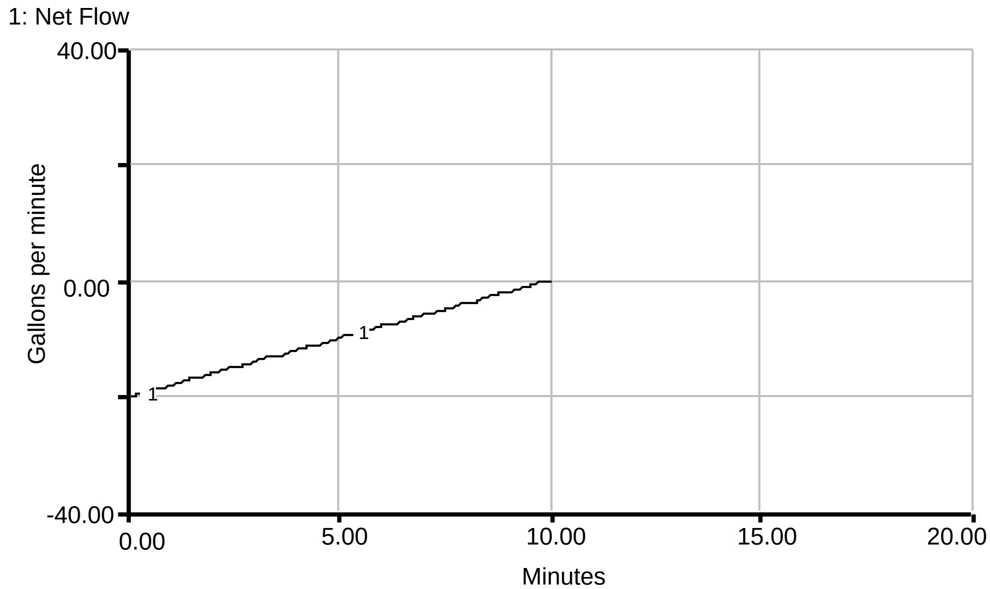


Figure 14: Graph of Net Flow from 0 to 10 Minutes.

For 10 minutes to 20 minutes we can repeat the same procedure. We already know that the net flow at 10 minutes is 0 gallons per minute.

At 15 minutes, the net flow is:

Net Flow at 15 min= Inflow at 15 min - Outflow at 15 min
Net Flow at 15 min= 25 gal per min - 10 gal per min
= 15 gallons per min

At 20 minutes, the net flow is:

Net Flow at 20 min= Inflow at 20 min - Outflow at 20 min
Net Flow at 20 min= 30 gal per min - 0 gal per min
= 30 gallons per min

With this information, and the fact that lines subtracted from one another always give us a line, we can draw the rest of the net flow graph. This is shown in Figure 15.

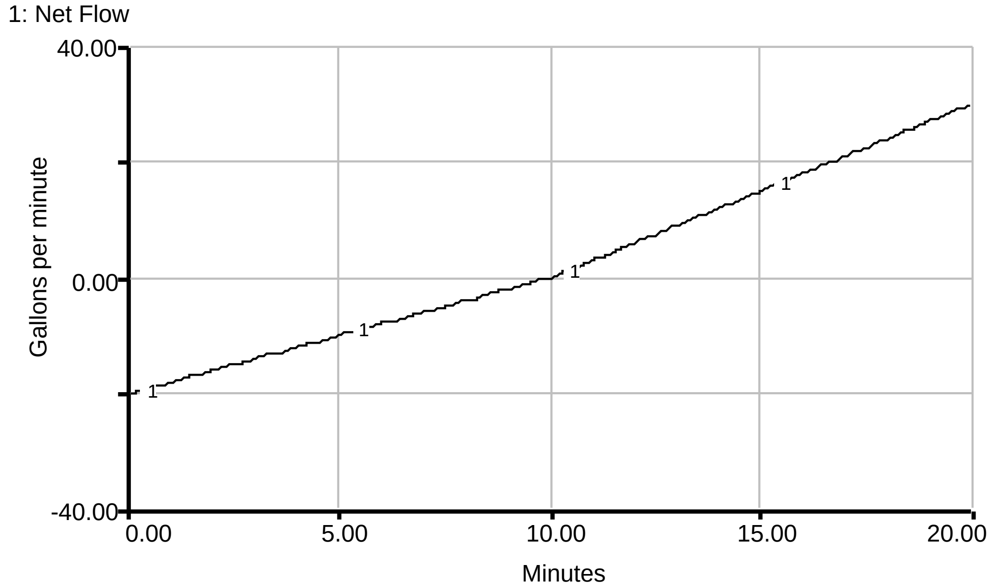


Figure 15: Graph of Net Flow

Now that we have the net flow, all we need to do to estimate the behavior of the system is calculate the areas between the net flow and the line flow = 0 gallons per minute. While you should now be able to do this calculation yourself, Figure 16 presents the net flow graphs with all the areas calculated for you. (Remember, a negative net flow, which is an area below the line flow = 0, empties a stock, while a positive flow fills a stock.)

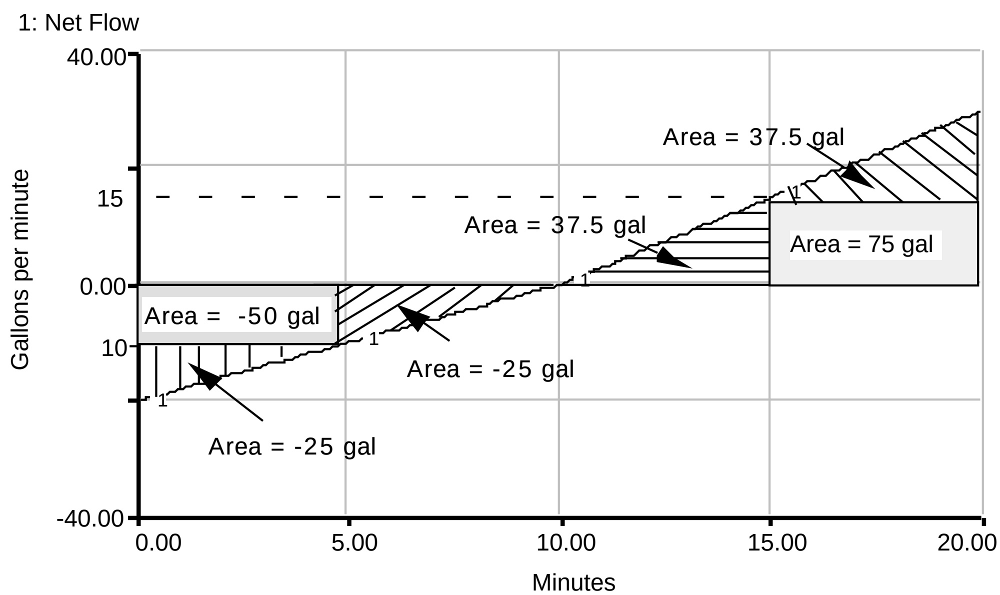


Figure 16: The Net Flow with Areas Calculated

With all the areas calculated for you, determining the behavior of the stock (the oil in the tank in this case) should be easy. For 0 to 5 minutes:

Change in value of Stock = area between Net Flow curve and Flow = 0
Change in value of Stock = Area of rectangle + Area of triangle
= (-50 gal) + (-25 gal) = –75 gall

Thus, the stock at 5 min is:

Stock at 5 min = Stock at 0 min + change in value of Stock from 0 to 5 min
Stock at 5 min = 100 gal + (–75 gal) = 25 gal

We can repeat this type of calculation for the remainder of the net flow graph. If we do, we will see that the stock's behavior over time is shown in Figure 17.

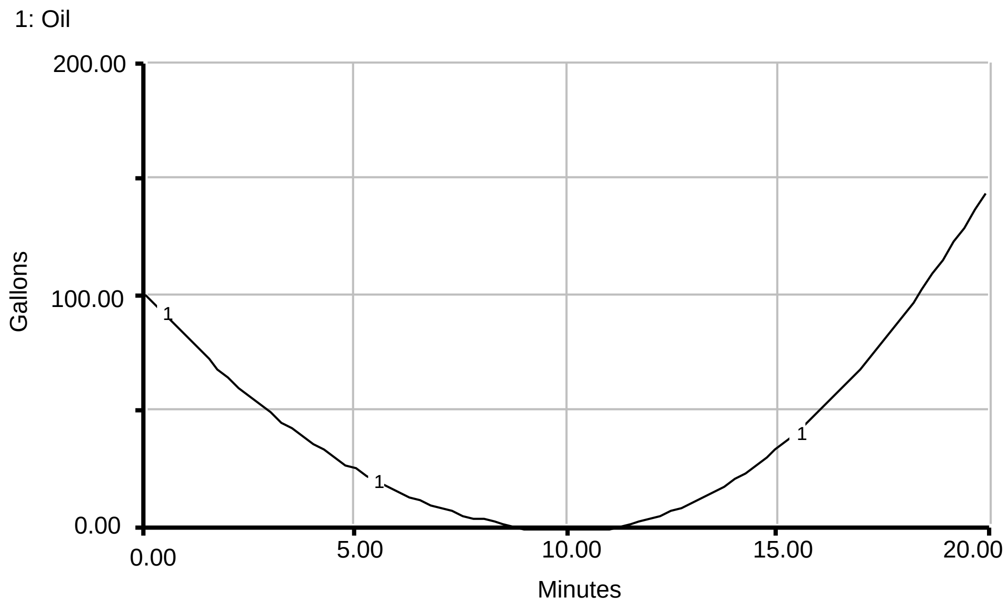


Figure 17: Behavior of the Stock with Variable Inflow and Outflow.

4. Key Ideas

Now that we have seen how stocks behave when the inflow and outflow are independent, we are ready to try some exercises. Anytime you are performing graphical integration, keep the following rules in mind:

1. **The area under the flow graph is equal to the change in the value of the stock over a period of time.**
2. **When the net flow is positive, stocks are filled; when the net flow is negative, stocks are emptied.**
3. **The value of the net flow gives the slope of the stock at that instant of time.**

4. **Constant flows cause the stock to increase linearly, with the slope of the stock equal to the value of the flow.**
5. **Linearly increasing and decreasing flows cause the stock to exhibit parabolic growth.**
6. **You can often break up complicated graphs into several small, simpler ones.**

And from this paper:

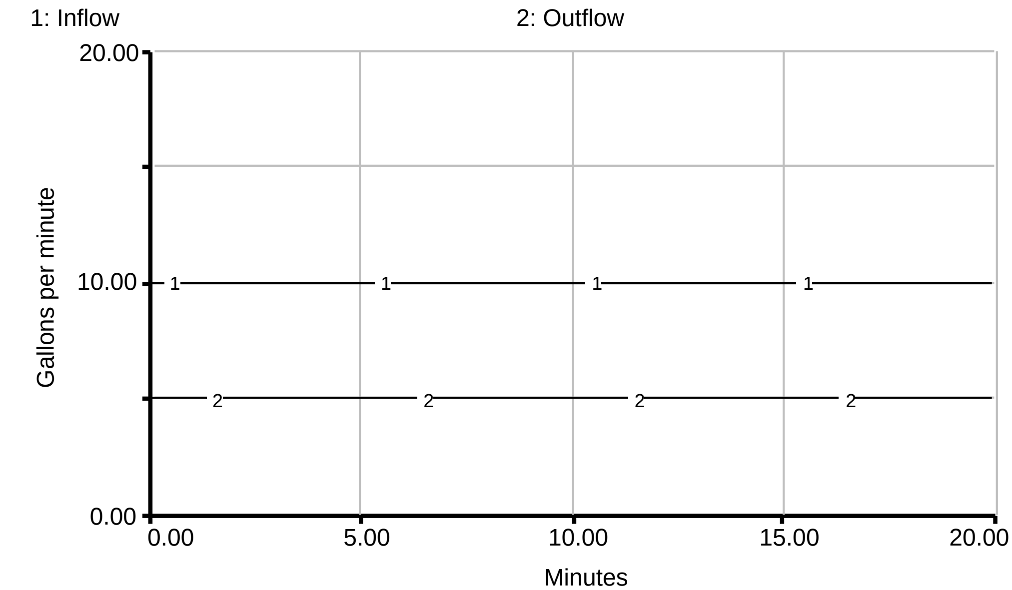
7. **Separate inflows and outflows can be used to form a net flow, which can then be graphically integrated by calculating the area between the flow curve and the line representing flow = 0. (Remember that area is the CHANGE in the value of the stock.)**
8. **Separate inflows and outflows can also be graphically integrated by simply finding the area between the two curves. (That area is the CHANGE in the value of the stock.)**

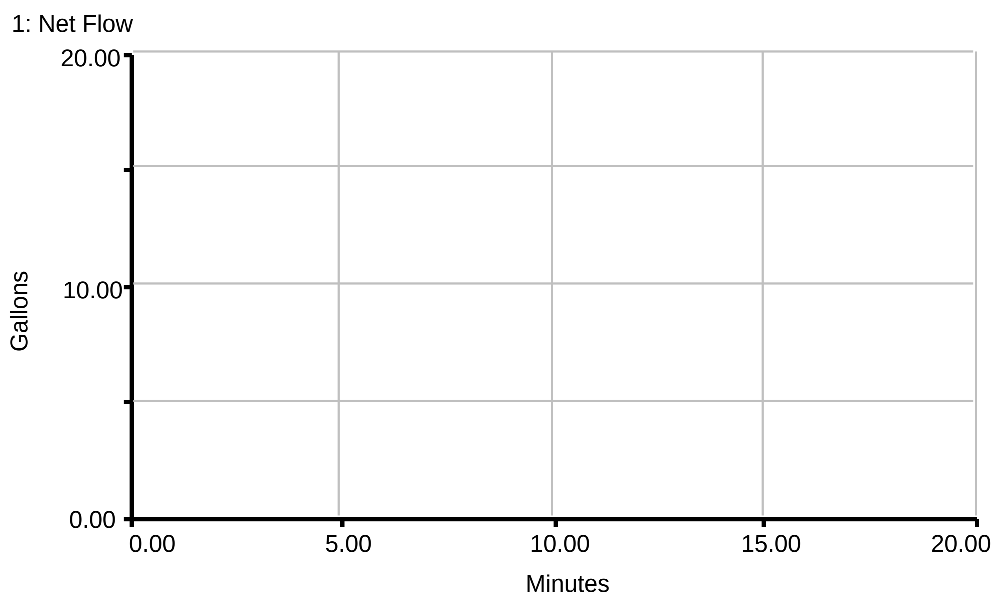
Now, let’s try some exercises!!!

5. Exercises

5.1 Calculating a Net Flow

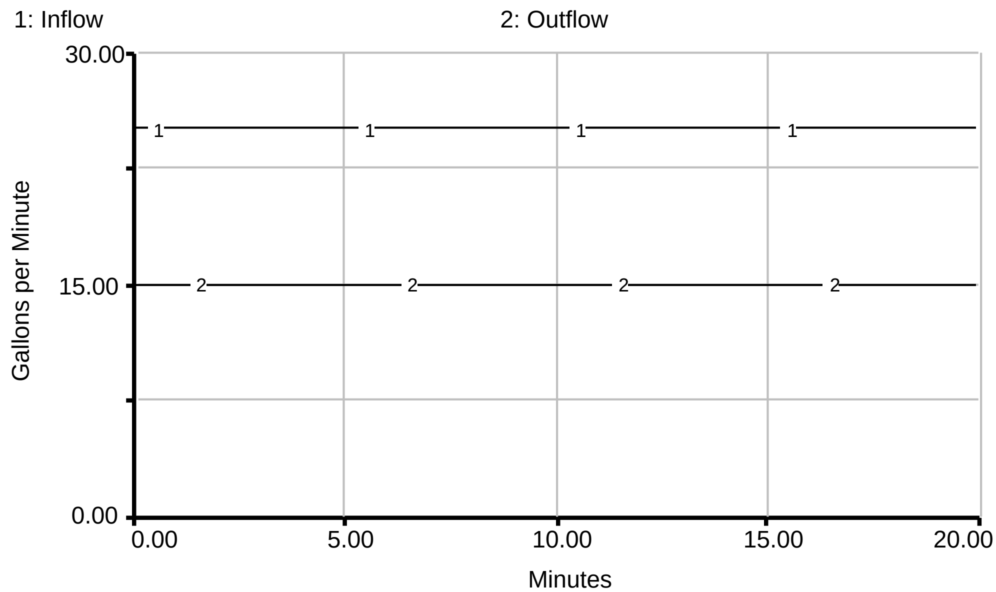
Calculate and sketch on the blank graph pad provided the net flow of oil into a storage tank where oil is being pumped in at a rate of 10 gallons per minute and being pumped out at a rate of 5 gallons per minute. These rates are shown below:

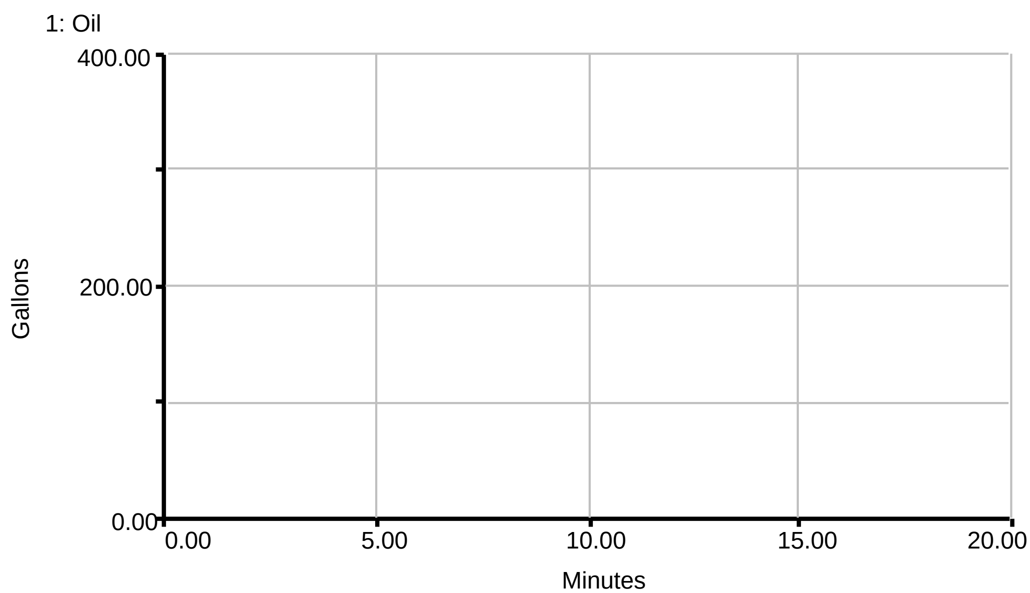




5.2 Constant Flows

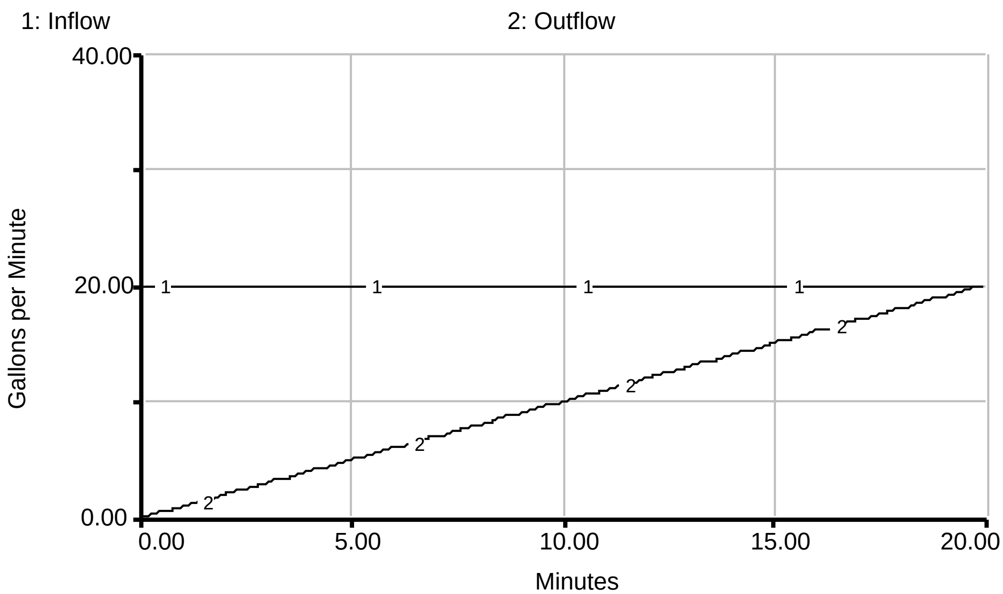
For this example, let us again look at an oil storage tank. However this time, determine the behavior of the stock (i.e. the amount of oil over time) if the pumping rate into the tank is 25 gallons per minute and oil is being drained out at 15 gallons per minute. Assume that there are initially 50 gallons of oil in the tank.

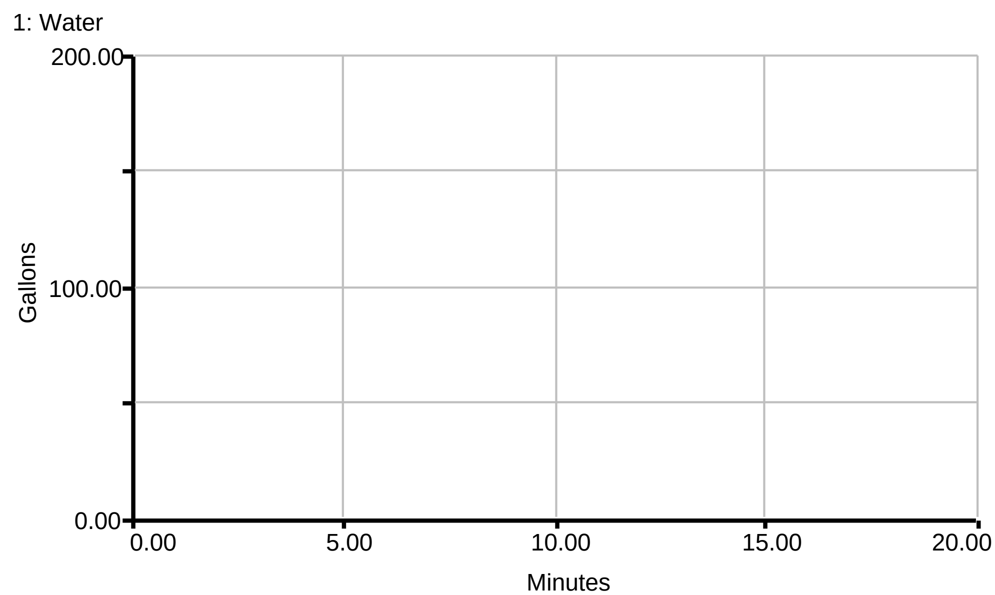




5.3 Constant Inflow, Variable Outflow

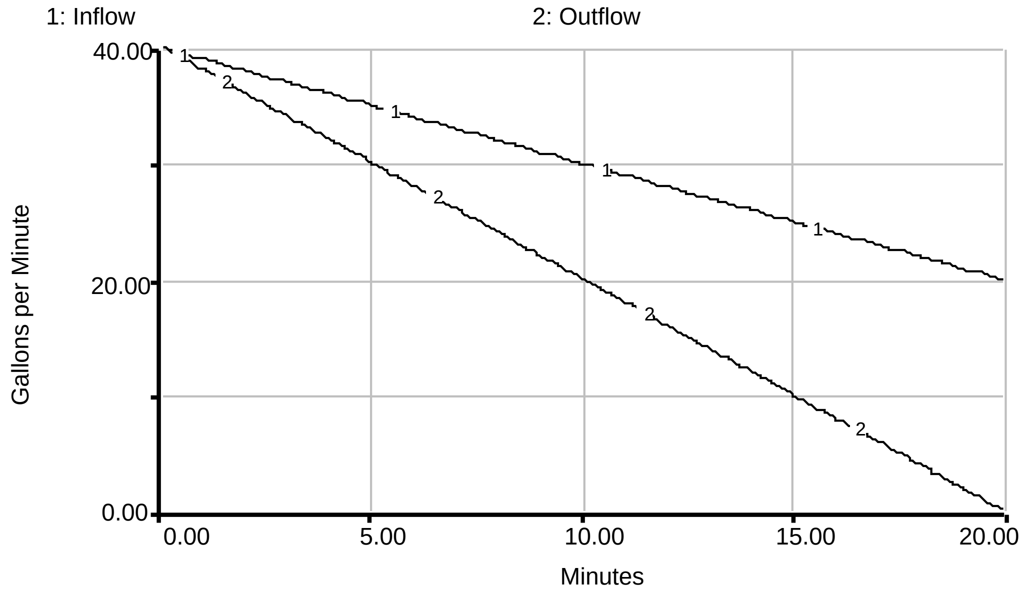
Now let us look at a slightly more complicated system. Consider a water storage tank that is filled at a constant flow of 20 gallons per minute. Water is pumped out of the tank at a linearly increasing flow, beginning at 0 gallons per minute at 0 minutes and increasing to 20 gallons per minute by 20 minutes (i.e. a slope of +1). These flows are graphed below. The tank was empty at time 0. Graph the level of water in the tank versus time.

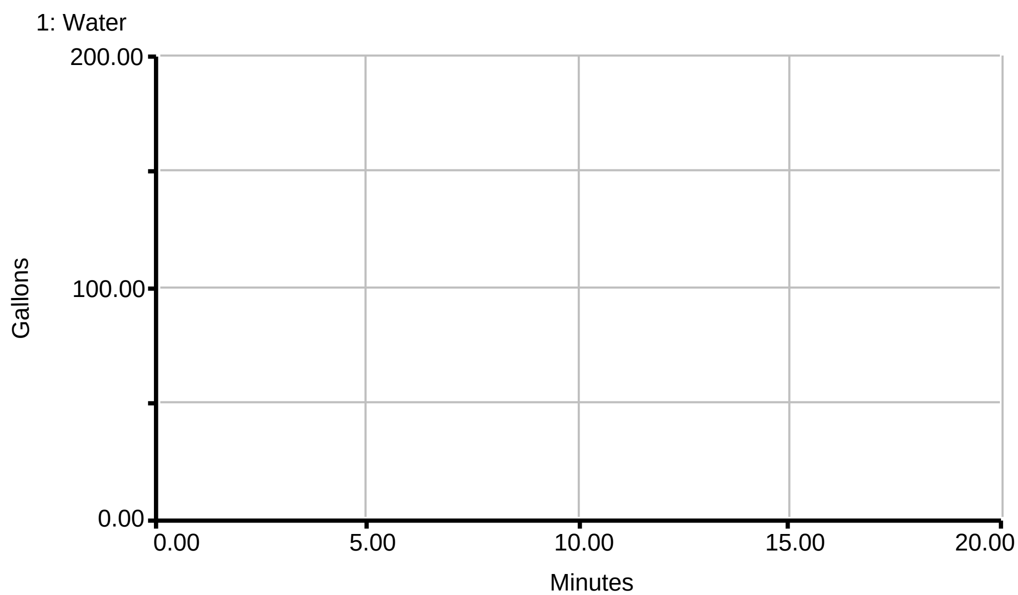




5.4 Variable Inflow and Outflow

For the last exercise, let's consider a water storage tank whose inflow is 40 gallons per minute at 0 minutes and decreases linearly to 20 gallons per minute by 20 minutes. The outflow also starts at 40 gallons per minute, but decreases (linearly) much faster, reaching zero by 20 minutes. These flows are shown below. Determine the behavior of the stock (the amount of water in the tank) over time and graph it on the graph pad provided. Assume the tank was initially empty.





6. Conclusion

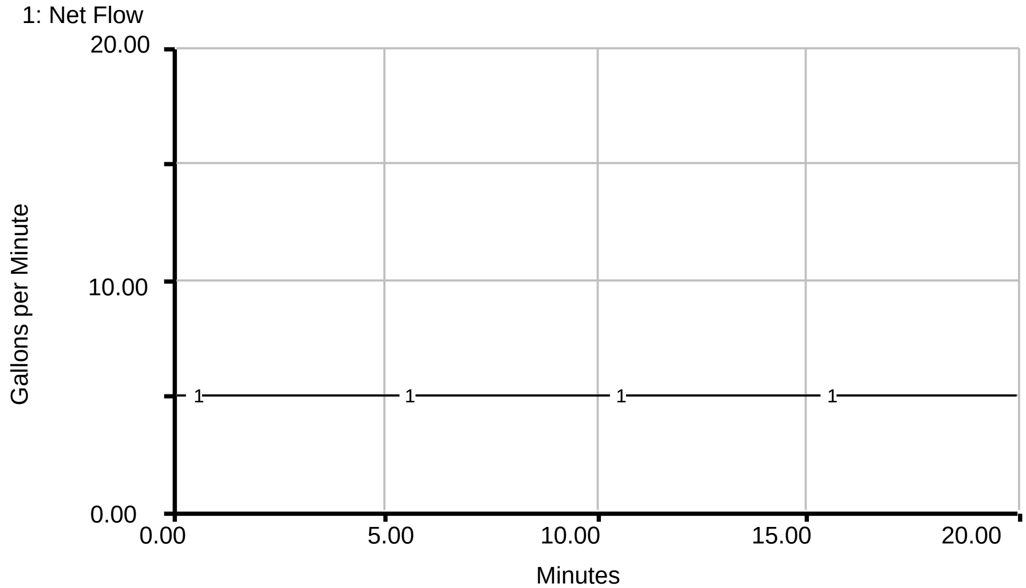
In this paper we looked at graphical integration with independent inflows and outflows. You should now feel comfortable using graphical integration as a method for estimating the behavior of systems with constant flows, step function flows, linearly increasing and decreasing flows with a single net inflow, or independent inflows and outflows.

If you do not feel you have a strong understanding of graphically integrating systems with separate inflows and outflows, please review this paper before continuing along in Road Maps.

7. Appendix: Answers to Exercises

7.1 Answer to Section 5.1

The net flow is shown below:



Remember, the net flow is simply:

Net Flow = Inflow - Outflow

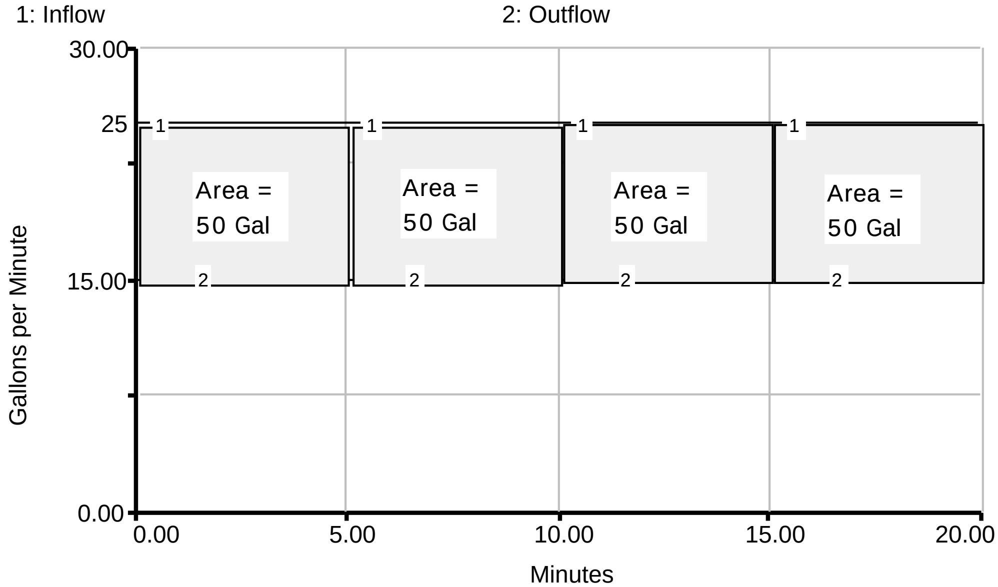
At all times in this example, the inflow was 10 gal per min and the outflow was 5 gal per min. Thus, the net flow, at all times, can be shown to be:

Net Flow = 10 gal per min - 5 gal per min = 5 gal per min

7.2 Answer to Section 5.2

For this example, let us solve it directly without calculating a net flow. Instead, we will use the fact that the area between the inflow and outflow is the change in the value of the stock. Below is the graph of the inflow and outflow divided into four sections. The area of the sections was calculated using:

Area = length * width = 5 minutes * 10 Gallons per minute = 50 gallons



Since this area is the change in the value of the stock (the amount of oil in the storage tank in this system), the value of the stock at 5 minutes can be computed from:

Stock at 5 min = Stock at 0 min + change in value of Stock from 0 to 5 min
= Stock at 0 min + area between inflow and outflow curve from 0 to 5 min
= 50 gallons + 50 gallons = 100 gallons

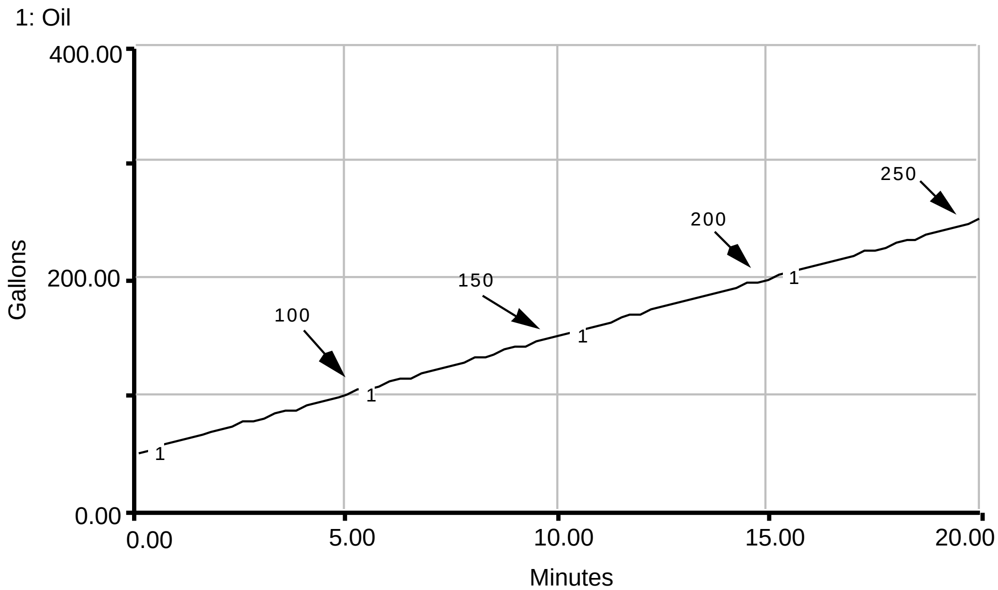
To find the value of the stock at 10 minutes, simply repeat this process:

Stock at 10 min = Stock at 5 min + change in value of Stock from 5 to 10 min
= Stock at 5 min + area between inflow and outflow curve from 5 to 10 min
= 100 gallons + 50 gallons = 150 gallons

To find the value of the stock at 15 minutes, again repeat this process:

Stock at 15 min = Stock at 10 min + change in value of stock from 10 to 15 min
= Stock at 10 min + area between inflow and outflow curve from 10 to 15 min
= 150 gallons + 50 gallons = 200 gallons

Finally, by the same method the value of the stock at 20 minutes is 250 gallons. The behavior is thus shown below.



7.3 Answer to Section 5.3

Let's solve this problem by first calculating the net flow. To do this, all we need to do is use the formula:

Net Flow = Inflow - Outflow

At 0 minutes:
Net Flow at 0 min = Inflow at 0 min - Outflow at 0 min
= 20 gal per min - 0 gal per min = 20 gal per min

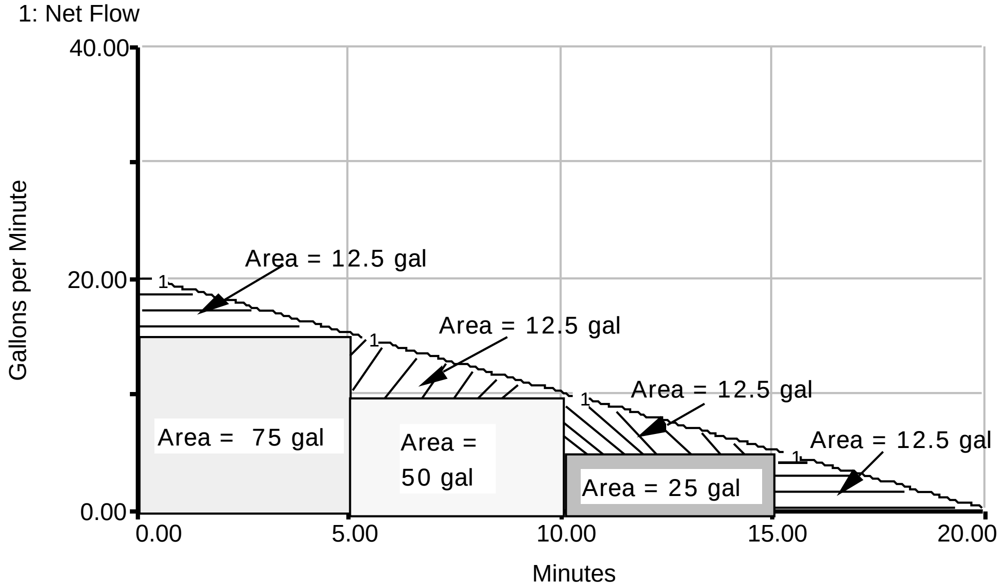
At 5 minutes:
Net Flow at 5 min = Inflow at 5 min - Outflow at 5 min
= 20 gal per min - 5 gal per min = 15 gal per min

At 10 minutes:
Net Flow at 10 min = Inflow at 10 min - Outflow at 10 min
= 20 gal per min - 10 gal per min = 10 gal per min

At 15 minutes:
Net Flow at 15 min = Inflow at 15 min - Outflow at 15 min
= 20 gal per min - 15 gal per min = 5 gal per min

At 20 minutes:
Net Flow at 20 min = Inflow at 20 min - Outflow at 20 min
= 20 gal per min - 20 gal per min = 0 gal per min

Now, we can draw the net flow graph (below) and calculate the area under it to determine the change in the stock value.



Remember, the area between the net flow and the line flow = 0 is the change in the value of the stock. Thus:

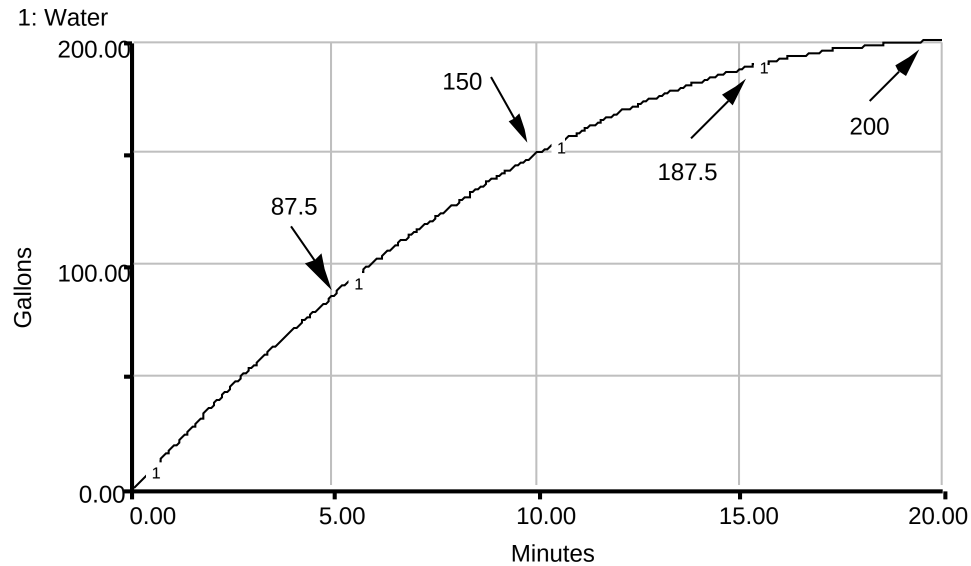
Stock at 5 min **= Stock at 0 min + Change in value of Stock from 0 to 5 min**
= Stock at 0 min + Area under curve from 0 to 5 min
= Stock at 0 min + (Area of Rectangle + Area of Triangle)
= 0 gal + (75 gal + 12.5 gal) = 87.5 gallons

Stock at 10 min **= Stock at 5 min + Change in value of Stock from 5 to 10 min**
= Stock at 5 min + Area under curve from 5 to 10 min
= Stock at 5 min + (Area of Rectangle + Area of Triangle)
= 87.5 gal + (50 gal + 12.5 gal) = 150 gallons

Stock at 15 min **= Stock at 10 min + Change in value of Stock from 10 to 15 min**
= Stock at 10 min + Area under curve from 10 to 15 min
= Stock at 10 min + (Area of Rectangle + Area of Triangle)
= 150 gal + (25 gal + 12.5 gal) = 187.5 gallons

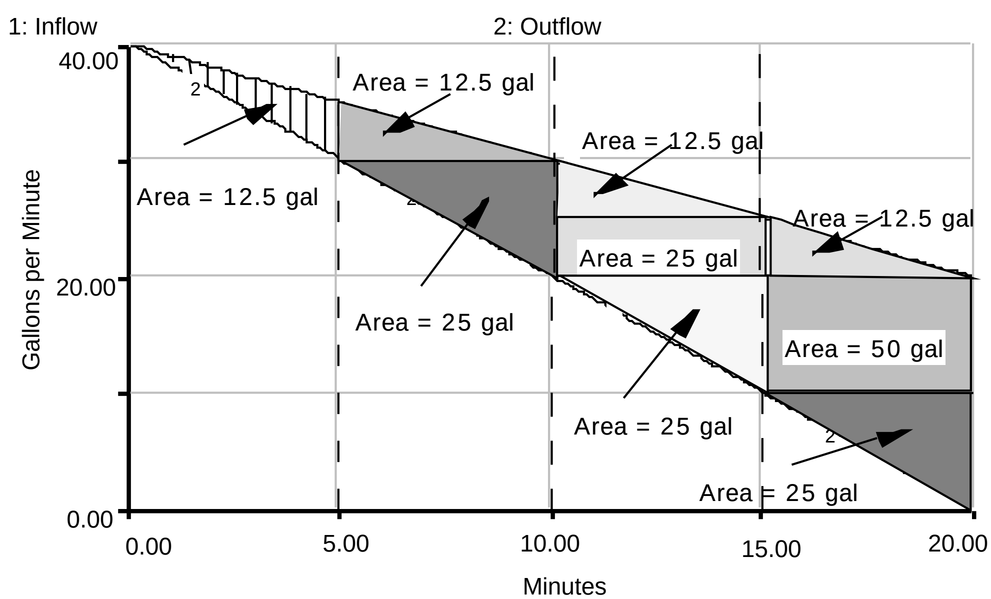
Stock at 20 min **= Stock at 15 min + Change in value of Stock from 15 to 20 min**
= Stock at 15 min + Area under curve from 15 to 20 min
= Stock at 15 min + Area of Triangle
= 187.5 gal + 12.5 gal = 200 gallons

We can now draw the behavior of the stock by plotting these points and connecting them.



7.4 Answer to Section 5.4

Let us solve this problem by finding the area between the inflow and outflow curves, and not bother calculating a net flow. Below is a picture of the flows with the area under the curve divided into rectangles and triangles. Also, the areas of these rectangles and triangles are calculated.



We will use these areas to calculate the change in the value of the stock for various time periods, just as we did for the previous examples.

To calculate the value of the stock at 5 minutes, we can use:

Stock at 5 min = Stock at 0 min + change in value of Stock from 0 to 5 min
Stock at 5 min = 0 gallons + Area between inflow and outflow
= 0 gallons + Area of Triangle
= 0 gallons + 12.5 gallons = 12.5 gallons

Stock at 10 min = Stock at 5 min + change in value of Stock from 5 to 10 min
Stock at 10 min = 5 gallons + Area between inflow and outflow
= 12.5 gallons + Area of Top Triangle + Area of Bottom Triangle
= 12.5 gallons + 12.5 gallons + 25 gallons = 50 gallons

Stock at 15 min = Stock at 10 min + change in value of Stock from 10 to 15 min
Stock at 15 min = 50 gallons + Area between inflow and outflow
= 50 gallons + Area of Top Triangle + Area of Rectangle + Area of Bottom Triangle
= 50 gallons + 12.5 gallons + 25 gallons + 25 gallons
= 112.5 gallons

Stock at 20 min = Stock at 15 min + change in value of Stock from 15 to 20 min
Stock at 20 min = 112.5 gallons + Area between inflow and outflow
= 112.5 gallons + Area of Top Triangle + Area of Rectangle + Area of Bottom Triangle
= 112.5 gallons + 12.5 gallons +50 gallons + 25 gallons
= 200 gallons

This behavior is shown below.