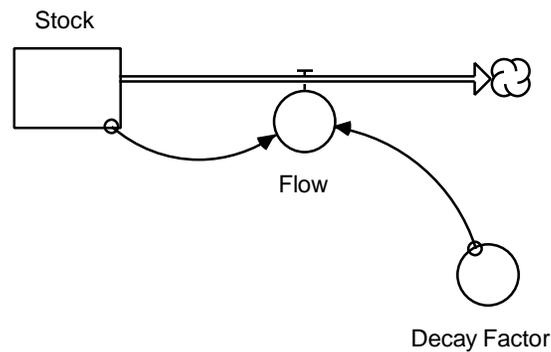


Beginner Modeling Exercises

Section 3

Mental Simulation of Simple Negative Feedback



Prepared for
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Helen Zhu
Vensim Examples added October 2001

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Introduction

Feedback loops are the basic structural elements of systems. Feedback in systems causes nearly all dynamic behavior. To use system dynamics successfully as a learning tool one must understand the effects of feedback loops on dynamic systems. One way of using system dynamics to understand feedback is with simulation software on your computer¹. Computer simulation is a very useful tool for exploring systems. However, one should be able to use the other simulation tool of system dynamics: *mental simulation*. A strong set of mental simulation skills will enhance ability to validate, debug and understand dynamic systems and models.

This paper deals primarily with negative feedback and begins with a review of some key concepts. A set of exercises at the end will help reinforce understanding of the feedback dynamics in a simple negative feedback loop. Solutions to these exercises are also included. It is assumed that the reader has already studied “Beginner Modeling Exercises: Mental Simulation of Positive Feedback,”² and is familiar with fundamental system dynamics terms.

Negative Feedback

Compare negative feedback to letting air out of a balloon. At first, air pressure inside the expanded balloon pushes air out at a high rate, allowing the balloon to deflate. As air escapes, the balloon gets smaller, the air pressure dies down, and the deflation rate drops. This continues until deflation stops completely. Negative feedback occurs when change in a system produces less and less change in the same direction until a goal is reached. In this circumstance, the goal is equal air pressure inside and outside the balloon.

¹There are several commercial system dynamics simulation packages available for both Windows and Macintosh. Road Maps is geared towards the use of STELLA II which is available from High Performance Systems (603) 643-9636. Road Maps can be accessed through the internet at <http://sysdyn.mit.edu/>.

² “Beginner Modeling Exercises: Mental Simulation of Positive Feedback” (D-4487) by Joseph G. Whelan, is part of the Road Maps series.

Systems exhibiting negative feedback are present everywhere, ranging from a population facing extinction to the simple thermostat. In each circumstance, negative feedback exhibits goal-seeking behavior. In other words, the difference between the current state of the system and the desired state causes the system to move towards the desired state. The closer the state of the system is to its goal, the smaller the rate of change, until the system reaches its goal. This goal can differ from system to system. In a population extinction model, the goal is zero — animals continue dying until there are none left. A thermostat, on the other hand, has a goal of a desired room temperature.

Example 1: Rainfall on the sidewalk (Goal = 0)

This section focuses on how to simulate and visualize stock behavior mentally over time for a zero-goal system. The first step explores initial behavior.

The system and its initial behavior

An example of negative feedback is the amount of wet sidewalk from rain. Initially, a 10 square foot block of sidewalk is completely dry. Rain begins to fall at a constant rate. Within the first minute, rain covers 50% of the block surface area, or 5 square feet total. In the next minute, the same amount of rain falls, but some of the raindrops fall on areas that are already wet. The chance of rain falling on a dry spot is one out of two, or 50%. Rainfall covers five square feet in the second minute, but 50% of that is already wet, so only an additional 2.5 square feet becomes wet.

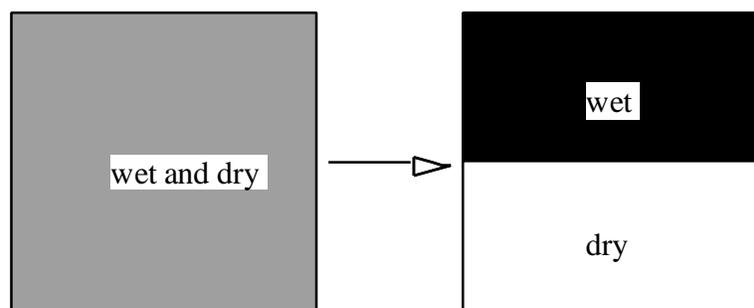


Figure 1: Rain falls on sidewalk, first minute

Although rainfall is evenly distributed in “wet and dry,” it quantitatively covers half of the surface area of the block. It is easier to visualize this, for modeling purposes, as its equivalent of one-half wet and one-half dry. Figure 1 above shows both the natural rainfall pattern and its equivalent schematic representation.

Rain continues to fall. Half of it lands on places where the sidewalk is already wet. The other half covers 50% of the remaining dry areas.

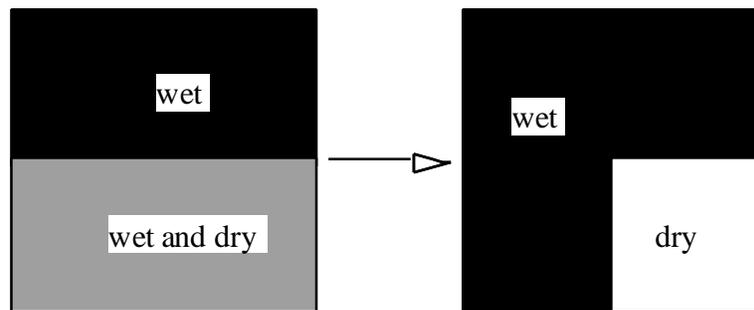


Figure 2: Sidewalk, minute two

Net rainfall is the equivalent of 75% wet, 25% dry. The same process of 50% of the remaining dry surface becoming wet continues, advancing towards covering the entire surface area.

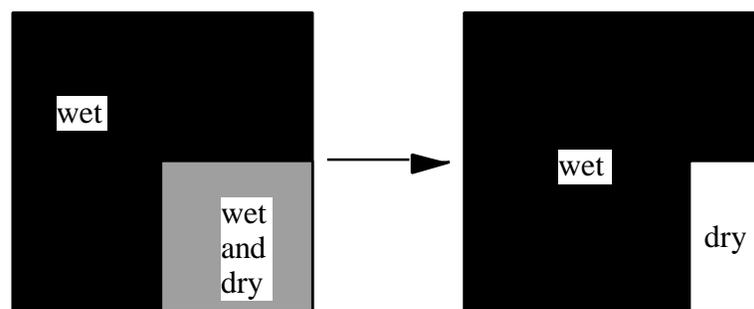


Figure 3: Sidewalk, minute three

Given this description of the rainfall system, we can model it in STELLA as shown in the figure below.

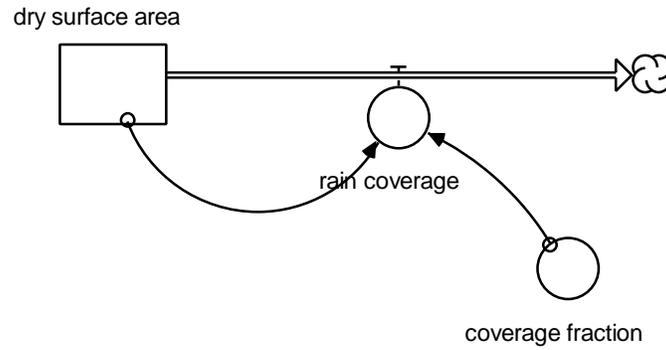


Figure 4: STELLA model of rainfall system

$$\text{rain coverage} = \text{dry surface area} * \text{coverage fraction}$$

$$[\text{square feet}/\text{min}] \quad [\text{square feet}] \quad [/\text{minute}]$$

We know that:

$$\text{initial } \textit{dry surface area} = 10 \text{ square feet}$$

$$\textit{coverage fraction} = 1/2 \text{ /minute}$$

Thus, initial $\textit{rain coverage} = 10 * 1/2 = 5$ square feet/minute. In other words, as Figure 5 shows, the initial slope of the $\textit{dry surface area}$ is -5 square feet per minute. This negative term indicates that $\textit{dry surface area}$ decreases by $\textit{rain coverage}$ rate.

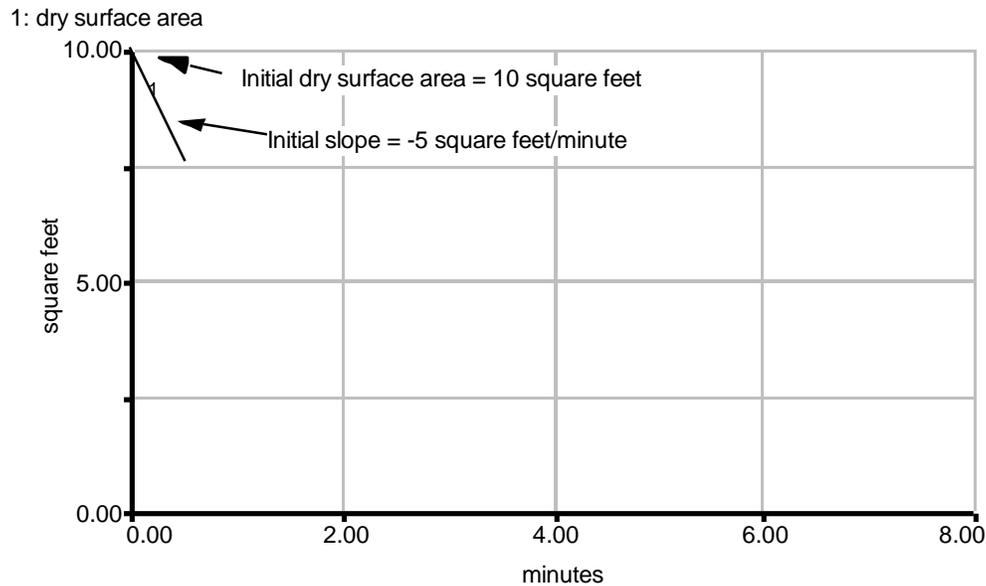


Figure 5: Behavior plot of rainfall system

What happens past the initial point? Eventually rain will cover the entire sidewalk. The system moves toward the goal of zero *dry surface area*. In order to mentally simulate the quantitative behavior of the system over time, one should perform half-life calculations.

Half-life Calculations for rainfall system

Negative feedback systems exhibit asymptotic behavior. Asymptotic decay of a stock has a constant halving time, or half-life, of the stock. Half-life is a measure of the rate of decay. It is the time interval during which distance to the goal decreases by a fraction of one-half. Half-life is the counterpart to the doubling time in positive feedback, and is likewise independent of initial stock value. Approximate the half life by:

$$\text{Half-life} = 0.7 * \text{time constant}$$

$$\text{where time constant} = 1 / \text{decay fraction}$$

Is *coverage fraction* a time constant or decay fraction? Both terms refer to a constant value, one being the reciprocal of the other, and vice versa. They differ only in how they are used. In a rate equation, the stock is either multiplied by a decay fraction or

divided by a time constant³. In this case, we multiply *coverage fraction* and *dry surface area* to get *rain coverage*, so *coverage fraction* is a decay fraction. The half-life is:

$$\text{Half-life} = 0.7 * (1/(1/2)) = 1.4 \text{ minutes}$$

This means that the *dry surface area* halves every 1.4 minutes. Knowing the half-life value, it is easy to plot the curve of the stock by connecting the points representing the value of the stock at each half-life. Figure 6 shows this, in graph form.

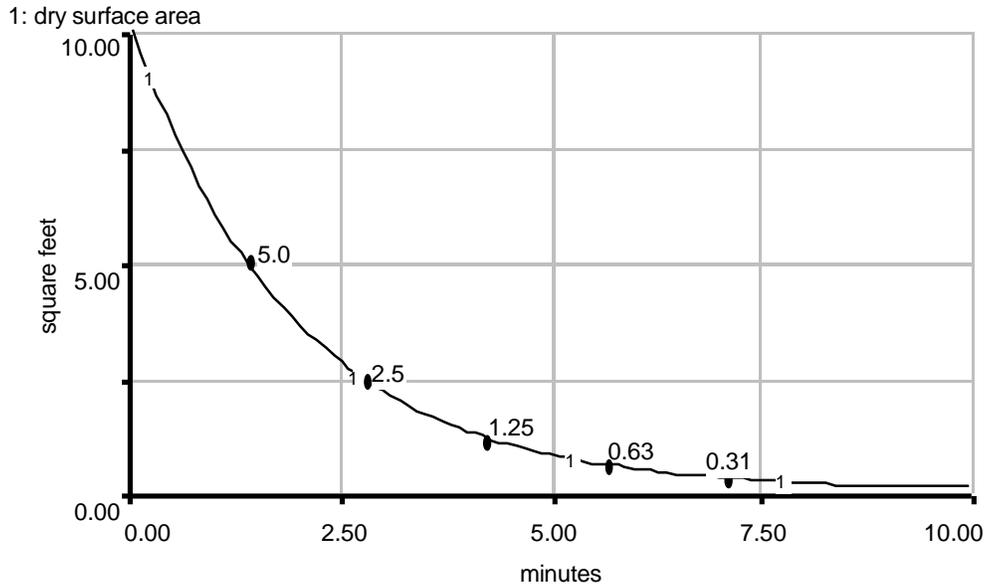


Figure 6: Mental simulation graph of rainfall system

The mental simulation is complete. It is evident from the graph that the system asymptotically declines toward a goal of zero.

Example 2: Chromatography (Goal ? 0)

So far, zero-goal negative feedback systems have been discussed. However, this is a special case and many systems have goals that are not equal to zero. A common experiment in biology classrooms is chromatography, on paper or on silica gel (thin layer chromatography). Chromatography is a method used to separate the component

³ Notice that multiplying by the time constant is the same as dividing by the decay fraction because:
 $0.7 * \text{time constant} = 0.7 * (1/\text{decay fraction}) = 0.7/\text{decay fraction}$

substances in an unknown liquid such as plant pigment or oils. The experimenter places drops of the substance to be tested about 4 cm from the base of the paper. The chromatography paper is placed standing upright in a container of solvent about 3 cm deep (water or dilute acid). As solvent is absorbed and travels upwards on the paper, it carries molecules of the specimen with it. These molecules separate into components according to differing masses and molecule sizes. When the paper is initially placed in the solvent, the dry paper quickly absorbs the solvent, and the solvent rises rapidly. Gradually, as the solvent rises, the forces pulling it up (surface tension, water pressure) and the forces pulling it down (gravity, air pressure) cancel each other. Equilibrium is reached and the paper no longer absorbs any solvent. Figure 7 is an illustration of the chromatography process.

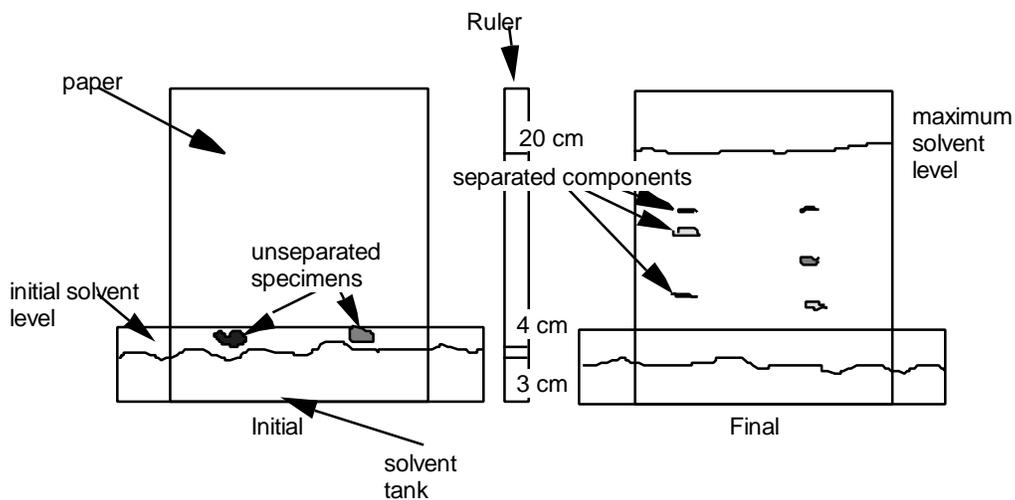


Figure 7: Chromatography: initial and final stages

The chromatography system can be generalized to say that the forces acting on the solvent in the paper are directly dependent on the height of the solvent on the paper. If *current height* (cm) is the level of the solvent mark on the paper at any time, and *max height* (cm) is the highest solvent level attained (through experiment), then the *absorption rate* (cm/minute) is a fraction (*absorption constant*) of the difference between the two heights (or *gap*). In a sample chromatography run, the maximum height (*max height*)

attained by the solvent was about 20 cm, and approximately 1/5 (*absorption constant*) of the *gap* absorbs solvent per minute. Thus the system equations are:

$$\text{gap} = \text{max height} - \text{current height}$$

$$[\text{cm}] \quad [\text{cm}] \quad \quad [\text{cm}]$$

$$\text{absorption rate} = \text{gap} * \text{absorption constant}$$

$$[\text{cm/minute}] \quad [\text{cm}] \quad [/\text{minute}]$$

The solvent absorption system can be modeled in STELLA as shown in Figure 8:

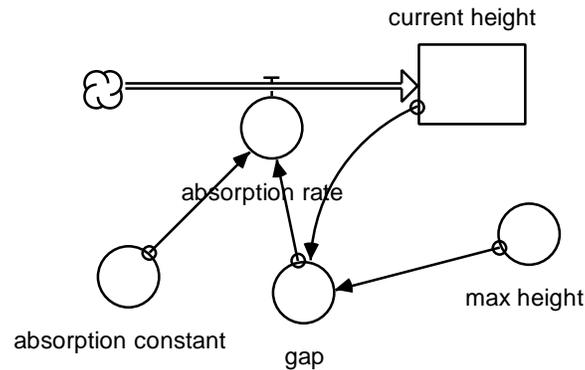


Figure 8: STELLA model of solvent absorption

Try to picture how the system works. The *current height* is initially 3 cm (the height of the initial solvent level), so the *gap* between the *current height* and the *max height* is 17 cm. Multiplying 17 cm by the fraction of 1/5 yields an initial absorption rate of 3.4 cm per minute.

Figure 9 shows the initial slope of the stock behavior to be 3.4 cm per minute:

1: current height

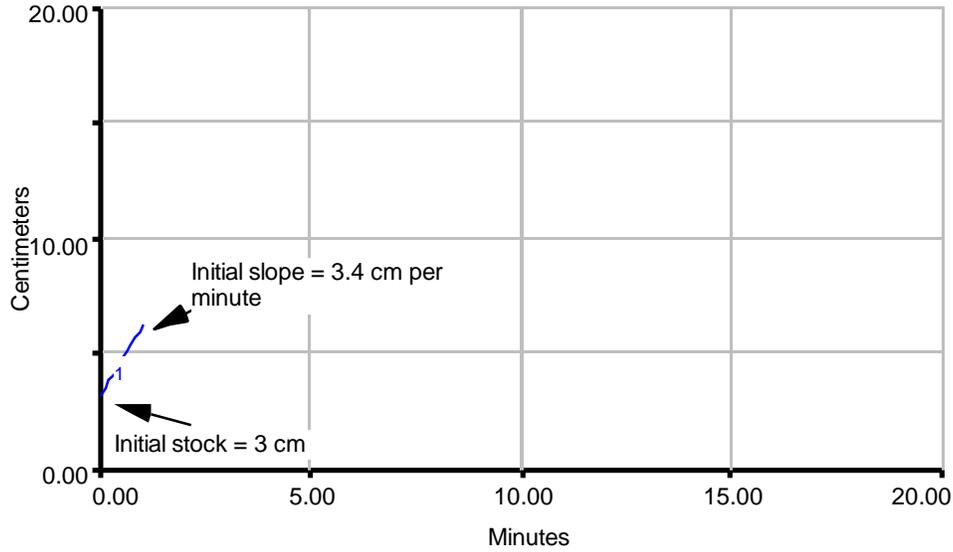


Figure 9: Behavior plot for chromatography system

The chromatography system differs from the rainfall example because the goal is no longer zero, but a set value. There is a gap involved equal to the initial stock value minus the goal, and that is why the system has a goal-gap formulation. The half-life can also be calculated: $\text{Half-life} = 0.7 * (1 / (1/5)) = 3.5$ minutes.

1: current height

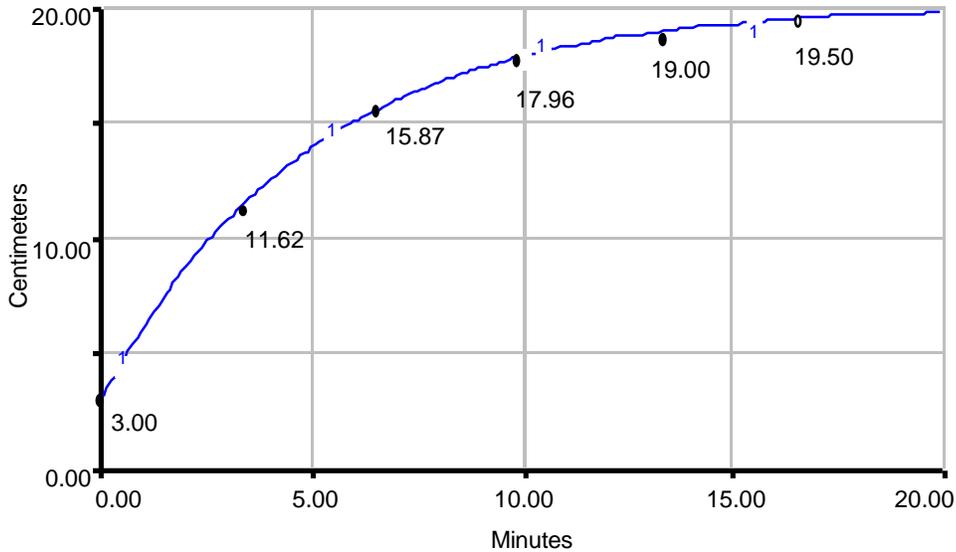


Figure 10: Mental Simulation graph of chromatography model

Review

The exercises in the following section assume a strong understanding of the terms below. If any of these terms is unfamiliar or confusing, please review this paper before attempting to correctly solve the exploration problems.

I. A starting point:

- Initial values of stocks and flows

II. From there on:

- Identifying negative feedback

III. Growth characteristics:

- Asymptotic growth
- Growth to equilibrium
- Above/below goal initial stock value

IV. Goal-gap structure of negative feedback

V. Halving time

The solutions to the following exercises are attached at the end of the paper. If you are having trouble with the exercises, refer to the examples in the paper before looking at the solution.

Exploration

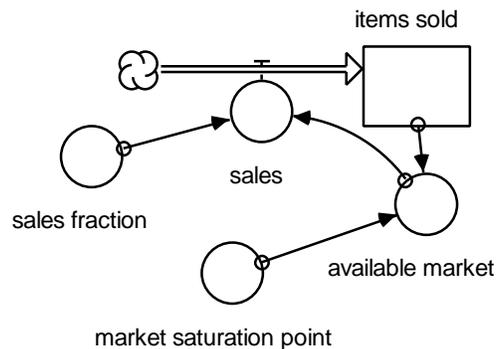
1. Negative feedback is more common in the everyday world than positive feedback. List some examples of negative feedback in the world around you.

\

2. Determine whether the following exhibits positive feedback or negative feedback. You may sketch the model structure if necessary.

The federal debt accumulates interest indefinitely.

3. You confront a STELLA model of the sale of a new invention, the widget. Because it is new, no one has this product initially. Assuming that each widget never needs to be replaced, and its reputation does not change so that the sales fraction is constant, the system can be modeled like this:



$$\text{items_sold}(t) = \text{items_sold}(t - dt) + (\text{sales}) * dt$$

$$\text{INIT items_sold} = 0 \text{ [widgets]}$$

$$\text{sales} = \text{available_market} * \text{sales_fraction} \text{ [widgets/month]}$$

$$\text{available_market} = \text{market_saturation_point} - \text{items_sold} \text{ [widgets]}$$

$$\text{market_saturation_point} = 10000 \text{ [widgets]}$$

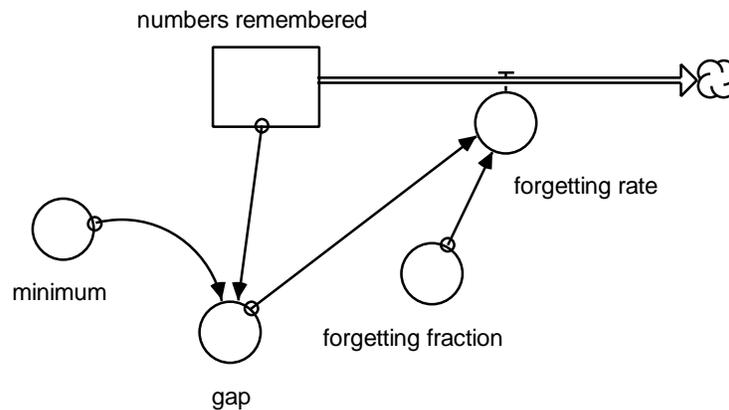
$$\text{sales_fraction} = 0.2 \text{ [/month]}$$

Is this model an example of negative feedback?

Why or why not?

Can you construct a model that would represent the same system but contains an outflow instead of an inflow? This requires choosing different variables.

4. The figure below shows a simple model for forgetfulness. The more you need to memorize, the more easily you forget. For instance, if you had to memorize 100 phone numbers, it is likely that you will forget about 80 of them, since your brain has to keep track of too many numbers. However, if you only had to memorize 10, you may not forget any.



$$\text{numbers_remembered}(t) = \text{numbers_remembered}(t - dt) + (- \text{forgetting_rate}) * dt$$

[numbers]

INIT numbers Remembered = 100 [numbers]

forgetting_rate = forgetting_fraction*gap [numbers/day]

forgetting_fraction = 0.8 [/day]

$\text{gap} = \text{numbers_remembered} - \text{minimum}$ [numbers]

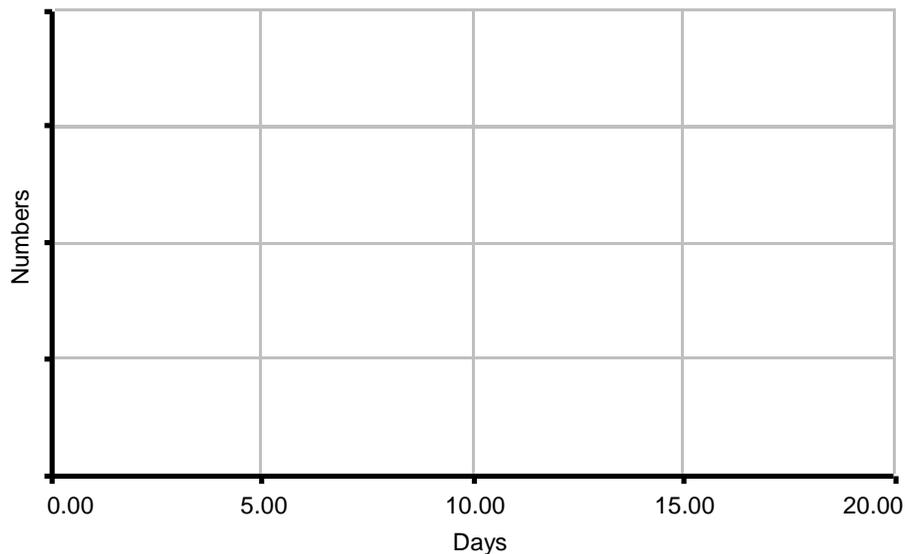
$\text{minimum} = 10$ (minimum is maximum number of phone numbers you can remember without forgetting any) [numbers]

What is the halving time for this model? (halving time = $0.7 * \text{time constant}$)

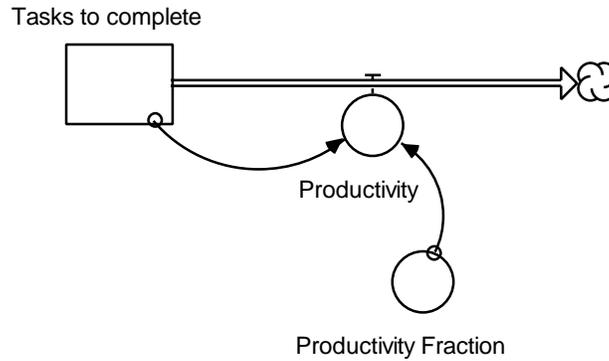
What is the goal in this model?

Use the halving time to approximate how many halving times it takes to reach the goal.

On the graph below, draw the behavior of the stock *Numbers Remembered* for 20 days:



5. Take the following STELLA model of human productivity. When faced with many projects to do, people work faster and harder. But when the demand for hard labor is lower, with fewer tasks to complete, productivity usually drops.



$\text{Tasks_to_complete}(t) = \text{Tasks_to_complete}(t - dt) + (- \text{Productivity}) * dt$ [tasks]

INIT $\text{Tasks_to_complete} = 100$ [tasks]

$\text{Productivity} = \text{Productivity_Fraction} * \text{Tasks_to_complete}$ [tasks/day]

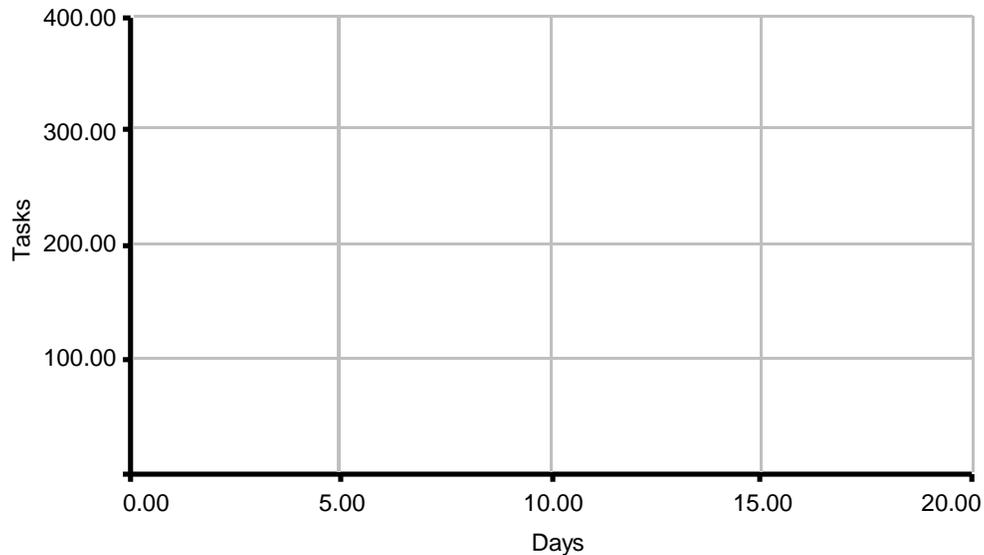
$\text{Productivity_Fraction} = 0.3$ [/day]

What is the goal of this model?

If the initial value of the stock was 200 tasks, what is the half life of this model?

If the initial value was 1000 tasks?

On the graph pad below, draw the behavior of the stock with initial value of 100, 200 and 300 (three lines on one graph) tasks. At what point does each curve reach equilibrium?



Solutions

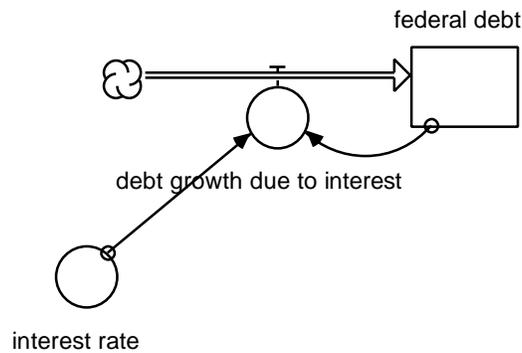
Exercise 1. Several examples of negative feedback that you may be familiar with are:

- running Initially you go fast, then slow down as you get more and more tired until you reach the point when you're so tired you stop completely.
- music When you get a new CD, you listen to it maybe ten times the first day, five times the second, and so on until you get sick of it. Eventually you only listen to it once a week or even less often.
- making pals A college freshman tries to make many friends during orientation, but then starts building a small group of pals to spend time with and is less eager to make more friends. Eventually the individual has "enough" friends, and stops trying to strike up random conversations all the time.
- business A company lays off a certain percentage of employees as part of cutting back, until it reaches the absolute minimum number needed to function.

- dress If the temperature is 100 degrees and you're in a huge fur coat with lots of sweaters and accessories, you will immediately throw off some large item to cool yourself down. As you approach comfort level, you gradually take off smaller items like scarves or gloves and make smaller adjustments until you arrive at an ideal temperature.

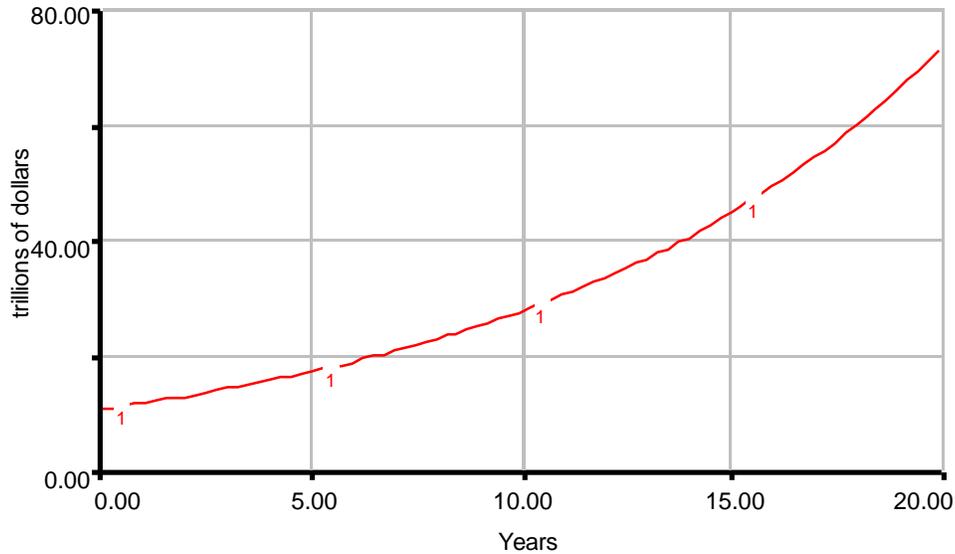
Exercise 2. The federal debt accumulates interest indefinitely: positive feedback.

The word “indefinitely” alone should be enough to tell you that this is positive feedback. Interest is always accrued as a certain percentage of the stock. The greater the initial debt, the more interest payments increase, and therefore the larger the debt becomes. Mental simulation shows that this kind of feedback yields exponential growth of the stock, so this is a positive feedback loop. The model is essentially the same as a bank account accumulating interest. It can be modeled as follows:



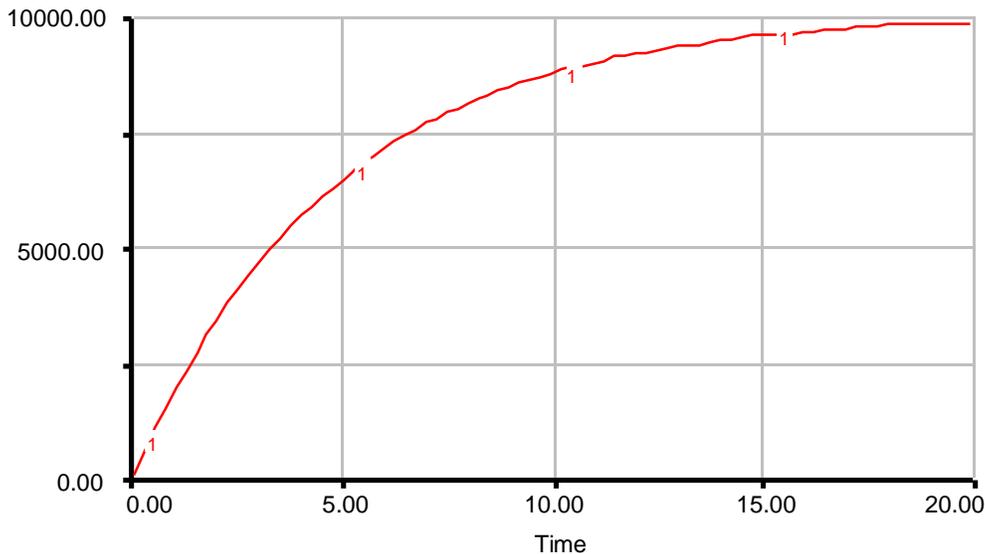
The exhibited exponential behavior, when initial *federal debt* is \$10 trillion, with an *interest rate* of 10% per year (0.1), is shown in the following graph of simulation results:

1: federal debt

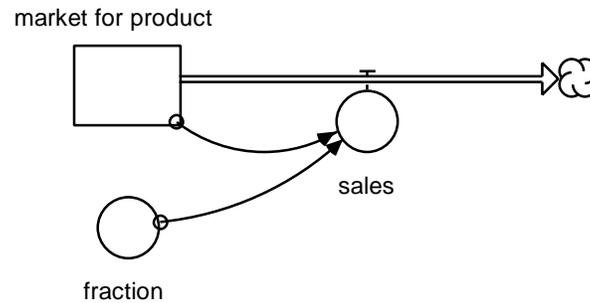


3. Yes, this model is definitely an example of negative feedback. It is easy to identify the goal-gap structure in this model. The goal is the *saturation of market*; the gap is the *available market*. As the *available market* gets smaller and smaller, the *sales* go down. The graph of items sold, therefore, exhibits asymptotic growth approaching the saturation point, in this case 10000 widgets. The following graph is a plot of the stock.

1: items sold



Another way to see this more clearly as a negative feedback loop is to build the model this way:



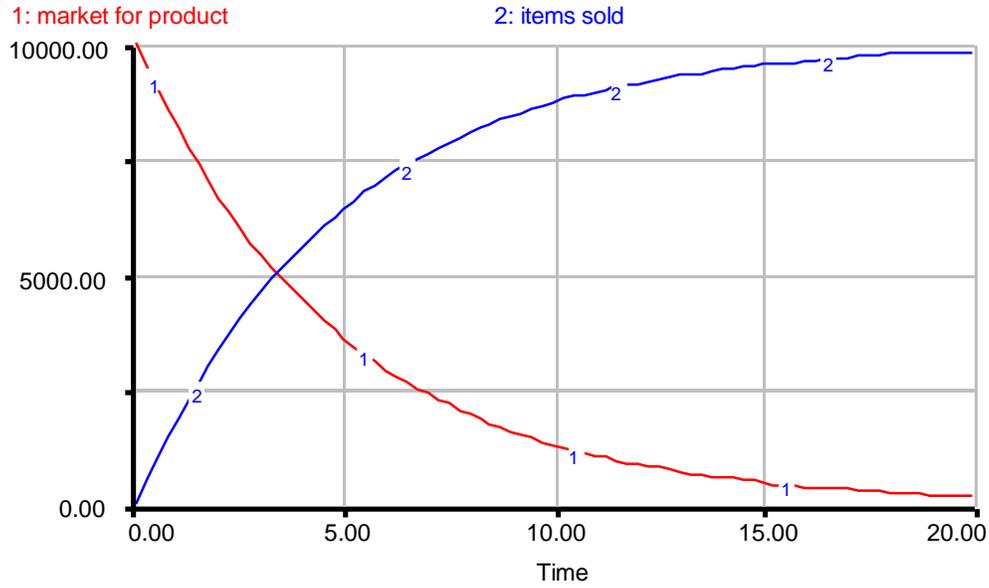
```
market_for_product(t) = market_for_product(t - dt) + (- sales) * dt [widgets]
```

```
INIT market_for_product = 10000 [widgets]
```

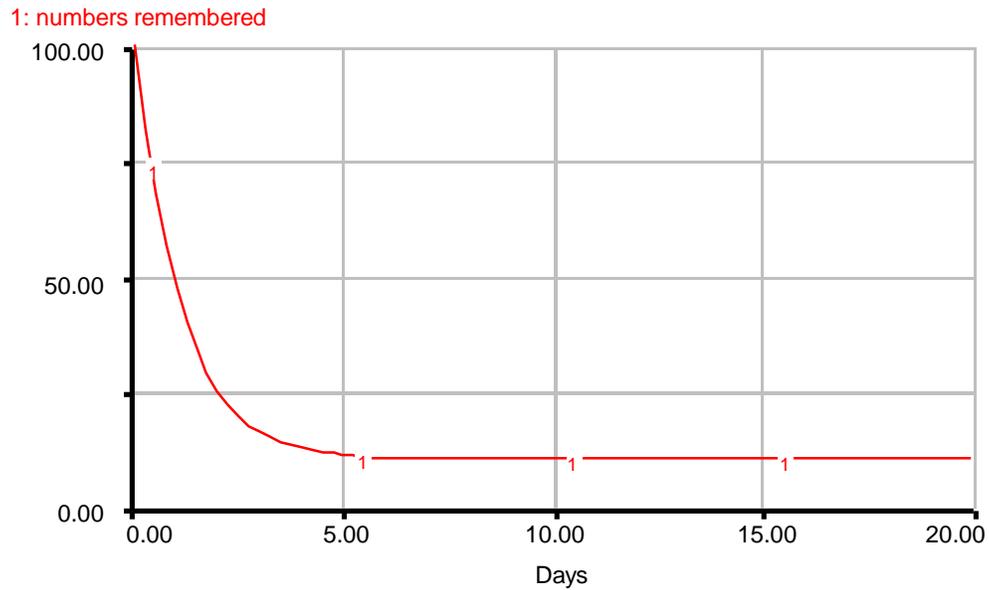
```
sales = market_for_product*fraction [widgets/month]
```

```
fraction = 1/5 [/month]
```

Notice that the decay fraction remains the same, and the maximum value is still 10000 widgets. Although the graph of the stock in this second case appears to be different, it actually exhibits the same type of behavior. One is asymptotic growth, the other asymptotic decay, and both have the same half-life because their decay fractions are exactly the same.



4. This system contains a negative feedback loop. The fewer phone numbers there are in your brain, the smaller the percentage of those phone numbers you will forget over a certain period of time. Below is a graph showing the asymptotic decline of the stock and rate. The stock approaches its goal of 10 phone numbers.



The halving time calculation is very simple.

$$\text{Halving time} = 0.7 * \text{time constant}$$

$$\text{Halving time} = 0.7 * (1 / \text{decay fraction})$$

$$\text{Halving time} = 0.7 * (1 / 0.8)$$

$$\text{Halving time} = 0.7 * 1.25 = 0.875 \text{ days}$$

Notice that if you halve the gap, 90, 7 times, the result is less than 1, which we consider a close enough approximation of 0. Thus it takes $7 * 0.875 = 6.125$ days for the stock to come to equilibrium.

5. The half life calculation is as follows:

$$\text{Half life} = 0.7 * \text{time constant}$$

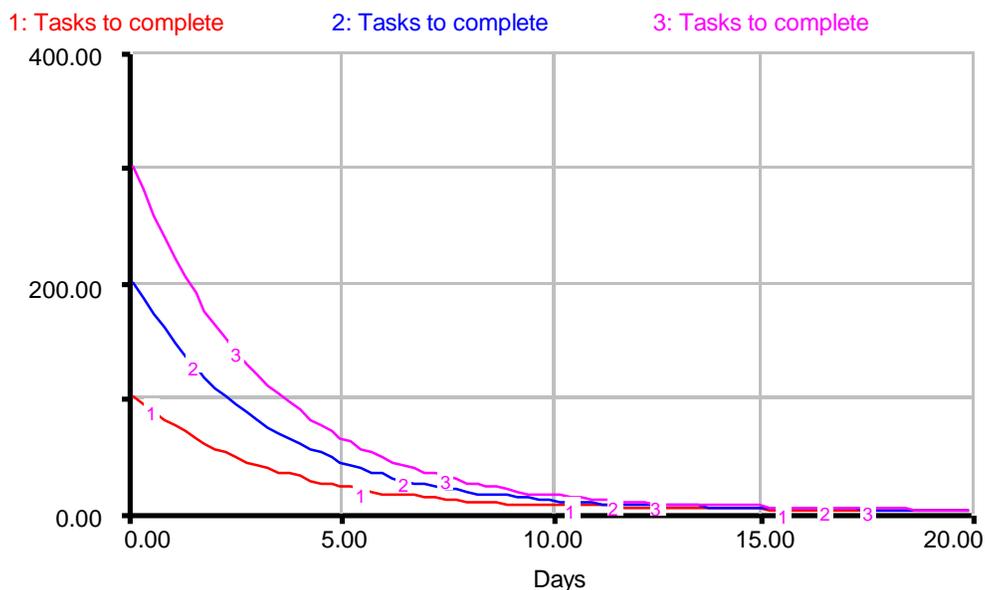
$$\text{Half life} = 0.7 * (1 / \text{decay fraction})$$

$$\text{Half life} = 0.7 * 3.3$$

$$\text{Half life} = 2.31 \text{ days}$$

Whether the initial value of the stock was 200 or 1000 tasks, the half life of this model would still be the same, because half-life does not depend on the initial value of the stock.

The graph of the three curves looks like this:



Evidently, it is a characteristic of negative feedback that the stock approaches equilibrium at about the same time, no matter what its initial value (this includes initial values above and below the goal). The stock asymptotically grows or decays towards a stable equilibrium.

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Vensim Examples: Beginner Modeling Exercises: Section 3 Mental Simulation of Simple Negative Feedback

By Lei Lei
October 2001

Example 1: Rainfall on the Sidewalk

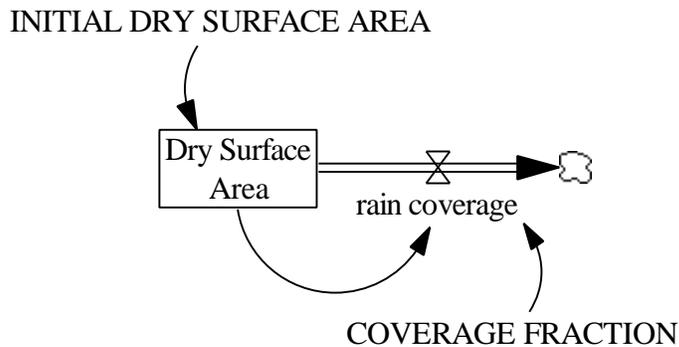


Figure 11: Vensim equivalent of Figure 4. Model of rainfall system

Documentation for Rainfall System model

- (1) $COVERAGE\ FRACTION = 0.5$
Units: 1/Minute
- (2) $Dry\ Surface\ Area = INTEG(-rain\ coverage, INITIAL\ DRY\ SURFACE\ AREA)$
Units: square feet
- (3) $FINAL\ TIME = 10$
Units: Minute
The final time for the simulation.
- (4) $INITIAL\ DRY\ SURFACE\ AREA = 10$
Units: square feet
- (5) $INITIAL\ TIME = 0$
Units: Minute
The initial time for the simulation.
- (6) $rain\ coverage = Dry\ Surface\ Area * COVERAGE\ FRACTION$
Units: square feet/Minute

(7) SAVEPER = TIME STEP

Units: Minute

The frequency with which output is stored.

(8) TIME STEP = 0.0625

Units: Minute

The time step for the simulation.

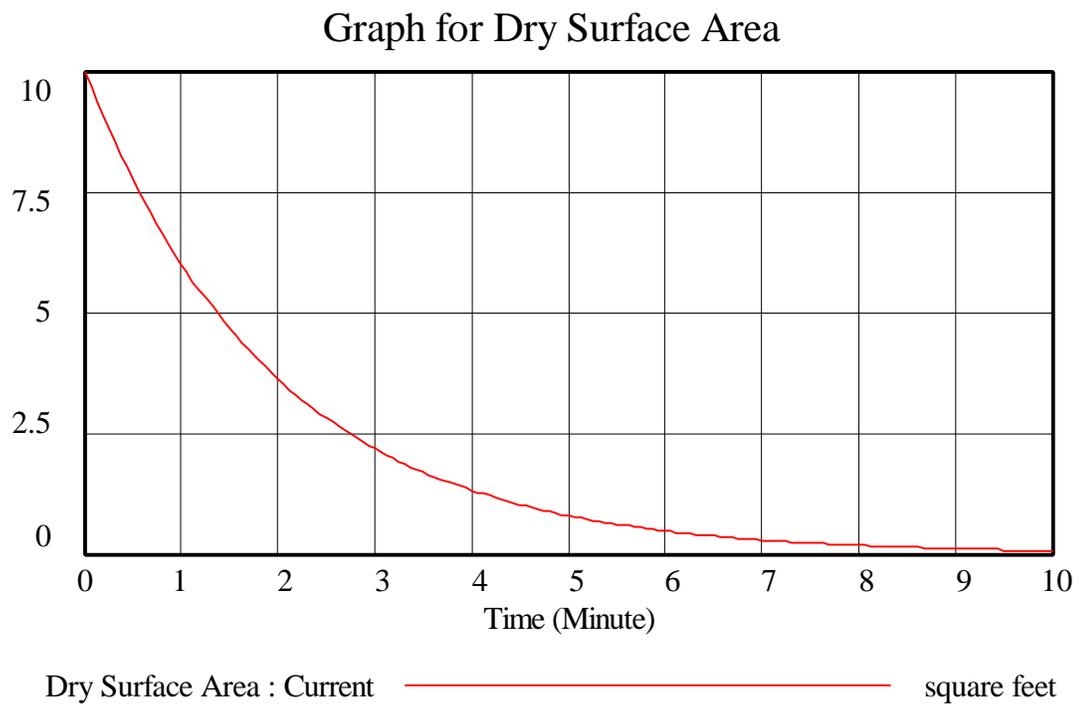


Figure 12: Vensim Equivalent of Figure 6. Mental simulation graph of rainfall system

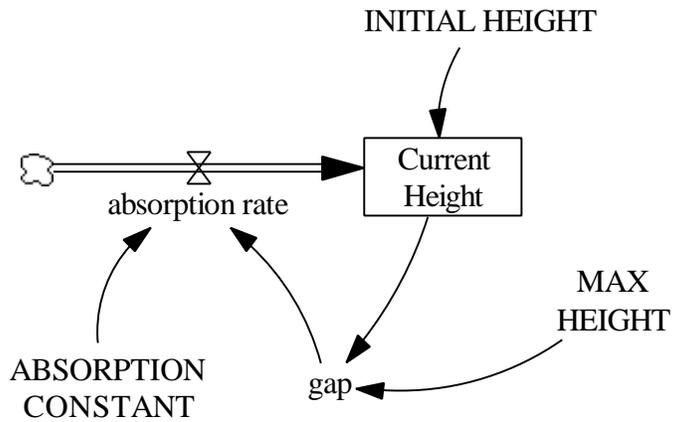
Example 2: Chromatography

Figure 13: Vensim equivalent of Figure 8. Model of solvent absorption

Documentation for Solvent Absorption model

- (01) ABSORPTION CONSTANT = 0.2
Units: 1/Minute
- (02) absorption rate = gap * ABSORPTION CONSTANT
Units: cm/Minute
- (03) Current Height = INTEG (absorption rate, INITIAL HEIGHT)
Units: cm
- (04) FINAL TIME = 20
Units: Minute
The final time for the simulation.
- (05) gap = MAX HEIGHT - Current Height
Units: cm
- (06) INITIAL HEIGHT = 3
Units: cm
- (07) INITIAL TIME = 0
Units: Minute
The initial time for the simulation.
- (08) MAX HEIGHT = 20
Units: cm

- (09) $SAVEPER = TIME\ STEP$
 Units: Minute
 The frequency with which output is stored.
- (10) $TIME\ STEP = 0.0625$
 Units: Minute
 The time step for the simulation.

Exploration 1: Widget

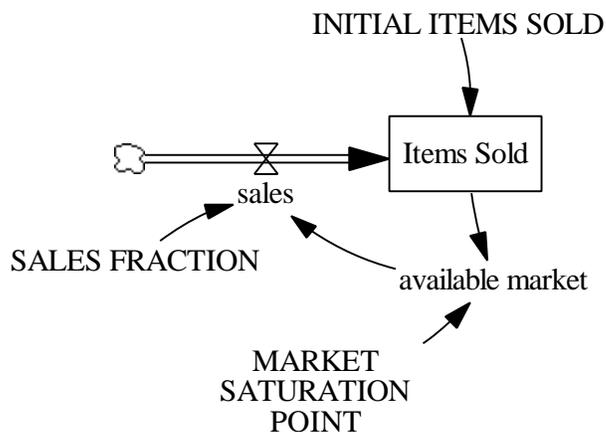


Figure 14: Vensim equivalent of Widget model

Documentation for Widget model

- (01) $available\ market = MARKET\ SATURATION\ POINT - Items\ Sold$
 Units: widgets
- (02) $FINAL\ TIME = 20$
 Units: Month
 The final time for the simulation.
- (03) $INITIAL\ ITEMS\ SOLD = 0$
 Units: widgets
- (04) $INITIAL\ TIME = 0$
 Units: Month
 The initial time for the simulation.
- (05) $Items\ Sold = INTEG (sales, INITIAL\ ITEMS\ SOLD)$
 Units: widgets

- (06) MARKET SATURATION POINT = 10000
Units: widgets
- (07) sales = available market*SALES FRACTION
Units: widgets/Month
- (08) SALES FRACTION = 0.2
Units: 1/Month
- (09) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.
- (10) TIME STEP = 0.0625
Units: Month
The time step for the simulation.

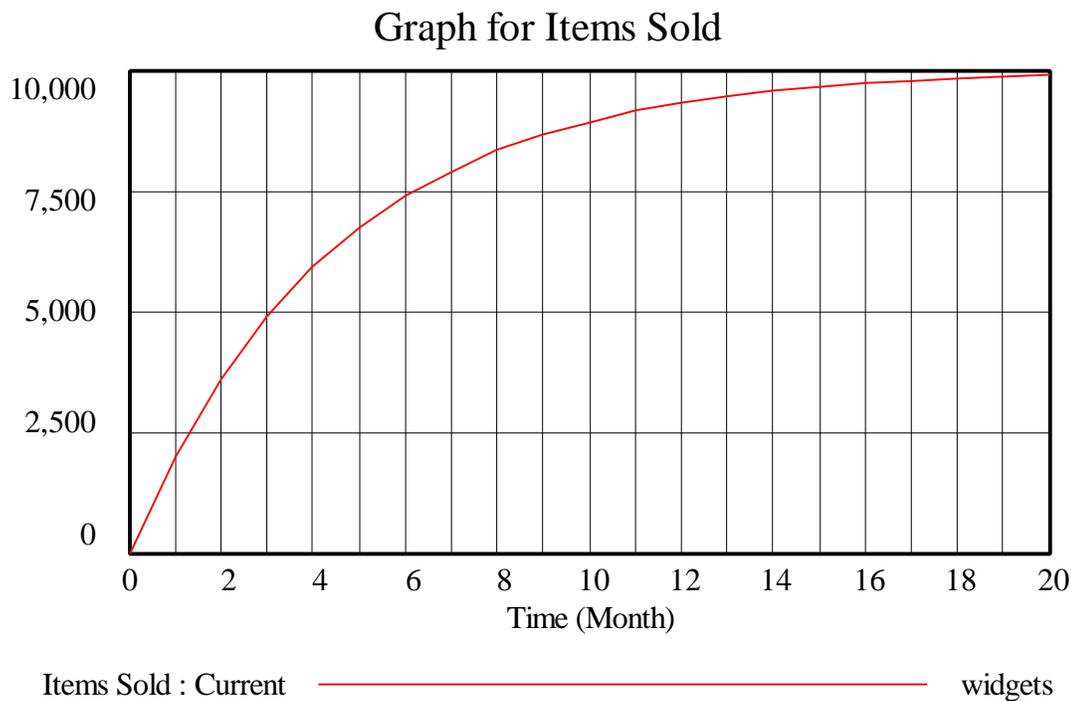


Figure 15: Vensim equivalent of Simulation for widget model

Exploration 2: Forgetting

INITIAL NUMBERS REMEMBERED

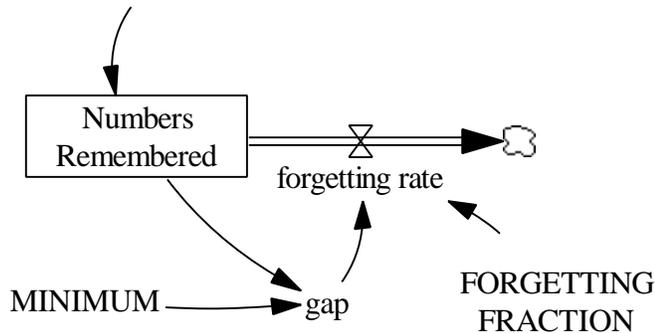


Figure 16: Vensim equivalent of Forgetting model

Documentation for Forgetting model

- (01) FINAL TIME = 20
Units: day
The final time for the simulation.
- (02) FORGETTING FRACTION = 0.8
Units: 1/day
- (03) forgetting rate = FORGETTING FRACTION * gap
Units: numbers/day
- (04) gap = Numbers Remembered - MINIMUM
Units: numbers
- (05) INITIAL NUMBERS REMEMBERED = 100
Units: numbers
- (06) INITIAL TIME = 0
Units: day
The initial time for the simulation.
- (07) MINIMUM = 10
Units: numbers
Minimum is maximum number of phone numbers you can remember without forgetting any.
- (08) Numbers Remembered = INTEG (-forgetting rate, INITIAL NUMBERS REMEMBERED)
Units: numbers

- (09) SAVEPER = TIME STEP
Units: day
The frequency with which output is stored.
- (10) TIME STEP = 0.0625
Units: day
The time step for the simulation.

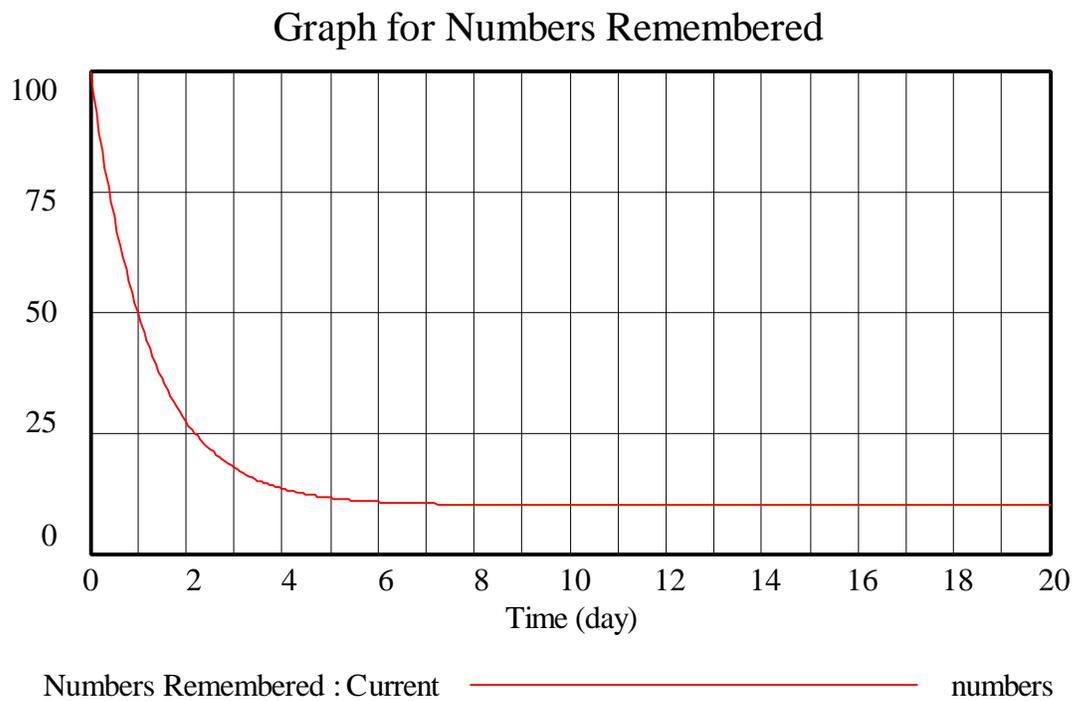


Figure 17: Vensim equivalent of Simulation of Forgetting model

Exploration 3: Productivity

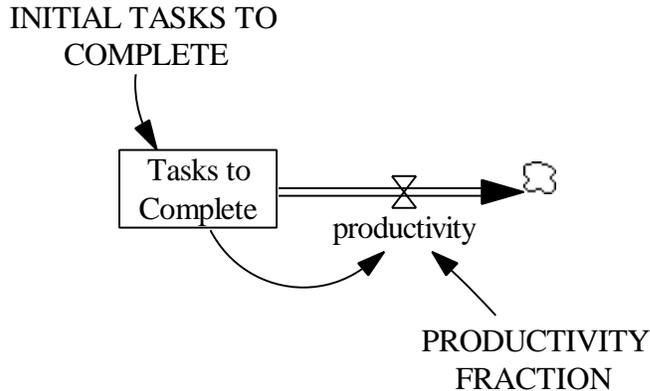


Figure 18: Vensim equivalent of Productivity model

Documentation for Productivity model

- (1) FINAL TIME = 20
Units: day
The final time for the simulation.
- (2) INITIAL TASKS TO COMPLETE = 300
Units: tasks
- (3) INITIAL TIME = 0
Units: day
The initial time for the simulation.
- (4) $\text{productivity} = \text{PRODUCTIVITY FRACTION} * \text{Tasks to Complete}$
Units: tasks/day
- (5) PRODUCTIVITY FRACTION = 0.3
Units: 1/day
- (6) SAVEPER = TIME STEP
Units: day
The frequency with which output is stored.
- (7) $\text{Tasks to Complete} = \text{INTEG} (-\text{productivity}, \text{INITIAL TASKS TO COMPLETE})$
Units: tasks
- (8) TIME STEP = 0.0625
Units: day
The time step for the simulation.

Graph for Tasks to Complete

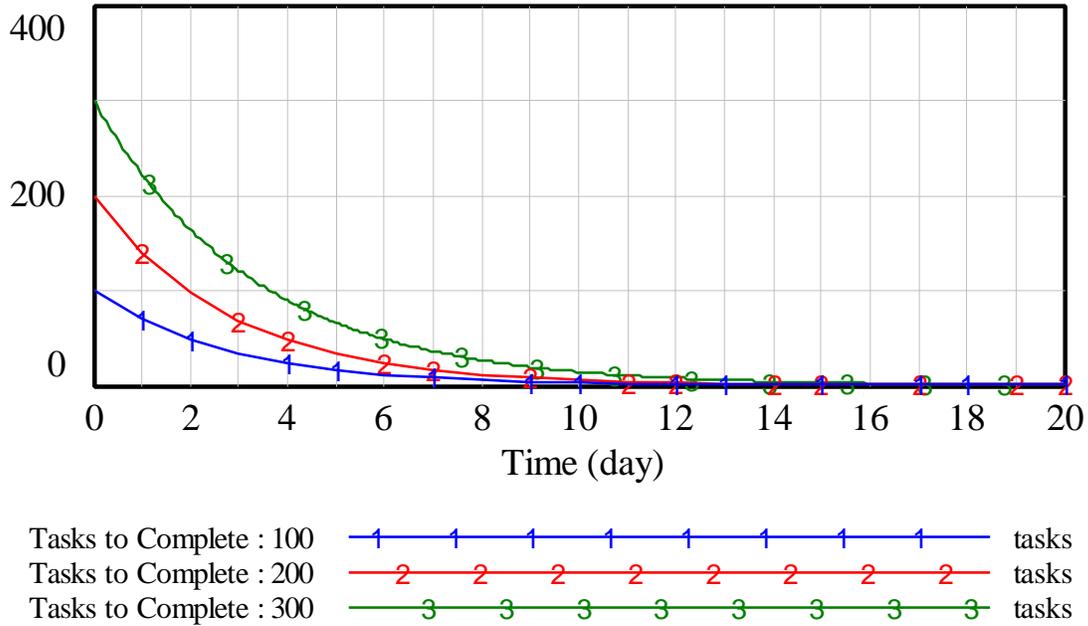


Figure 19: Vensim Equivalent of Simulation of Productivity model

Exploration 4: Federal Debt

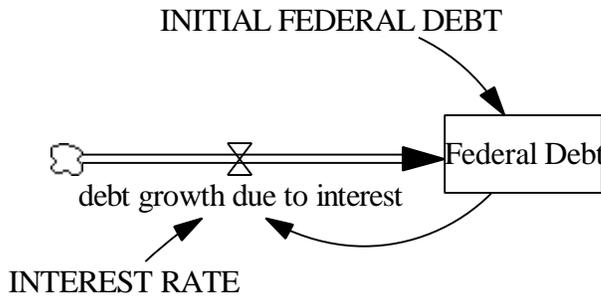


Figure 20: Vensim equivalent of Federal Debt model

Documentation for Federal Debt model

- (1) $\text{debt growth due to interest} = \text{Federal Debt} * \text{INTEREST RATE}$
 Units: trillion dollars/Year
- (2) $\text{Federal Debt} = \text{INTEG}(\text{debt growth due to interest}, \text{INITIAL FEDERAL DEBT})$

Units: trillion dollars

- (3) FINAL TIME = 20
Units: Year
The final time for the simulation.
- (4) INITIAL FEDERAL DEBT = 10
Units: trillion dollars
- (5) INITIAL TIME = 0
Units: Year
The initial time for the simulation.
- (6) INTEREST RATE = 0.1
Units: 1/Year
- (7) SAVEPER = TIME STEP
Units: Year
The frequency with which output is stored.
- (8) TIME STEP = 0.0625
Units: Year
The time step for the simulation.

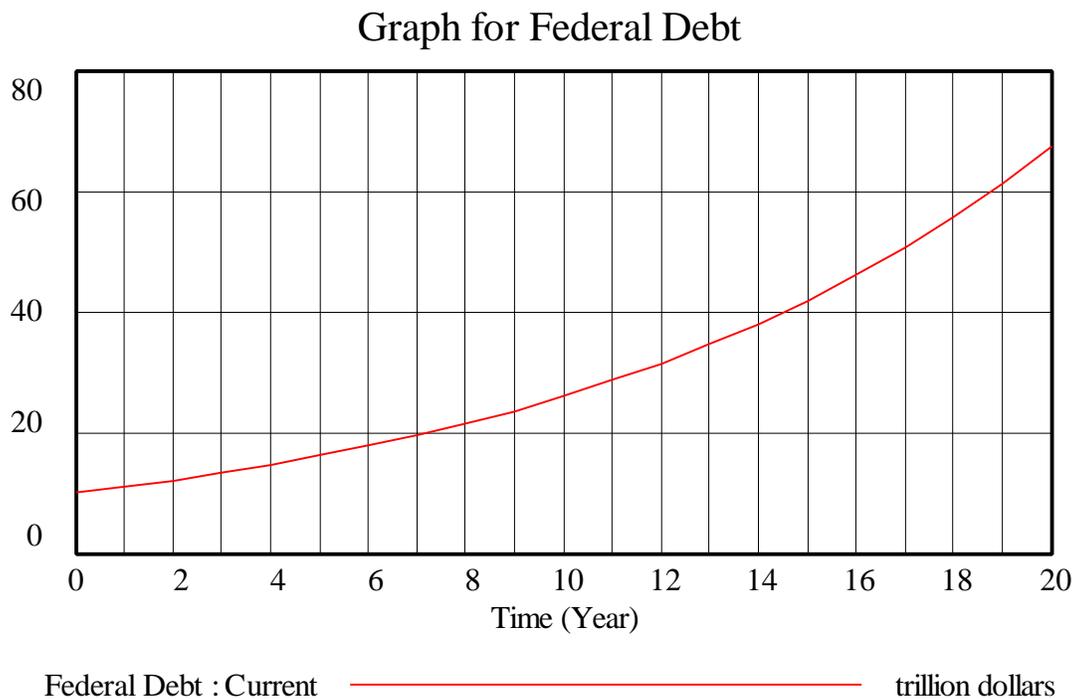


Figure 21: Vensim equivalent of Simulation of Federal Debt model

Exploration 5: Market for Widgets

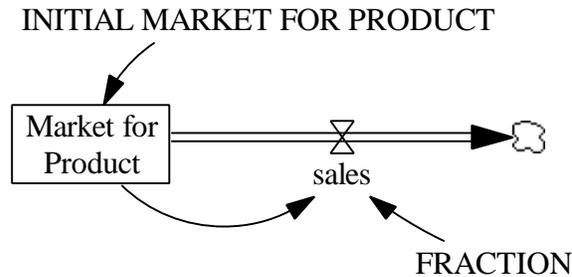


Figure 23: Vensim equivalent of Market for Widgets model

Documentation for Market for Widgets model

- (1) FINAL TIME = 20
Units: Month
The final time for the simulation.
- (2) FRACTION = 0.2
Units: 1/Month
- (3) INITIAL MARKET FOR PRODUCT = 10000
Units: widgets
- (4) INITIAL TIME = 0
Units: Month
The initial time for the simulation.
- (5) Market for Product = INTEG (-sales, INITIAL MARKET FOR PRODUCT)
Units: widgets
- (6) sales = Market for Product * FRACTION
Units: widgets/Month
- (7) SAVEPER = TIME STEP
Units: Month
The frequency with which output is stored.
- (8) TIME STEP = 1
Units: Month
The time step for the simulation.

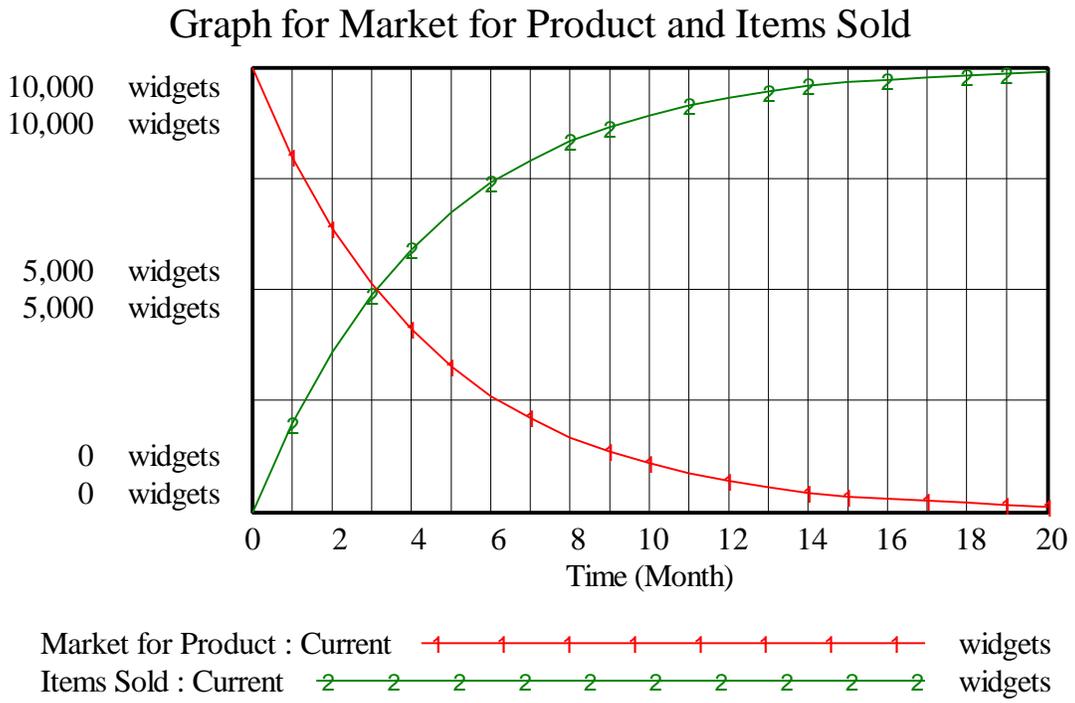


Figure 23: Vensim equivalent of Simulation of Market for Product and Items Sold