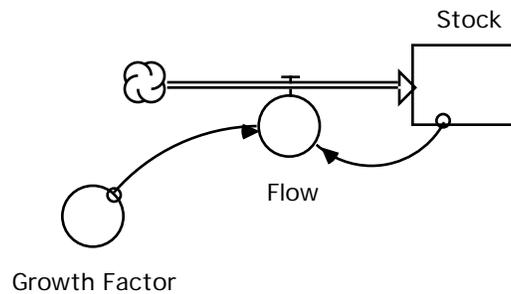# Beginner Modeling Exercises

Section 2

Mental Simulation of Simple Positive Feedback

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
Dr. Jay W. Forrester

by
Joseph G. Whelan
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Introduction

Feedback loops are the basic structural elements of systems. Feedback in systems is the cause of nearly all dynamic behavior. To successfully use system dynamics as a learning tool you must understand the effects of feedback loops on dynamic systems. One way of using system dynamics to understand feedback is with simulation software on your computer[1]. Computer simulation is a very useful tool for exploring systems. However, it is also crucial that you be able to use the other simulation tool of system dynamics: *Mental Simulation.* A strong set of mental simulation skills will enhance your ability to validate, debug and understand dynamic systems and models.

This paper begins with a review of some key concepts of simple feedback systems. A set of exercises is included to help reinforce your understanding of the feedback dynamics in a simple positive feedback loop. Solutions to the exercises are included in the appendix.

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[1]There are several commercial system dynamics simulation packages available for both Windows and Macintosh. Road maps is geared towards the use of STELLA II which is available from High Performance Systems. (603) 643-9636

Positive Feedback

One of the simplest feedback systems is the positive feedback loop. Positive feedback can be likened to a snowball rolling down a hill. As the ball rolls down the hill, it collects snow. The bigger the snowball gets, the more snow it collects and the faster it grows. Positive feedback occurs when change propagates through a system to produce *more* change in the *same* direction. This is the type of feedback that produces growth.

You see positive feedback every day in the world around you. Everyone knows about the ability of rabbits to multiply at an alarming rate. This is a good example of positive feedback. Every time a new pair of rabbits is born, they contribute to the reproductive capabilities of the total population. As the population grows, so does the rate at which rabbits are born; making the population grow even faster.

Another everyday example of positive feedback is a savings account. Let's say you deposit $10 into a savings account that earns 10% per year (It's just an example ☺) After 1 year, you would have $11 in the bank. In the first year, you earned $1.00 of interest. However, in the second year, your interest earnings are 10% of $11, or $1.10 leaving you with $12.10 after the second year. Each year as your bank account grows, so do the interest payments, causing your bank account to grow even faster. This simple system can be modeled with a STELLA model as shown in Figure 1.
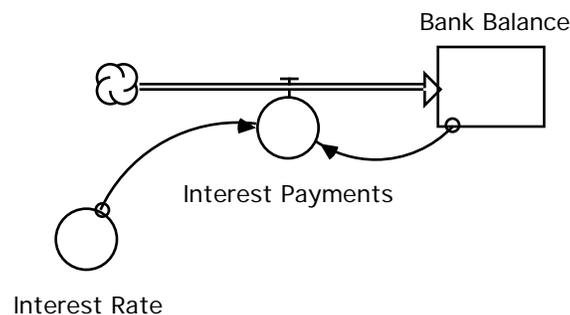
Figure 1: STELLA model of a savings account

The *interest payments* each year are equal to the *bank balance* multiplied by the *interest rate*.

Table 1 below shows the first four years of simulation of the savings account model.

Years	Bank Balance	Interest Payments	Interest Rate
1990	$10.00	1.00	0.10
1991	$11.00	1.10	0.10
1992	$12.10	1.21	0.10
1993	$13.31	1.33	0.10
1994	$14.64	1.46	0.10

Table 1: Model run for savings account

Computing the values of model elements by hand can be a very tedious process and is not the only way to obtain the behavior of a model. If you need a precise result for the behavior of a model, you should build it on the computer and run the simulation. However, often you will only need to know the general behavior pattern of a model, or how the behavior would change if the model were altered. For this, there are many mental simulation tools that will allow you to predict the behavior of a model without ever touching a computer. In this paper, you will learn some of the basics using the savings account model described above.

The first step in predicting the behavior is determining a starting point. Often, you are given the initial value(s) for the stock(s) in a model. If not, you should pick a reasonable value. If you are dealing with a model of a rabbit population you might start with 100 rabbits. If you were dealing with a model of the national debt, you would pick a much larger number, say a few trillion. You can calculate the beginning values for all the flows by using the initial values of the stocks.

In the savings account model, we are given that the initial value for the *bank balance* stock is $10. Since the equation for *interest payments* is:

Interest Payments = **Bank Balance Interest Rate**

We can calculate that the first value for the flow *interest payments* is $1.00.

We can take the information we have and begin to plot the graph of the model behavior as shown in Figure 2. Recall that the slope of a stock is equal to the net inflow. This means that the initial slope of the *bank balance* stock is equal to the first value of *interest payments*, or 1.
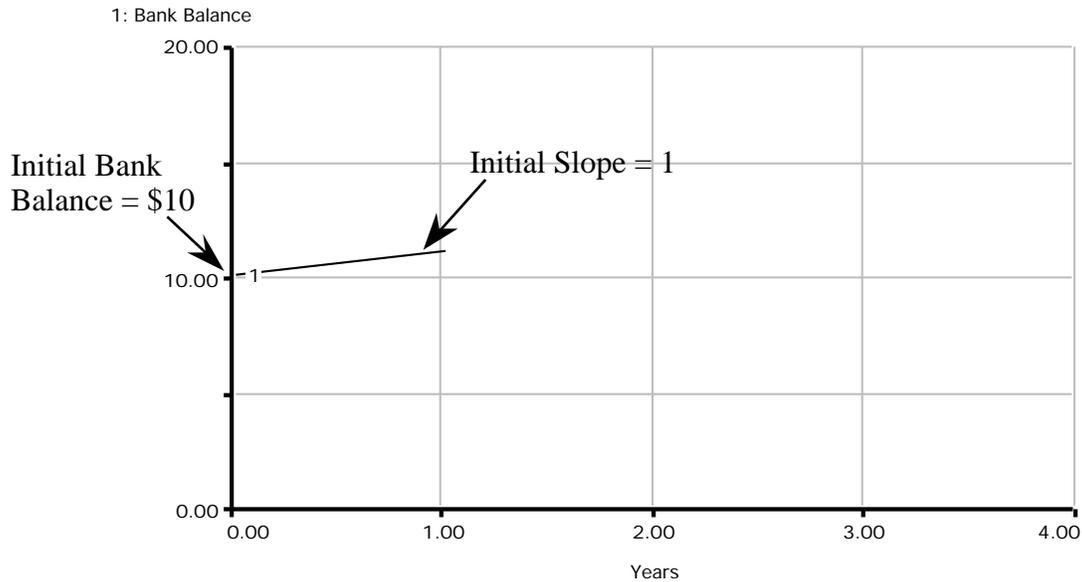
Figure 2: Initial bank balance behavior

Now that we know how the model starts to behave we need to figure out where it goes. We know that the bank balance model is an example of positive feedback, so the stock will continue to increase. But, what would happen if we didn't know that the model exhibited positive feedback? How could you tell? Here's a good test:

Imagine that you came upon a simple system and you wanted to know if it exhibited positive feedback. You know that positive feedback occurs when change propagates throughout a system to produce more change in the same direction. So, let's test the system and see if that happens. Assume that the bank account has $10 in it, so the interest payment for that year is $1. Now pretend that you deposited $5 into the account (a change in the system). What happens? That deposit will cause the interest payments to increase (propagate through the system) from $1 to $1.50. Now, the bank balance will start to increase at a faster rate because the interest payments are larger. So, a change propagated through the system and produced more change in the same direction. A positive feedback loop!

So far, we have established the starting point for the model and the fact that it contains a positive feedback loop. The characteristic behavior of a positive feedback loop is *exponential growth*. Exponential growth of a stock is characterized by the fact that the stock has a constant *doubling time*. This means that if the account balance doubled from $10 to $20 in 5 years it would double from $20 to $40 in the next five years and from $40 to $80 in the five years after that.
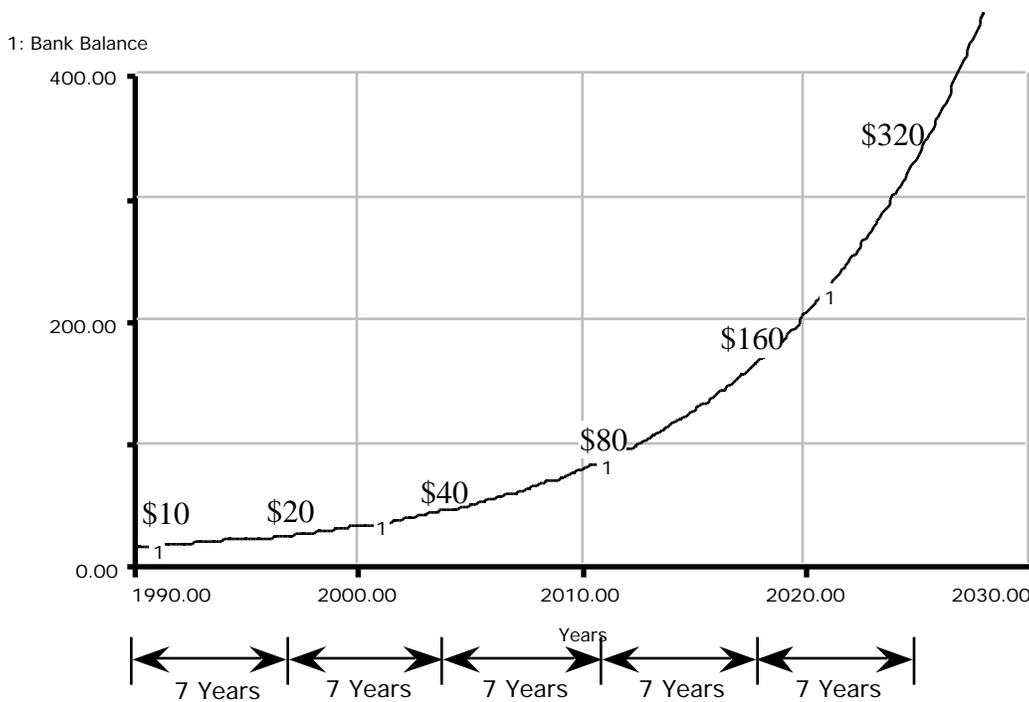
The next question to ask is, "How quickly will the growth occur?" The doubling time for a simple positive feedback loop can be approximated as follows:

$$\textbf{Doubling time} = \frac{\textbf{0.7}}{\textbf{interest rate}}$$

The doubling time for this model is:

$$\textbf{Doubling time} = \frac{\textbf{0.7}}{\textbf{.10}} = \textbf{7 years}$$

The graph in Figure 3 shows the exponential growth generated by the savings account model.

Figure 3: Behavior graph for the Bank Balance Model

The same equation for doubling time can be used for other systems as well. If the rate equation is formulated as follows:

$$\textbf{Inflow} = \textbf{Stock} \cdot \textbf{growth factor}$$

Then, simply substitute the growth factor for the interest rate in the doubling time equation.

There is one more thing about positive feedback that must be covered before going on to the exercises: Positive feedback cannot continue forever. There is *always* a factor or factors in any system that will limit the growth of its elements. In a population system there are limited food and water resources, the snowball will eventually reach the bottom of the hill, and your bank would probably fold sometime before your savings account began rivaling the size of the national debt. The presence of limiting factors in any system containing positive feedback is known as *limits to growth.*

Review:

Below is a brief list of the concepts you have learned so far. If you feel unclear about any of the topics mentioned below, you may wish to refer to the appropriate section of the paper before proceeding to the exploration exercises which follow.

1. A starting point:
    *   Initial values of stocks
    *   Initial values of flows
2. Where does is go from there:
    *   identifying feedback
    *   positive feedback
3. How does the stock grow:
    *   exponential growth
4. Doubling time

The exercises which follow will help to review the skills you have learned and give you practice in using them. The solutions to the exercises begin on page 12.

Exploration:

#1: All growth processes have some kind of underlying positive feedback. Name some examples of positive feedback in the world around you:

_____

_____

_____

_____

_____

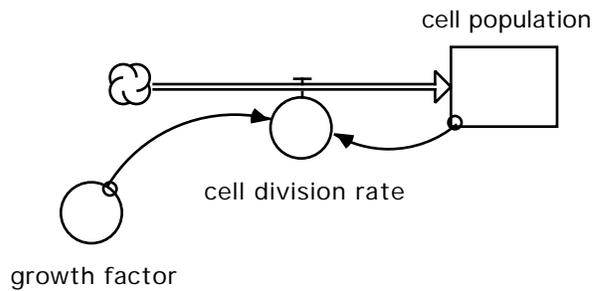**#2:** A friend approaches you with the following model of a yeast cell population. He is new to system dynamics and wants you to check for him if this model will produce positive feedback as he believes it will. Using your knowledge of positive feedback, verify or refute your friend's guess.

cell population

Model Equations:

growth factor = 0.2 (UNITS = 1/days)

**cell division rate =
cell population * growth factor**

cell division rate

Initial cell population = 40 cells

growth factor

Is your friend's model an example of positive feedback? _____

Why or why not? _____

_____

_____

_____

_____

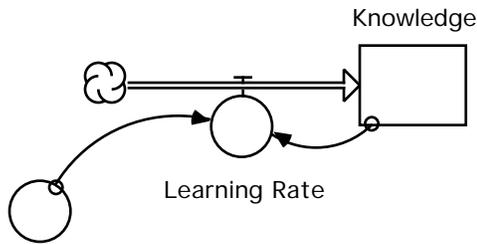**#3:** The figure below shows a simple model for the growth of knowledge in a developing civilization. The rate of learning is proportional to the current level of knowledge.

Knowledge   **Model Equations**

Ability to Learn = .02
Learning Rate =
   Knowledge   Ability to Learn
INIT Knowlege = 1

Learning Rate
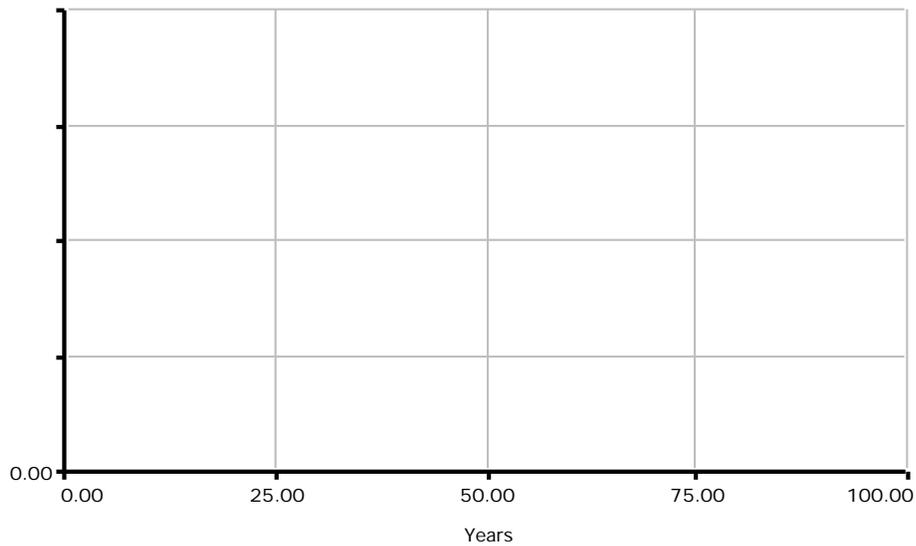
Ability to Learn

What is the doubling time for the model? (doubling time = 0.7/growth factor) _____

Use the doubling time to find the value of the stock after 100 years. Now, set an appropriate scale on the graph.

On the graph below, draw the behavior of the stock *Knowledge* for 100 years.

1: Knowledge

0.00
   0.00   25.00   50.00   75.00   100.00

Years

#4: The savings account model used in this paper can also be used to model debt. The only difference is that instead of the *balance* being positive, it takes on negative values, representing your debt. The *interest payments* now represent accumulating interest charges and the *interest rate* represents the interest rate you are charged for your debt. (Note that an inflow with a negative value is the same as an outflow.)

We will use this model to explore the accumulation of debt on a credit card. Assume that you owe $100 to a credit card that charges 18% annually. What would happen to the amount that you owed if you didn't make any payments for 10 years. (Not a good idea, but it is possible.)

Balance

Interest Accumulation
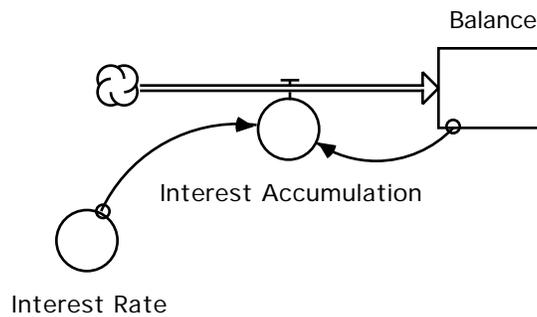
Interest Rate

Verify that this new use of the model is still a positive feedback loop.

Assume that your owe $100, so the initial value for the *Balance* is -$100. If the interest rate is 18%, what is the first value for *interest accumulation?* _____
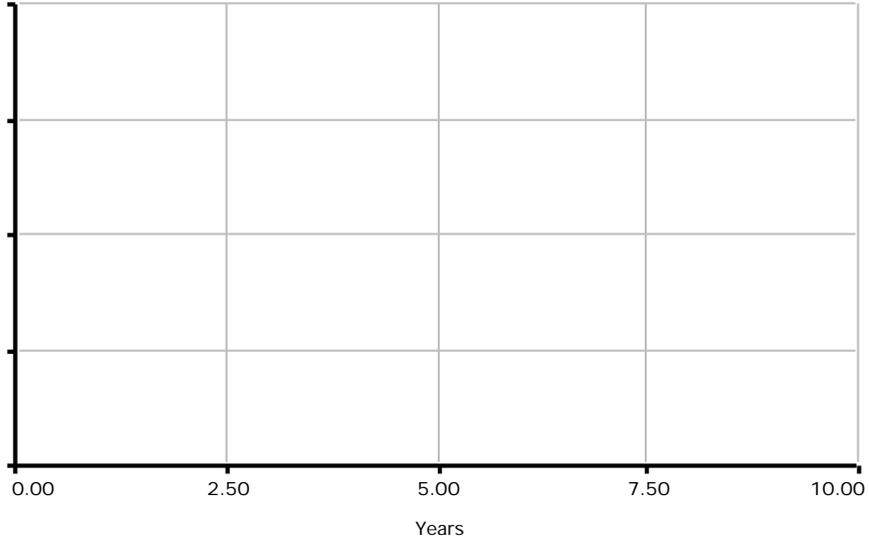
Will the *balance* increase or decrease? _____

Will your debt increase or decrease? _____

What is the doubling time for this scenario? _____

Draw on the graph below what you think the behavior of the *balance* will be over the next 10 years.

1: Balance

Years

#5: What would happen if the bank *balance* were started at zero? (i.e., you opened up a savings account, but didn't deposit any money into it.) _____

_____

_____

_____

_____

Solutions:

#1: There are countless examples of positive feedback in the world around us. Here are just a few:

- cell division — As the population grows, so does the number of cell divisions per hour.

- expansion of a population — Similar to the cell population. The larger the population, the greater the number of births per year.

- spread of a rumor — As the number of people that know the rumor increases, more people are able to spread the rumor.

- spread of a disease — This feedback loop works the same way as the spread of a rumor.

- growth of a company — As the size of a company increases, the funding available for expansion also increases allowing it to grow at an increasing rate.

- spread of a forest fire — A forest fire's ability to ignite the land around it increases with the size of the fire.

Recall the definition given for positive feedback: Positive feedback exists in a system when a change propagates through the system to produce more change in the same direction.

#2: Your friend is correct. This model will produce positive feedback. In fact, this model has exactly the same structure as the bank balance model. The only differences are the names of the model elements and the values of the *growth factor* parameter and the initial value of the *cell population*.

#3: The doubling time is calculated as shown:

$$\textbf{doubling time} = \frac{.7}{\textbf{growth factor}} = \frac{0.7}{.02} = \textbf{35 years}$$

100 years is slightly less than 3 doubling times of growth. So, after 100 years the stock *Knowledge* is approximately equal to 1*2*2*2 = 8. So an appropriate scale to choose is one that goes from 0 to 8. You can use the doubling time to find a few more points on the graph and then draw in the curve by hand.

The behavior of the learning model over a 100 year period is shown below.

1: Knowledge

| Years	Knowledge
0	1
35	2
70	4
105	8
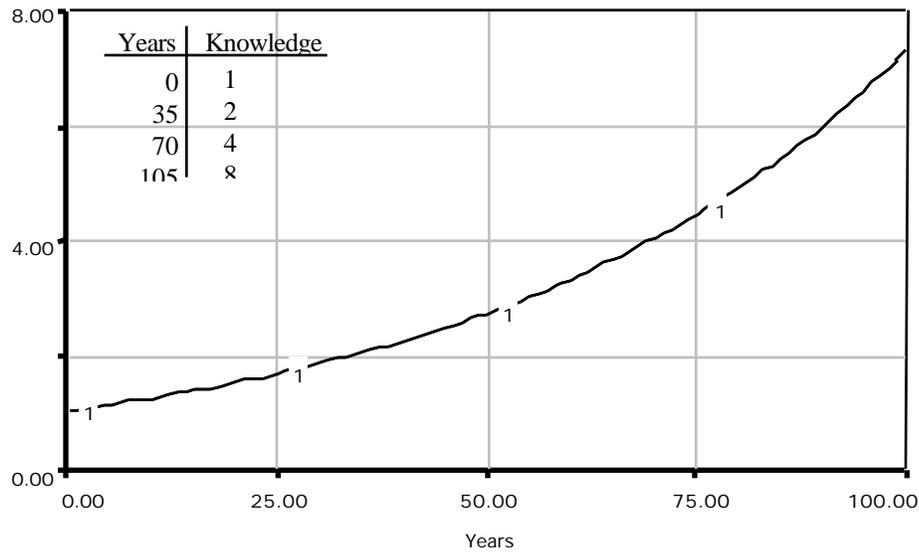
#4: This system contains a positive feedback loop because as the *balance* becomes more negative, so does the rate of *interest accumulation* causing the *balance* to decrease even faster which results in even higher *interest accumulation.*

Assuming you owe $100 and you are paying an *interest rate* of 18%, the first year's worth of *interest accumulation* is $18.   After 1 year, the *bank balance* will decrease to -$118, making your debt equal to $118.   The balance will continue to decrease exponentially. The doubling time is calculated as follows:

$$\textbf{doubling time} \ = \ \frac{\textbf{.7}}{\textbf{growth factor}} = \frac{\textbf{0.7}}{\textbf{.18}} = \textbf{3.9 years}$$

Since 10 years is between 2 and 3 doubling times, the scale on the graph can be set to -$800: the balance after 3 doubling times. The doubling time can then be used to plot a few more points to complete the curve.

The graph below shows the behavior of the *balance* over a 10 year period.

1: Balance

Years	Balance ($)
0	-100
3.9	-200
7.8	-400
11.7	-800
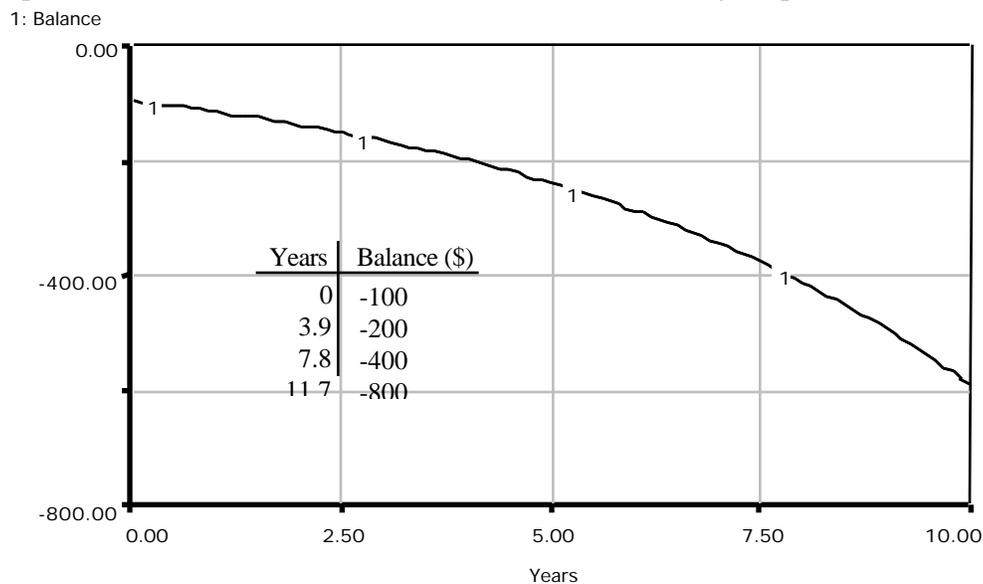
#6: If the bank balance were to start at zero, this would mean that the value of *interest payments* would also be zero. Since the only flow that affects the *bank balance* stock is equal to zero, the stock will never change and neither will your *bank balance.* This state is called equilibrium. It is a special kind of equilibrium called *unstable equilibrium.* The reason for this name is because a small change in the stock to either increase or decrease it will initiate exponential behavior. This can be likened to a ball balanced on top of a pin. If the ball is exactly centered over the pin, it will not move, but <u>any</u> disturbance at all will cause it to fall all the way off.

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