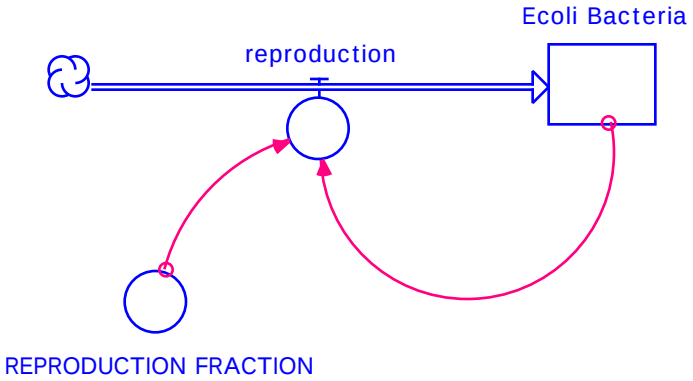
An Introduction to Feedback



Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
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1. ABSTRACT

This paper will develop the concept of feedback. Feedback is the process through which a signal travels through a chain of causal relations to reaffect itself. Feedback can be divided into two categories: positive and negative. Feedback is *positive* if an increase in a variable, after a delay, leads to a further increase in the same variable. Positive feedback is found in compounding, reinforcing, or amplifying systems that produce exponential behavior. On the other hand, feedback is *negative* if an increase in a variable eventually leads to a decrease in the variable. Negative feedback drives balancing or stabilizing systems that produce asymptotic or oscillatory behavior. This paper will reinforce the general theory of feedback with many different examples in the form of exercises. The exercises will involve the reader in making the distinction between positive and negative feedback.

2. WHAT IS FEEDBACK?

"Feedback is a process whereby an initial cause ripples through a chain of causation ultimately to reaffect itself."[1] The easiest way to explain the concept of feedback is with an example:

> "As a simple example, consider an initial disturbance in the temperature of a room caused by a sudden cold spell. This drop in temperature might in turn cause various types of activities. For example, persons in the room might put on sweaters or move to a warmer part of the house. Also, the thermostat might turn on the furnace. The activity of the furnace might in turn cause a number of things to happen. Furnace activity might cause the level of fuel oil in the storage tank to go down, which in turn might cause the future purchase of more fuel oil. Furnace activity might also cause wear and tear on the burner unit, which might cause further repairs to be made. However, none of these causal chains feeds back to influence room temperature. The important effect of furnace activity, for our purposes of analyzing the control of room temperature, is the heating up of the radiators in the room, which eventually causes the room temperature to rise."[2]

The thermostat example provides instances of both open-loop and closed-loop systems. An open-loop system is a simple chain of causality: the room temperature drops so people put on sweaters. Notice that the act of wearing a sweater does not at all affect the temperature in the room. A closed-loop system, on the other hand, is a circular chain of causality that "feeds back" to itself: the room temperature drops and activates the thermostat, which turns on the furnace, which heats up the radiators, which raises the room temperature. A change in room temperature automatically causes another change in room temperature. In the thermostat example, the effect of the two changes is balancing: a drop in room temperature causes a rise in room temperature. If a thermostat is working correctly, the temperature in the room should never deviate very far from the ideal setting.

Closed-loop systems are most commonly called **feedback systems**. In a feedback system, each variable is—at the same time—cause and effect. Room temperature is *effected by* the activity of the radiators and *affects* the thermostat, which affects the furnace and the future activity of the radiators. In a feedback system, a change in the environment (for example, a change in room temperature) leads to a decision (made by the thermostat) that results in an action (turning on the furnace) that affects the

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[1] Nancy Roberts, et al, 1983. Introduction to Computer Simulation. Reading, MA: Addison-Wesley, p. 16.
[2] ibid.

environment (the radiators raise the room temperature) and thereby influences future decisions (the thermostat can shut off once the desired room temperature has been restored).[3]

Feedback can be categorized as either positive or negative. Nothing is especially great about "positive" feedback and nothing is especially depressing about "negative" feedback. The designations "positive" and "negative" indicate whether changes in the feedback system move in the same direction to produce compounding, reinforcing behavior, or move in opposite directions to produce balancing, stabilizing behavior.

3. POSITIVE FEEDBACK

Positive feedback drives growth and change. Figure 1 demonstrates an example of positive feedback. A biology research assistant is growing bacteria in a shaking flask. Reproduction increases the number of *E. coli* bacteria. The higher the reproduction rate, the more bacteria in the flask. What determines the reproduction rate? *E. coli* bacteria reproduce by dividing and multiplying. The reproduction rate depends directly on how many bacteria are already in the flask.

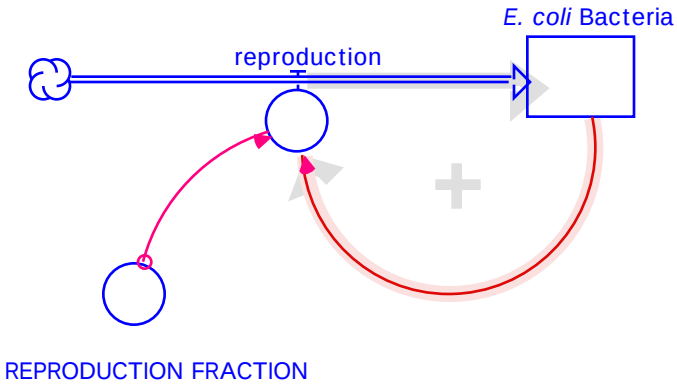


Figure 1: Positive feedback: *E. coli* bacteria

The shaded arrows in Figure 1 are not information links or material links but *causal* links. The reproduction flow of bacteria each hour into the stock of accumulated bacteria is a *material* link. The connector which ties the reproduction rate to the current number of bacteria is called an *information* link because it carries the information about the current state of the system (the value of the stock) to the mechanisms that drive the system to change (the flow). The material and information links help determine the

[3] Jay W. Forrester, 1961. Industrial Dynamics. Portland, OR: Productivity Press, p. 14.

behavior of the system over time. The shaded arrows, on the other hand, do not drive the system but simply describe the flow of causality. The number of E. coli bacteria in the flask influences the reproduction rate which, in turn, influences the number of E. coli bacteria. You can trace through the feedback loop by following the causal arrows. Increasing the number of *E. coli* bacteria will in turn increase the reproduction rate, which compounds the initial increase of the number of *E. coli* bacteria in the shaking flask.

What do you think happens to the number of bacteria over time? Assume the shaking flask holds initially 100 bacteria cells. It takes approximately a half an hour for a population of *E. coli* to double. Assume that the bacteria will have enough room and nourishment to grow unhindered. How many bacteria will the research assistant find in her shaking flask four hours from now? Sketch your guess below:

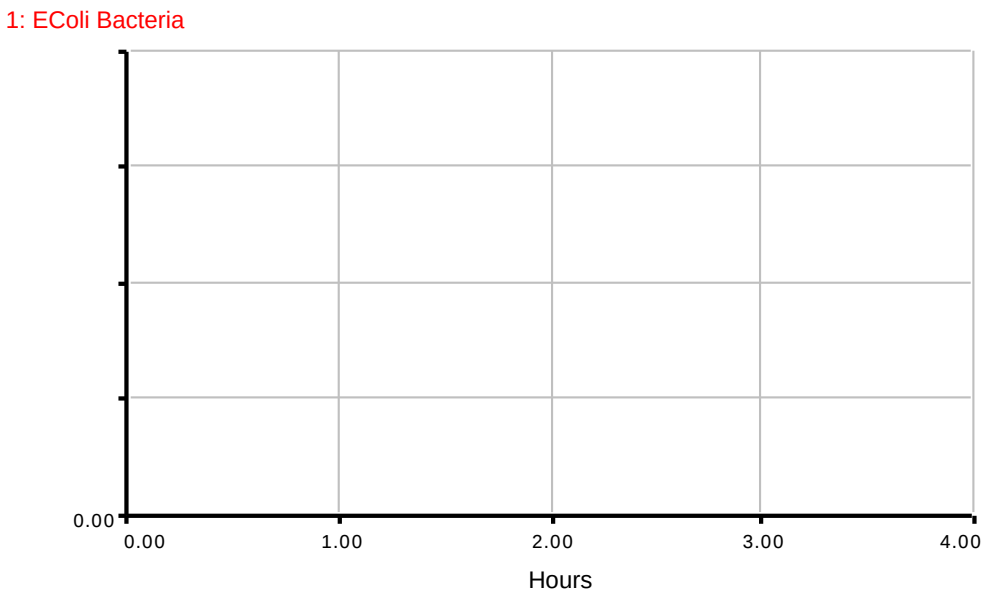


Figure 2 shows the behavior of the *E. coli* bacteria when the model in Figure 1 is run in STELLA. In four hours the population grows from 100 to 25,600 bacteria!

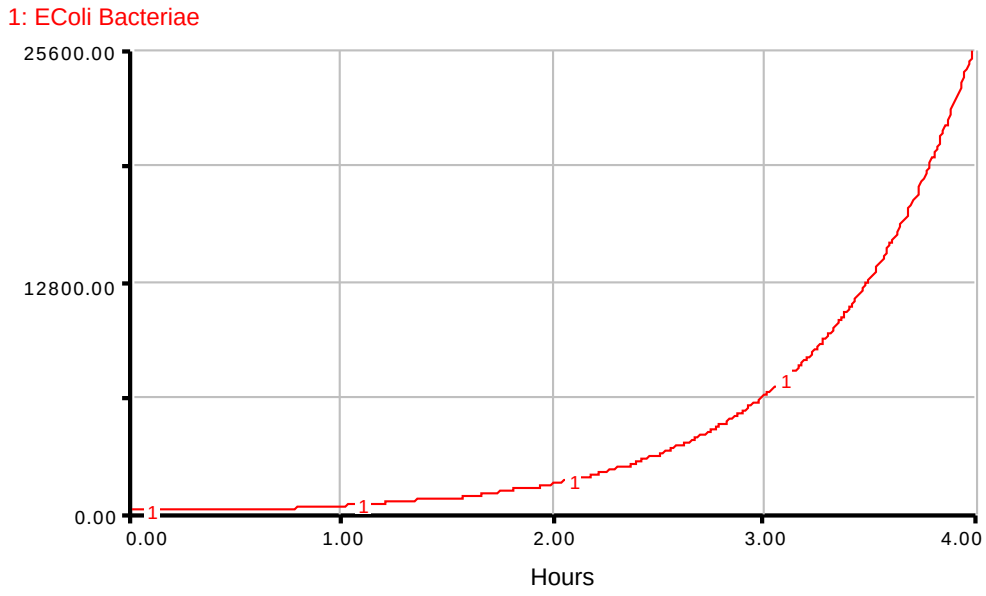


Figure 2: Exponential growth

The behavior depicted in Figure 2 is called **exponential growth**. Exponential growth is compounding and reinforcing. Another positive feedback system that exhibits exponential growth is represented in the model in Figure 3.

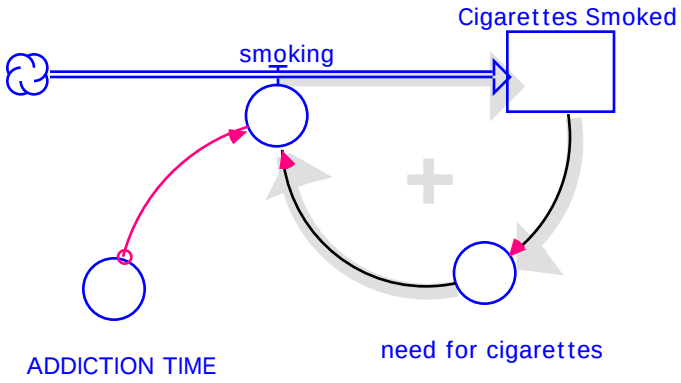


Figure 3: Positive feedback: addiction

The more my uncle smokes, the more addicted he becomes to the nicotine in his cigarettes. After smoking a few cigarettes a long time ago, my uncle began to develop a need for cigarettes. The need caused him to smoke even more, which produced an even

stronger need to smoke. The reinforcing behavior in the addiction process is characteristic of positive feedback.

4. NEGATIVE FEEDBACK

Negative feedback negates change and stabilizes systems. In positive feedback, an increase in a variable eventually leads to a further increase in the same variable. In negative feedback, on the other hand, an increase in a variable eventually leads to a decrease in that variable. The thermostat example contains a negative feedback system: an increase in room temperature eventually causes a decrease in room temperature. A child who learns to ride a bicycle is also using a negative feedback process: when the bike sways too much to the left, the child corrects by swaying to the right, and then when the bicycle sways too much to the right, he compensates again by swaying to the left. Negative feedback loops are balancing and stabilizing.

One example of a negative feedback system is the gradual decay of radioactive nuclei. Every year, a fraction of the total number of radioactive nuclei decays.

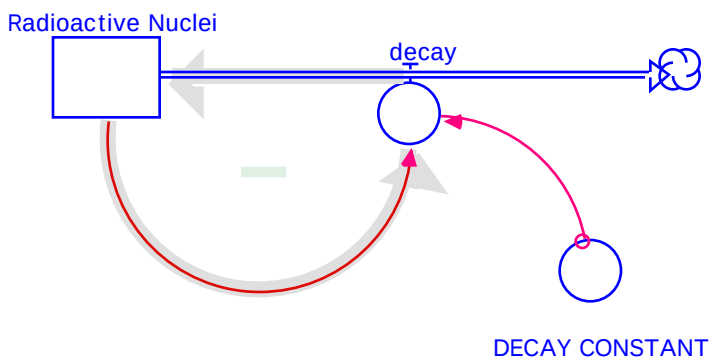
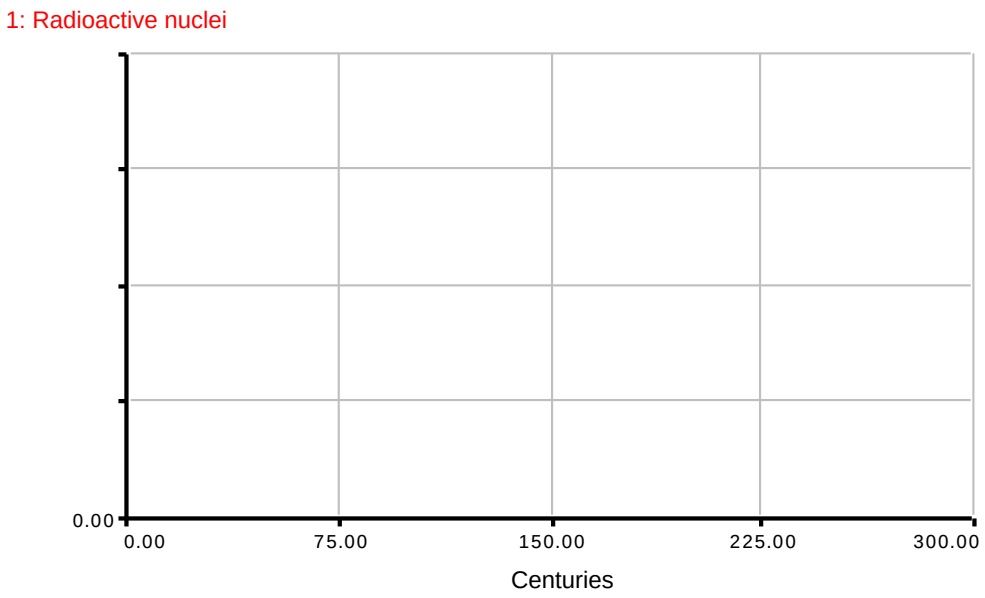


Figure 4: Negative feedback: radioactive nuclei

Figure 4 depicts a radioactive decay structure. Be careful to distinguish between the material flow out of the stock and the causal arrow into the stock. The decay rate constantly draws nuclei *out* of the stock of accumulated nuclei. So the decay rate is affecting the stock by causing it to decrease. Trace through the feedback loop, starting with the stock. At first the number of radioactive nuclei is large. Because so many nuclei are present, the decay rate is large. The decay reduces the number of nuclei. Loop through a second time. Now a smaller number of nuclei remain, but the number is still significant. The decay rate is still high, but not as high as before. The number of nuclei

is further reduced, but not as drastically. Loop through a several more times. Nuclei continue to decay, so the nuclei become fewer and fewer. As the nuclei decay, the decay rate becomes weaker and weaker, so the decaying process gets slower and slower.

Given that carbon-14 has a half-life of 5230 years, what do you think that the behavior of 1000 radioactive carbon-14 nuclei would be over a period of 300 centuries?



The behavior of the radioactive nuclei, modeled in STELLA, is replicated in Figure 5. The behavior in Figure 5 is called **exponential decay** or **asymptotic decline**.

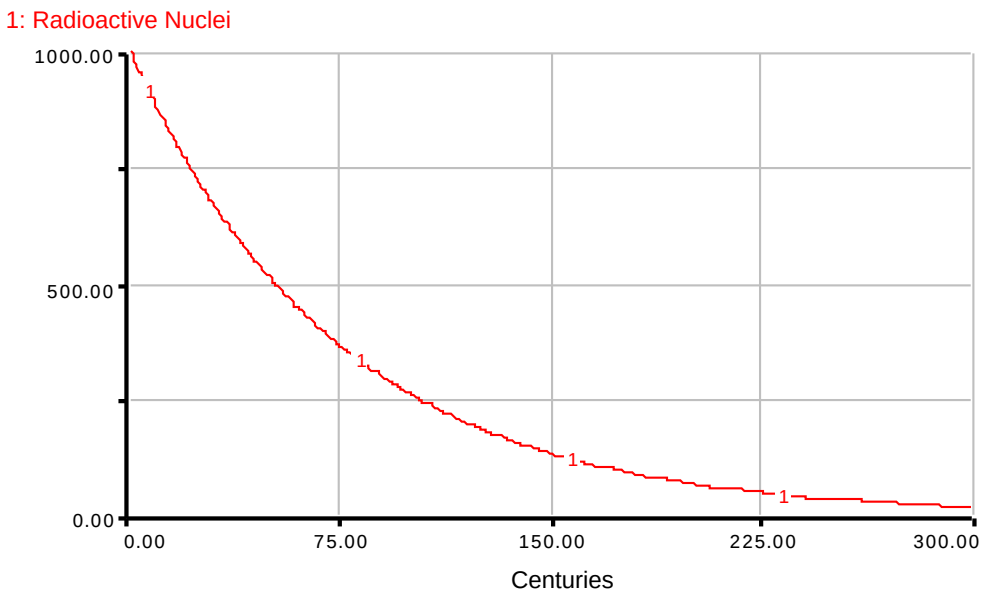


Figure 5: Asymptotic decline

My dieting is another example of a negative feedback system. The model representing how I diet is shown in Figure 6. The more I am overweight, the more I diet, which makes me less overweight. As my weight approaches my desired weight, I feel less pressured to diet.

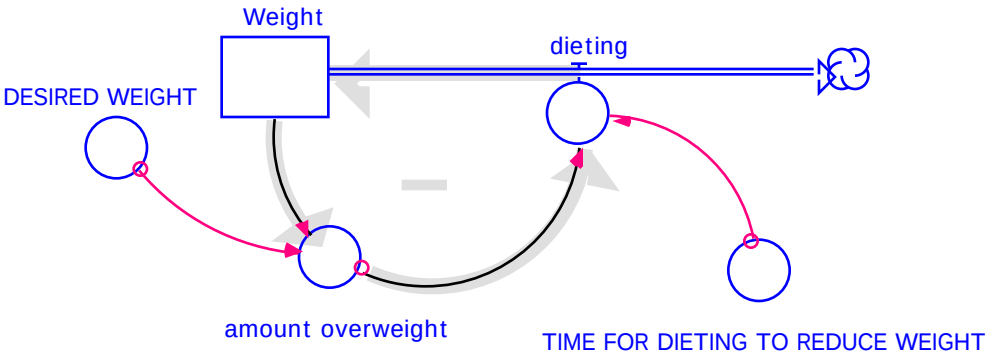


Figure 6: Negative feedback: dieting

As an example, today I weigh 200 pounds. I would really love to weigh 150 pounds, so I begin a strict diet. Figure 7 demonstrates how my weight decreases over a

period of 24 weeks, or 6 months. My weight drops, sharply at first and then more and more slowly, as it approaches the 150 pounds that I consider to be ideal.

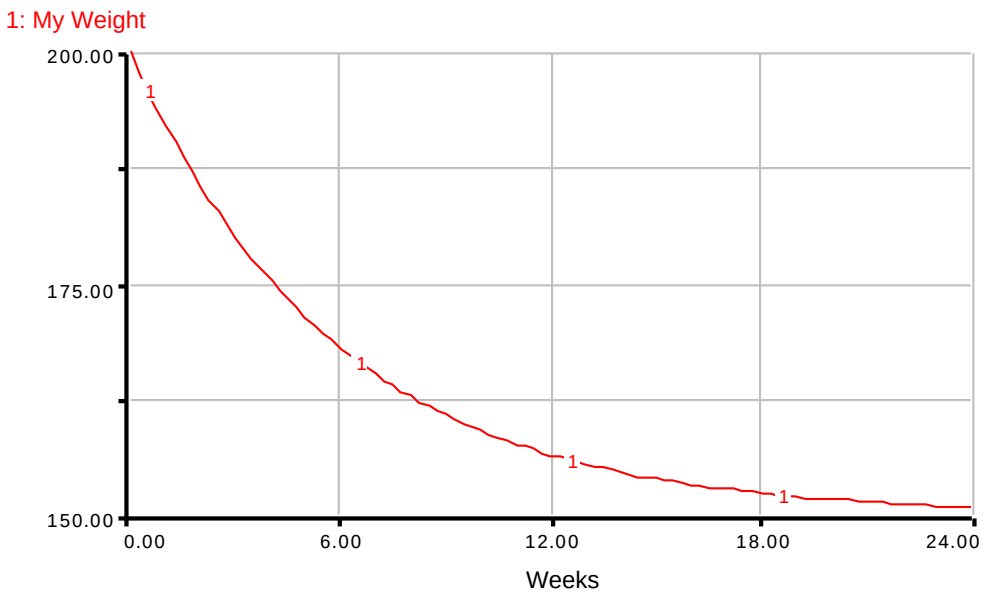


Figure 7: Asymptotic decline

Notice the similarities between the behavior of my weight and the behavior of the radioactive nuclei. Negative feedback exhibits goal-seeking behavior. I diet in order for my weight to reach a goal of 150 pounds. Radioactive nuclei decay towards a goal of zero; they decay until none remain.

In negative feedback, the stock in question does not need to *fall* towards its goal. The stock can also *climb* towards the goal. Take dieting as an example. I may be overweight, but my younger brother is underweight. He wants to weigh 150 pounds but currently weighs a puny 100 pounds. So whereas I begin to diet, he starts to eat heartily. Figure 8 shows how both weights change over the same 24-week period.

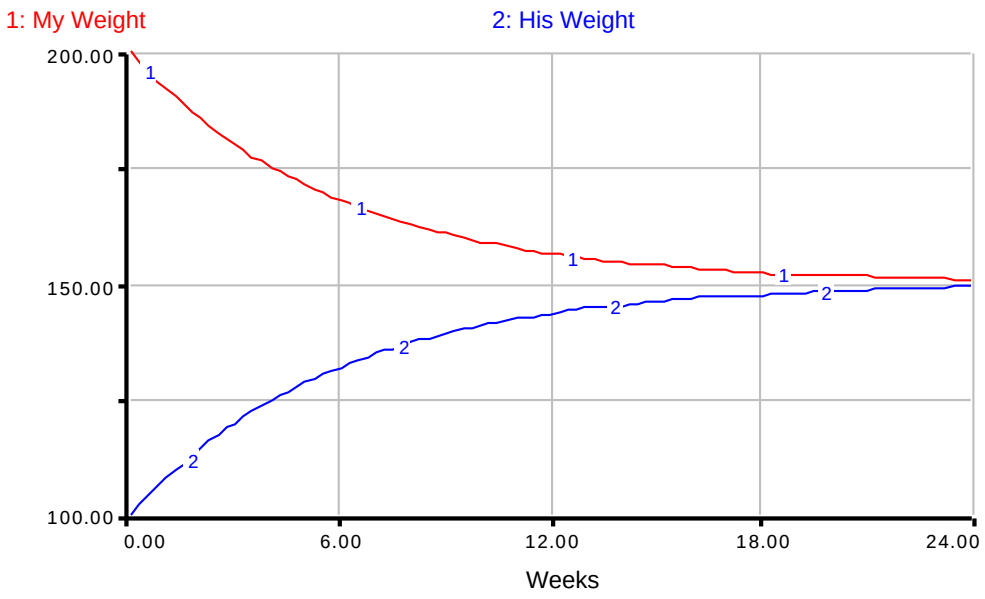


Figure 8: Asymptotic behavior

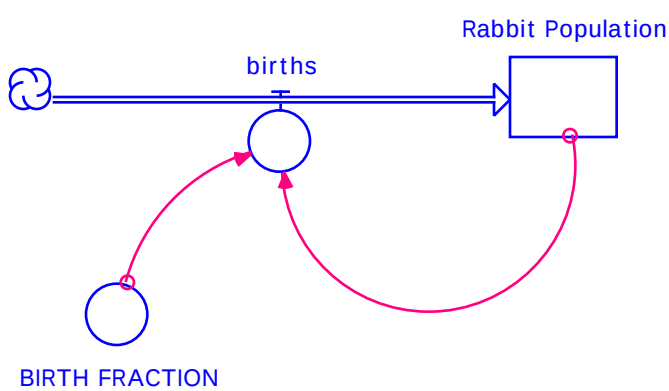
The behavior of both of the stocks in Figure 8 is asymptotic. The curves tend towards a goal, the desired weight. Negative feedback causes goal-seeking behavior. In simple systems like the ones presented in this paper, goal-seeking behavior will be asymptotic; in complex systems, goal-seeking behavior may be oscillatory. Positive feedback, in contrast, causes exponential, amplifying behavior. The two types of feedback, positive and negative, combine to create all of the behavior observed in complex systems.

5. EXERCISES

The goal of these exercises is to learn to differentiate between positive and negative feedback. For each exercise, first trace through the causal links by identifying whether each variable causes an increase or a decrease in the next. Circle the correct choice. Then determine whether the feedback loop is positive or negative: is a change in the stock reinforced or balanced? Circle the correct choice. Then trace through the feedback loop on the stock and flow diagram with your pen and label the loop as positive or negative. When you are done, flip to the next page to check your answers.

5.1 Rabbit Population

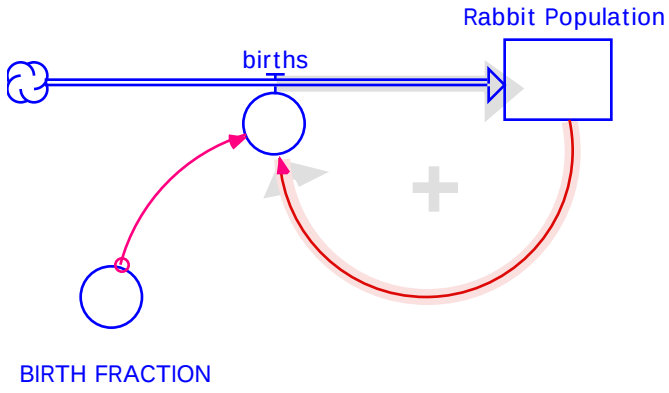
- An increase in the rabbit population *increases* *decreases* the birth rate.
- An increase in the birth rate *compounds* *counterbalances* the initial increase in the rabbit population.
- The loop, therefore, is a *positive* *negative* feedback loop.



births = Population * BIRTH FRACTION
UNITS: people/year

Rabbit Population: Answers

- An increase in the rabbit population *increases* the birth rate.
- An increase in the birth rate *compounds* the initial increase in the rabbit population.
- The loop, therefore, is a *positive* feedback loop.



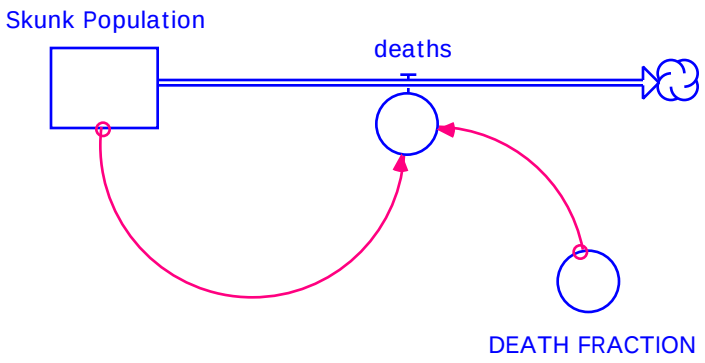
The larger the population, the greater the number of births. The more the birth rate increases, the faster the population increases.

5.2 Skunk Population

- An increase in the skunk population *increases* *decreases* the death rate.

- An increase in the death rate *compounds* *counterbalances* the initial increase in the skunk population.

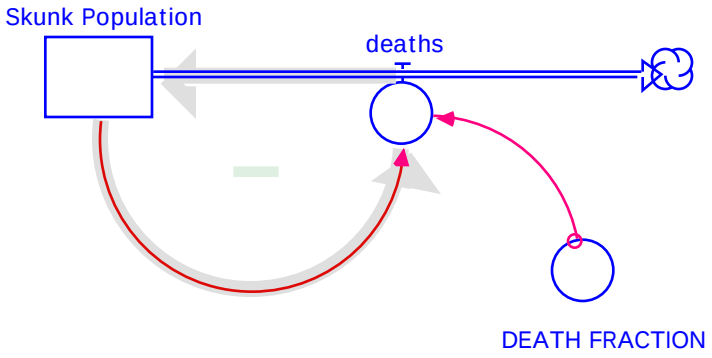
- The loop, therefore, is a *positive* *negative* feedback loop.



deaths = Skunk Population * DEATH FRACTION
UNITS: skunks/year

Skunk Population: Answers

- An increase in the skunk population *increases* the death rate.
- An increase in the death rate *counterbalances* the initial increase in the skunk population.
- The loop, therefore, is a *negative* feedback loop.



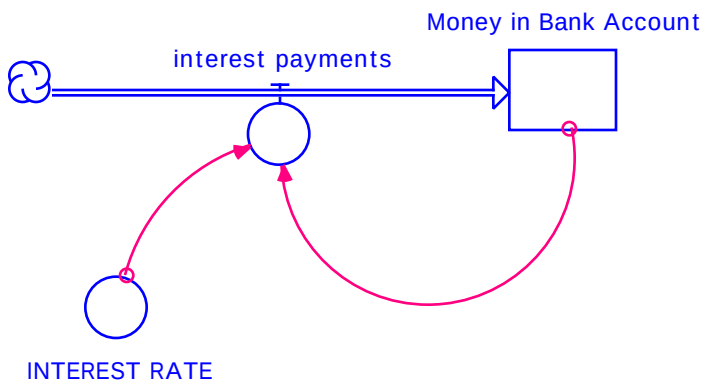
The number of deaths each year is a certain fraction of the skunk population. The death fraction represents the mortality of the population. A large skunk population has a high death rate, which sharply reduces the population. As the population decreases, the death rate becomes less drastic, causing the population to continue to decrease, but not as severely. Eventually, all the skunks die.

5.3 Bank Account

- An increase in the amount of money in a bank account *increases* *decreases* the interest payments.

- An increase in the interest payments *compounds* *counterbalances* the initial increase in the amount of money in the bank account.

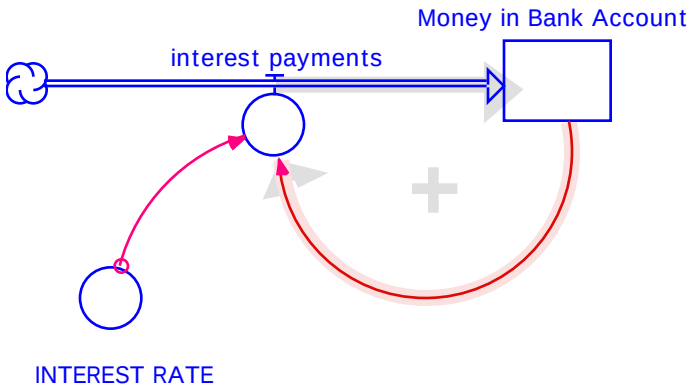
- The loop, therefore, is a *positive* *negative* feedback loop.



interest payments = Money in Bank Account * INTEREST RATE
UNITS: dollars/year

Bank Account: Answers

- An increase in the amount of money in a bank account *increases* the interest payments.
- An increase in the interest payments *compounds* the initial increase in the amount of money in the bank account.
- The loop, therefore, is a *positive* feedback loop.



Interest payments are calculated by multiplying the amount of money in a bank account by a given interest rate. The more money is in the account, the higher the interest payments, which adds even more money into the account. The rich get richer.[4]

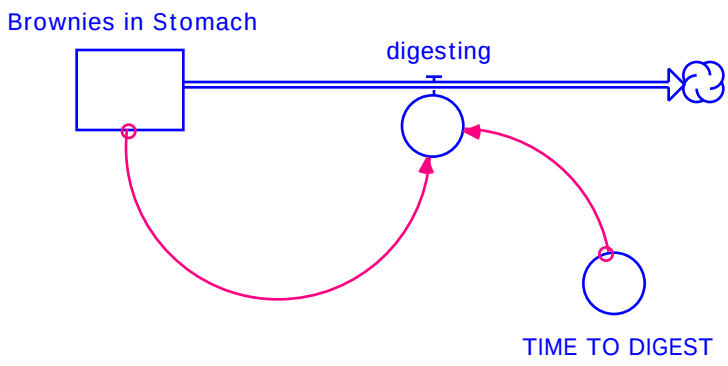
[4] Careful: Note that an interest *rate* is not actually a rate in the system dynamics sense of the word! An interest rate is a certain percentage per year. For example, the money in my bank account might be subject to 5% interest each year. A *rate* that changes the balance of my bank account is the flow of money into the account, my interest payments. If initially my bank account has $1,000 and if my account is subject to 5% interest, compounded annually, then a *rate* of $50 in interest payments flow into my account the first year.

5.4 Brownies

- An increase in the number of brownies in your stomach *increases* *decreases* your digestion rate.

- An increase in your digestion rate *compounds* *counterbalances* the initial increase in the number of brownies in your stomach.

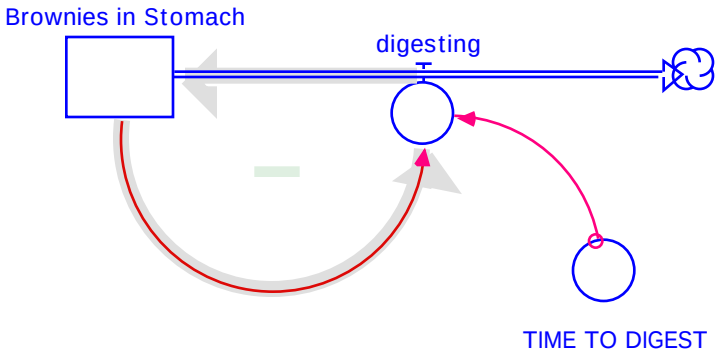
- The loop, therefore, is a *positive* *negative* feedback loop.



digesting = Brownies in Stomach / TIME TO DIGEST
UNITS: brownies/hour

Brownies: Answers

- An increase in the number of brownies in your stomach *increases* your digestion rate.
- An increase in your digestion rate *counterbalances* the initial increase in the number of brownies in your stomach.
- The loop, therefore, is a *negative* feedback loop.



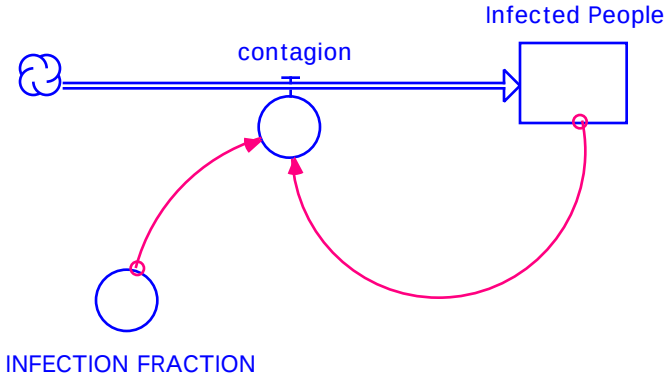
Food takes time to digest. The more brownies you have in your stomach, the more brownies you digest, which increases the rate at which brownies leave your stomach. As the number of brownies in your stomach decreases, the digesting rate slows down until all of the brownies have been digested.

5.5 Infection

- An increase in the number of infected people *increases* *decreases* the contagion rate.

- An increase in the contagion rate *compounds* *counterbalances* the initial increase in the number of infected people.

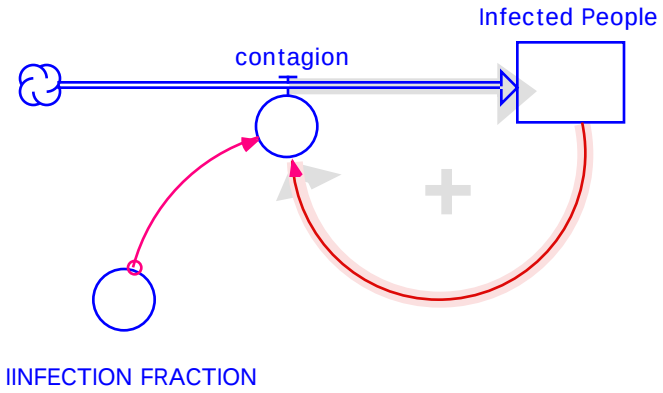
- The loop, therefore, is a *positive* *negative* feedback loop.



contagion = Infected People * INFECTION FRACTION
UNITS: people/day

Infection: Answers

- An increase in the number of infected people *increases* the contagion rate.
- An increase in the contagion rate *compounds* the initial increase in the number of infected people.
- The loop, therefore, is a *positive* feedback loop.



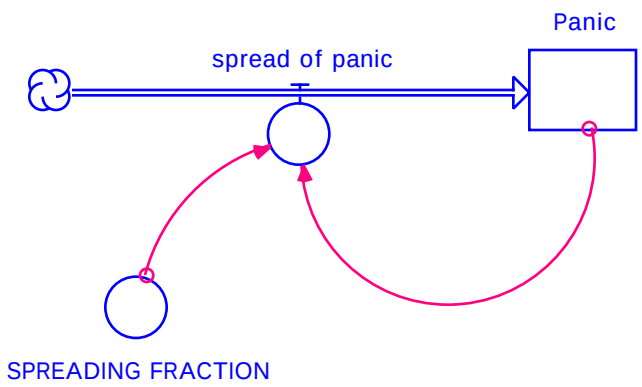
When many students in a classroom come to school with the flu, the remaining students have a higher chance of becoming sick. As the number of infected people increases, so does the rate of contagion. In turn, more and more people will be infected with the illness.

5.6 Panic

- An increase in the amount of panic *increases* *decreases* the rate at which panic spreads.

- An increase in the rate at which panic spreads *compounds* *counterbalances* the initial increase in the amount of panic.

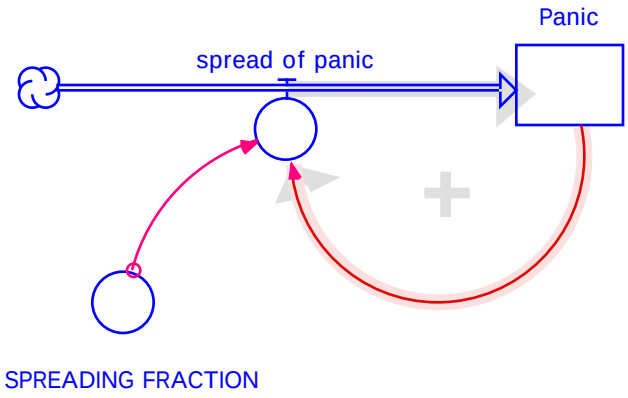
- The loop, therefore, is a *positive* *negative* feedback loop.



increase in panic = Panic * SPREADING FRACTION
UNITS: panic/minute

Panic: Answers

- An increase in the amount of panic *increases* the rate at which panic spreads.
- An increase in the rate at which panic spreads *compounds* the initial increase in the amount of panic.
- The loop, therefore, is a *positive* feedback loop.



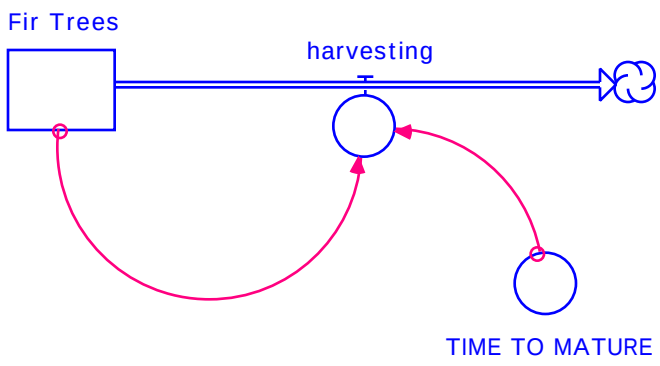
Someone shouts "fire!" in a theater. As people panic, the spread of panic increases. Across the theater hall, more and more people try to rush out. As the spread of panic increases, the total amount of panic skyrockets.

5.7 Fir Trees

- An increase in the number of fir trees *increases* *decreases* the harvesting rate.

- An increase in the harvesting rate *compounds* *counterbalances* the initial increase in the number of fir trees.

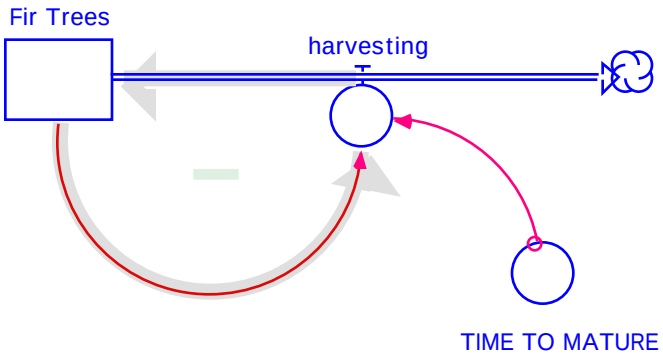
- The loop, therefore, is a *positive* *negative* feedback loop.



harvesting = Fir Trees / TIME TO MATURE
UNITS: trees/year

Fir Trees: Answers

- An increase in the number of fir trees *increases* the harvesting rate.
- An increase in the harvesting rate *counterbalance*s the initial increase in the number of fir trees.
- The loop, therefore, is a *negative* feedback loop.



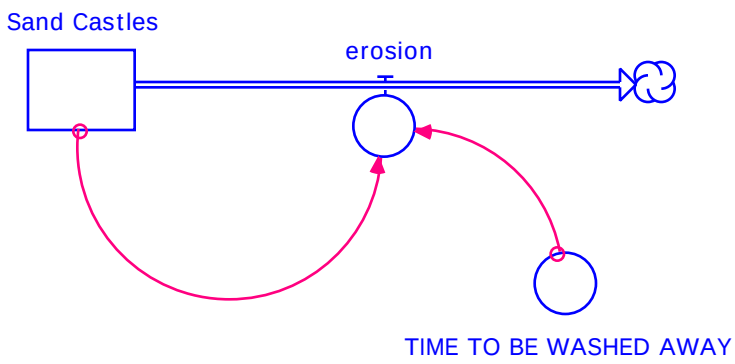
Timberjacks chop down great, towering trees; they harvest the mature trees in the forest. The more fir trees in the forest, the more trees that the timberjacks can harvest. As the number of fir trees in the forest decreases, the timberjacks can harvest fewer and fewer trees. Eventually, the trees will all get chopped down.

5.8 Sand Castles

- An increase in the number of sand castles *increases* *decreases* the erosion rate.

- An increase in the erosion rate *compounds* *counterbalances* the initial increase in the number of sand castles.

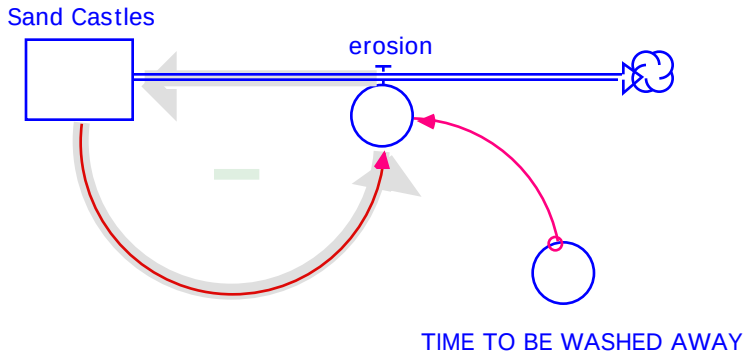
- The loop, therefore, is a *positive* *negative* feedback loop.



erosion = Sand Castles / TIME TO BE WASHED AWAY
UNITS: castles/hour

Sand Castles: Answers

- An increase in the number of sand castles *increases* the erosion rate.
- An increase in the erosion rate *counterbalances* the initial increase in the number of sand castles.
- The loop, therefore, is a *negative* feedback loop.



As the tide rises, waves wash away sand castles. The more sand castles are on the beach, the more castles the waves erode. As there are fewer sand castles left, the erosion rate becomes less dramatic until finally the last surviving castle loses its last walls.

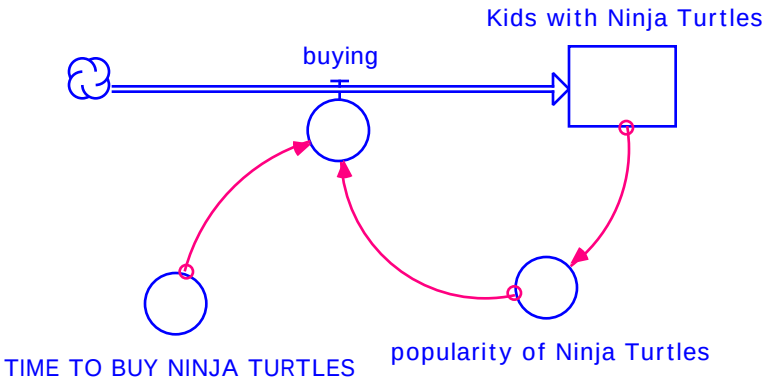
5.9 Kids with Ninja Turtles

- An increase in the number of kids with Ninja Turtles *increases* *decreases* the popularity of Ninja Turtles.

- An increase in the popularity of Ninja Turtles *increases* *decreases* the rate at which kids buy Ninja Turtles.

- An increase in the buying rate *compounds* *counterbalances* the initial increase in the number of kids with Ninja Turtles.

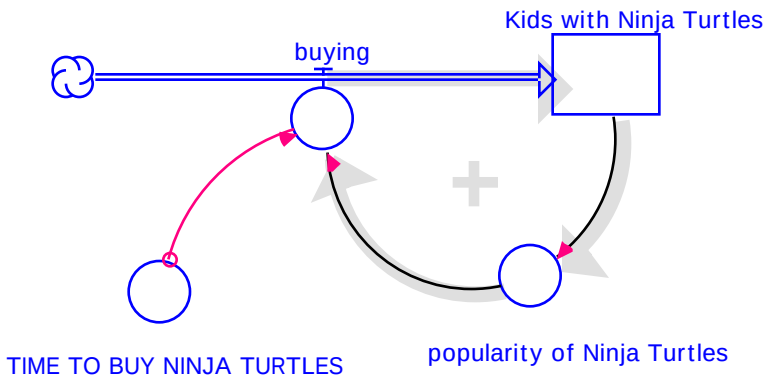
- The loop, therefore, is a *positive* *negative* feedback loop.



buying = Kids with Ninja Turtles / TIME TO BUY NINJA TURTLES
UNITS: kids/month

Kids with Ninja Turtles: Answers

- An increase in the number of kids with Ninja Turtles *increases* the popularity of Ninja Turtles.
- An increase in the popularity of Ninja Turtles *increases* the rate at which kids buy Ninja Turtles.
- An increase in the buying rate *compounds* the initial increase in the number of kids with Ninja Turtles.
- The loop, therefore, is a *positive* feedback loop.



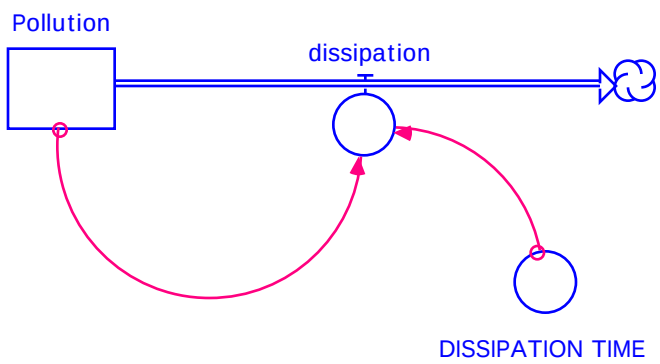
The spread of a fad is a reinforcing process. As more and more kids have Ninja Turtles, the toys become more and more popular; children (and adults) want to have the same toys as their friends. As the popularity of the toys increases, more and more people buy the Ninja Turtles.

5.10 Pollution

- An increase in the amount of pollution *increases* *decreases* the dissipation rate.

- An increase in the dissipation rate *compounds* *counterbalances* the initial increase in the amount of pollution.

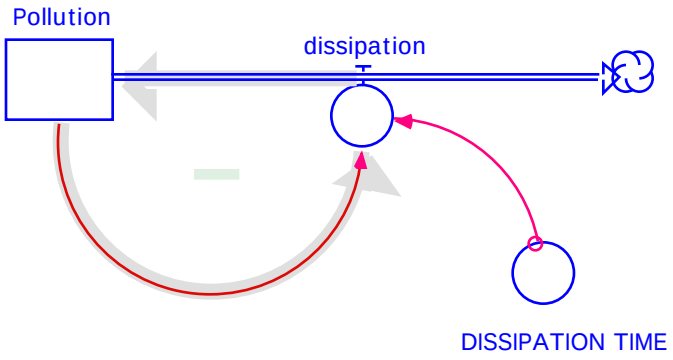
- The loop, therefore, is a *positive* *negative* feedback loop.



dissipation = Pollution / DISSIPATION TIME
UNITS: pollution/year

Pollution: Answers

- An increase in the amount of pollution *increases* the dissipation rate.
- An increase in the dissipation rate *counterbalances* the initial increase in the amount of pollution.
- The loop, therefore, is a *negative* feedback loop.



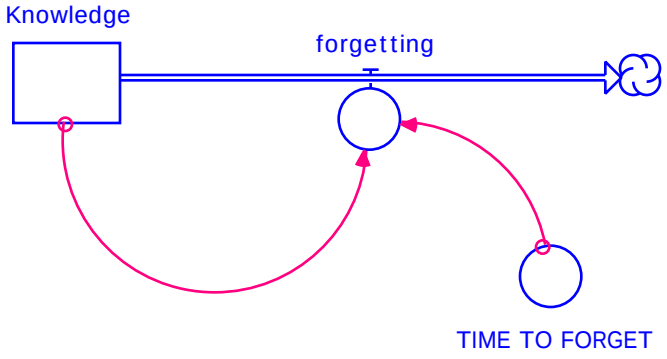
Pollution takes a long time to dissipate. The more pollution, the higher the dissipation rate, which, after a long delay, will reduce the overall level of pollution by a greater amount. As the amount of pollution decreases, the dissipation rate also decreases, until all the pollution dissipates away.

5.11 Knowledge

- An increase in the extent of your knowledge *increases* *decreases* the rate at which you forget.

- An increase in the rate at which you forget *compounds* *counterbalances* the initial increase in the extent of your knowledge.

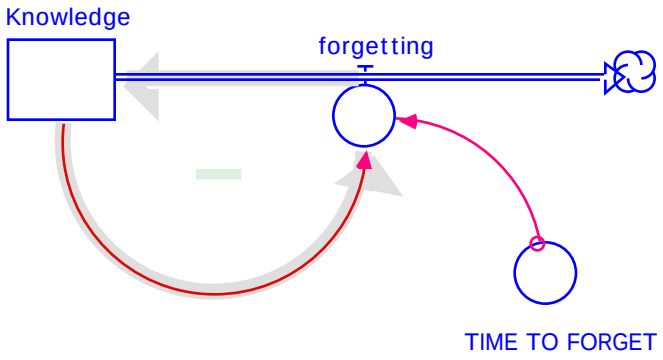
- The loop, therefore, is a *positive* *negative* feedback loop.



forgetting = Knowledge / TIME TO FORGET
UNITS: facts/year

Knowledge: Answers

- An increase in the extent of your knowledge *increases* the rate at which you forget.
- An increase in the rate at which you forget *counterbalances* the initial increase in the extent of your knowledge.
- The loop, therefore, is a *negative* feedback loop.



Sherlock Holmes used to choose not to remember certain facts because he knew that the more facts you know, the more you forget. As the number of facts you know decreases, the rate at which you forget also decreases. You may easily forget the phone numbers of the first 500 listings in the phone book that you memorized to impress your friends. On the other hand, a few select facts, like your phone number or the date of your birth, are more difficult to forget and remain with you for a longer time.

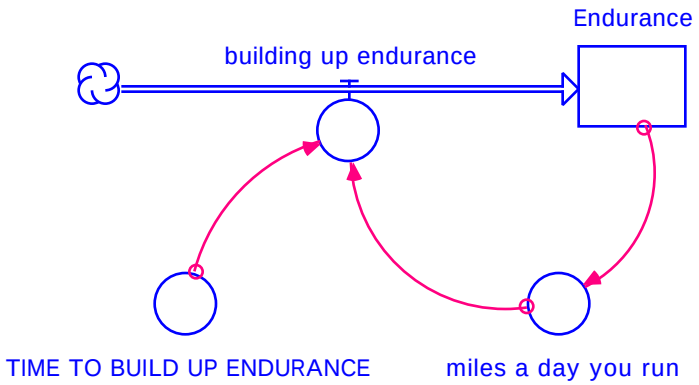
5.12 Endurance

- An increase in your endurance *increases* *decreases* the number of miles a day you run.

- An increase in the number of miles a day you run *increases* *decreases* the rate at which you build up your endurance.

- An increase in the rate at which you build up your endurance *compounds* *counterbalances* the initial increase in your endurance.

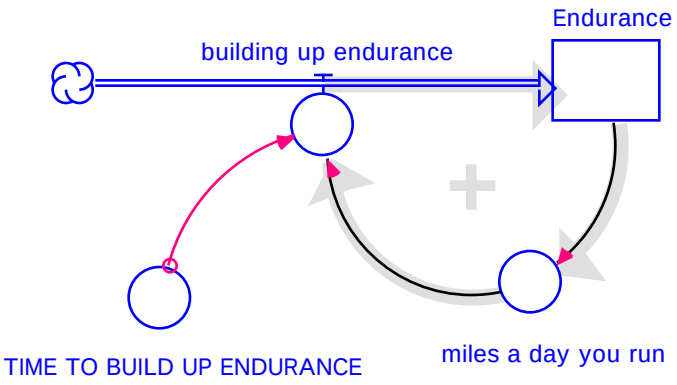
- The loop, therefore, is a *positive* *negative* feedback loop.



building up endurance = endurance / TIME TO BUILD UP ENDURANCE
UNITS: miles per day /week

Endurance: Answers

- An increase in your endurance *increases* the number of miles a day you can run.
- An increase in the number of miles a day you can run *increases* the rate at which you build up your endurance.
- An increase in the rate at which you build up your endurance *compounds* the initial increase in your endurance.
- The loop, therefore, is a *positive* feedback loop.



The more endurance you have, the more miles you can run every morning. The more you run, the faster you build up your endurance. Your ability to run improves with practice.

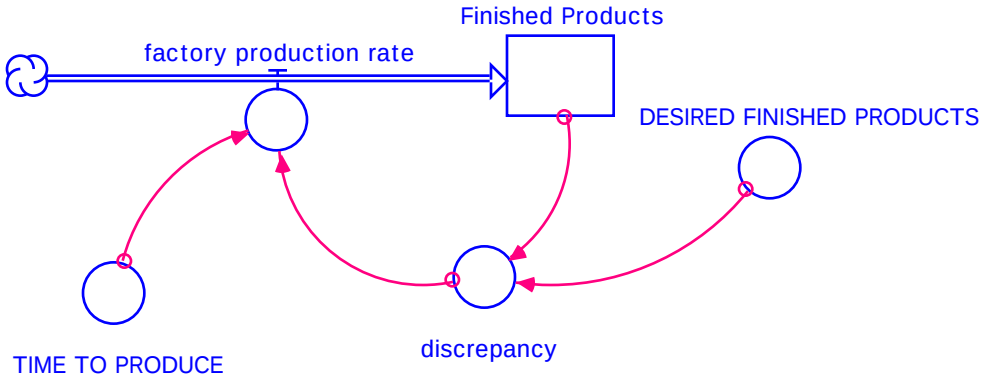
5.13 Finished Products

- An increase in the number of finished products *increases* *decreases* the discrepancy between the desired and actual number of finished products.[5]

- A decrease in the discrepancy *increases* *decreases* the factory production rate.

- A decrease in the factory production rate *compounds* *counterbalances* the initial increase in the accumulation of finished products.

- The loop, therefore, is a *positive* *negative* feedback loop.

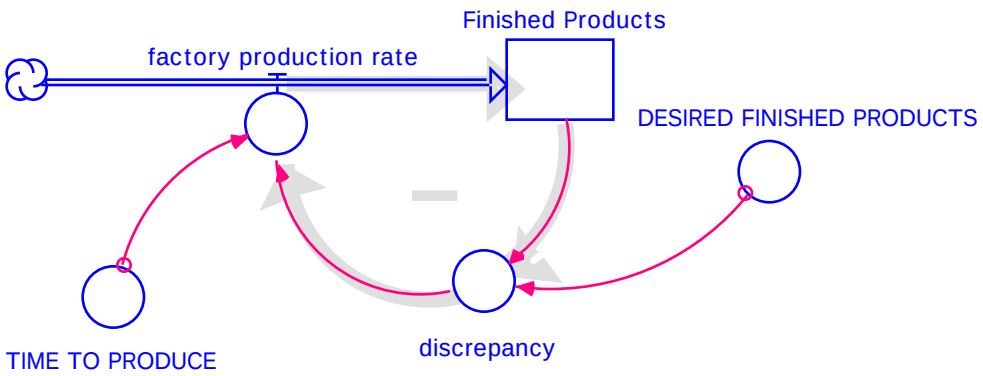


factory production rate = discrepancy / TIME TO PRODUCE
UNITS: widgets/year
discrepancy = DESIRED FINISHED PRODUCTS - Finished Products
UNITS: widgets

[5] Assume that initially the factory has fewer "Finished Products" than "DESIRED FINISHED PRODUCTS."

Finished Products: Answers

- An increase in the number of finished products *decreases* the discrepancy between the desired and actual number of finished products.
- A decrease in the discrepancy *decreases* the factory production rate.
- A decrease in the factory production rate *counterbalances* the initial increase in the accumulation of finished products.
- The loop, therefore, is a *negative* feedback loop.



The managers of a factory decide how many finished products they want to produce. They then compare the actual number of finished products already in their warehouses to the desired number of finished products and work to eliminate any discrepancy between what they have and what they want. At first, the factory runs full strength because it has many products to make. As it gets closer and closer to finishing the job, the discrepancy decreases and production slackens until finally the desired number of finished products has been produced.

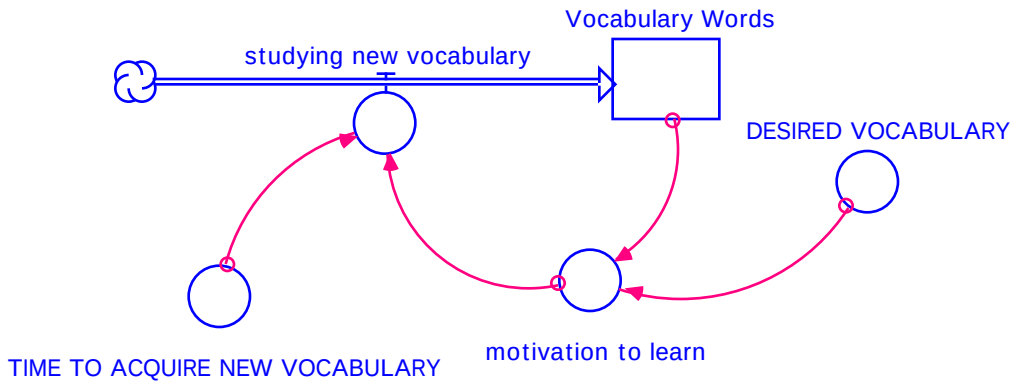
5.14 Vocabulary Words

- An increase in the number of vocabulary words that you know, compared to the number of vocabulary words you want to know, *increases* *decreases* your motivation to learn new words.[6]

- A decrease in your motivation to learn new words *increases* *decreases* the rate at which you study new vocabulary.

- A decrease in the rate at which you study new vocabulary *compounds* *counterbalances* the initial increase in the number of vocabulary words that you know.

- The loop, therefore, is a *positive* *negative* feedback loop.



studying new vocabulary = motivation to learn / TIME TO ACQUIRE NEW VOCABULARY
UNITS: words/week

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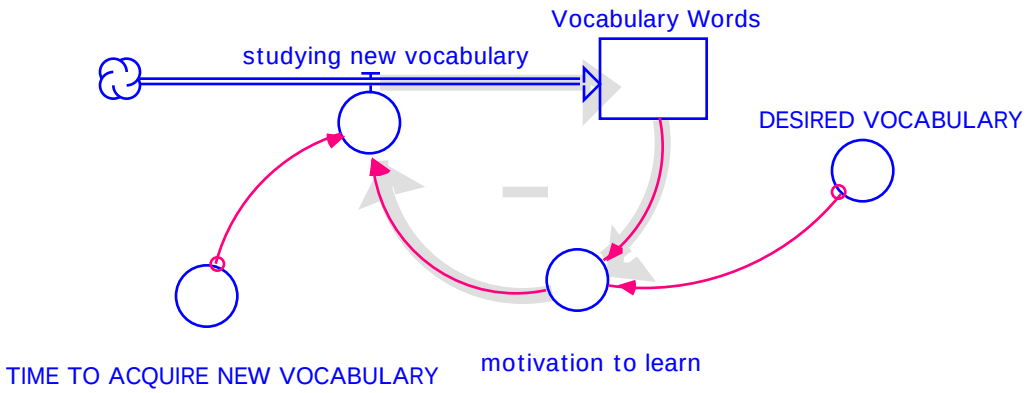
[6] Assume that initially your "DESIRED VOCABULARY" is larger than the number of "Vocabulary Words" that you know.

Vocabulary Words: Answers

- An increase in the number of vocabulary words that you know, compared to the number of vocabulary words you want to know *decreases* your motivation to learn new words.
- A decrease in your motivation to learn new words *decreases* the rate at which you study new vocabulary.
- A decrease in the rate at which you study new vocabulary *counterbalances* the initial increase in the number of vocabulary words that you know.
- The loop, therefore, is a *negative* feedback loop.



I am getting ready for a trip to France. I bought a beginner's language book that contains 1000 of the most important French words that I will need to know. I want to learn all of them. At first I study really hard and learn many new words because I am very motivated. As I learn more and more words, however, I start to feel satisfied with the words that I do know, my motivation to learn decreases, and it is only slowly and grudgingly that I commit to memory the last words.

6. SUMMARY

Feedback is the process through which a signal travels through a chain of causal relations to reaffect itself. Feedback can be divided into two categories: positive and negative. Feedback is *positive* if an increase in a variable leads to a further increase in the variable. Positive feedback is a reinforcing, compounding, or amplifying process. The reproduction of *E. coli* bacteria and the behavior of the development of a nicotine addiction are examples of positive feedback. Positive feedback systems produce exponential behavior. On the other hand, feedback is *negative* if an increase in a variable eventually leads to a decrease in the variable. Examples of negative feedback include the decay of radioactive nuclei or the process of dieting to reach an ideal weight. Negative feedback drives balancing, stabilizing systems that produce asymptotic or oscillatory behavior. The two types of feedback, positive and negative, combine to create all of the behavior observed in complex systems.