

Building a System Dynamics Model Part II: Formulation

The Fruit Fly System

Prepared for the
MIT System Dynamics in Education Project
Under the Supervision of
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by

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The three fruit fly models that accompany this paper are in VensimPLE software.

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1. INTRODUCTION

Formulation describes how to build and formulate models. The paper follows the modeling techniques as outlined in *Building a System Dynamics Model Part 1: Conceptualization*¹. This paper describes and builds a fruit fly system by first presenting a description of the system, then conceptualizing the model, its structure and components, then finally formulating the variables, constants, flows and stocks in the model. After building a model based solely on the description, the model will be run. After we analyze the results and find that they are not plausible, we will make assumptions about the behavior of the system and a corrected model will be built and formulated that more realistically resembles the system.

2. THE SYSTEM

Let's try to build a simple one-stock model from the following description:

“A scientist wants to breed a population of fruit flies in a fish-tank sized container for use in his experiment. The flies can produce offspring, and the scientist knows that they multiply at a rate of 125 times per week. The average life span of a fruit fly is one and half weeks. The scientist initially buys 1000 flies and puts them in the container. The flies are of variable ages. Approximately how many flies will be in the container after 1.5 weeks?”

3. CONCEPTUALIZATION

3.1 Define the Purpose of the Model

The Fruit Fly system presents the modeler with the question of how fruit fly population grows in a container.

The purpose of the model is to estimate about how many fruit flies will be in the container after a period of 1.5 weeks.

The audience for this model is the person learning to convert a description of a system into a simulation model. The audience is interested in general trends in the fruit fly population rather than detailed statistics. All of the information about the system is given in the problem description.

¹ Stephanie Albin, 1997. *Building a System Dynamics Model Part 1: Conceptualization* (D-4597), System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, 36 pp.

3.2 Define Model Boundary

The system consists of fruit flies that multiply to create new flies and die after a period of time. We first identify **Fruit Flies** as a stock. Initially, the system is simplified, so we ignore different stages of the fly (eggs, larva, etc). The simplification of the system also leads to the principle that the flies will be able to and will begin to reproduce as soon as they are hatched. The flows that influence the number of **Fruit Flies** in the system are inflow of new fruit flies hatching called **multiplying** and the outflow of flies dying called **dying**. The rate of reproduction is called the **MULTIPLYING RATE**. Flies die after about a week and a half. We call this rate the **DYING RATE**. A review of the Fruit-Fly description leads to the following initial components list:

Endogenous Components	Exogenous Components
Fruit Flies (Stock)	MULTIPLYING RATE (constant)
multiplying (Flow)	DYING RATE (constant)
dying (Flow)	

Figure 1. Fruit Fly model boundary

3.3 Behavior Description

The fruit flies multiply and die based on the number of fruit flies in the container. Ignoring the dying of the fruit flies, we notice that the **Fruit Flies** stock should experience growth: as each week passes, more fruit flies will multiply to create more new fruit flies than were created the previous week.

The dying on average of each fly after one and a half weeks will slow the multiplying of the flies, but will not stop growth from occurring. Because the flies multiply at such a high rate, we expect the population to grow.

3.4 Basic Mechanisms

The growth of the number of **Fruit Flies** is caused by the multiplication of the existing number of **Fruit Flies**. As the number of **Fruit Flies** grows, the inflow of flies grows, which increases the number of **Fruit Flies** more. Thus, there is a positive feedback loop between **multiplying** and **Fruit Flies**. The decay of the number of **Fruit Flies** is caused by the deaths of the existing number of **Fruit Flies**. As the number of **Fruit Flies** grows, the **dying** outflow grows, which decreases the number of **Fruit Flies**. Thus, there is a negative feedback loop between **dying** and **Fruit Flies**. Figure 2 below shows a stock and flow diagram of the basic mechanisms of the fruit fly system. For this paper, we use Vensim modeling software.

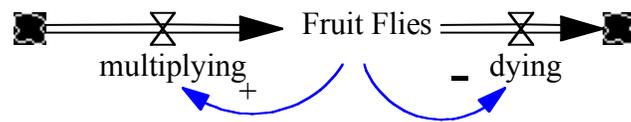


Figure 2. Stock and Flow diagram of basic mechanisms

4. INITIAL FORMULATION

4.1 Units for Stocks and Flows, Settings

Stocks: The only stock in the Fruit Fly model is **Fruit Flies**. Because the stock keeps a count of the number of fruit flies in the container, the units for **Fruit Flies** is fruit flies.

Flows: The inflow and outflow to **Fruit Flies** must have the same units and both indicate a changing number of fruit flies over a period of time. Because the units of time measurement in this system is weeks, both **multiplying** and **dying** have units fruit flies/week.

Settings: Be sure that under model settings in Vensim, units for time are specified for week.

4.2 Initial Stock Values

It is good modeling practice to explicitly show the initial values of the stocks on the diagram. Thus, we connect **INITIAL FRUIT FLIES** to **Fruit Flies**. The value of **INITIAL FRUIT FLIES** is 1000 and its units, fruit flies, are consistent with those of the **Fruit Flies** stock.

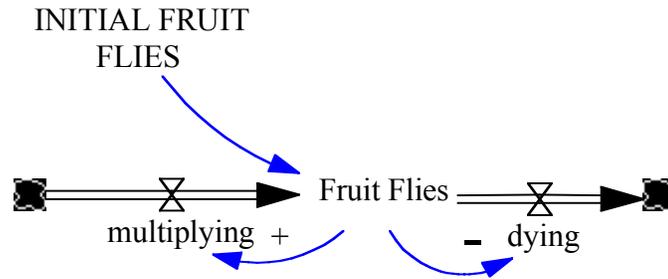


Figure 3. Addition of **INITIAL FRUIT FLIES** to Fruit Fly system. Note, some Vensim packages may not allow a line to be drawn from **INITIAL FRUIT FLIES** to **Fruit Flies**.

4.3 Equating the Inflows

multiplying: In this system, flies multiply at a rate of 125 times per week. Hence,

$$\text{multiplying} = \text{Fruit Flies} * 125/\text{week}$$

The constant 125/week is called the **MULTIPLYING RATE**.

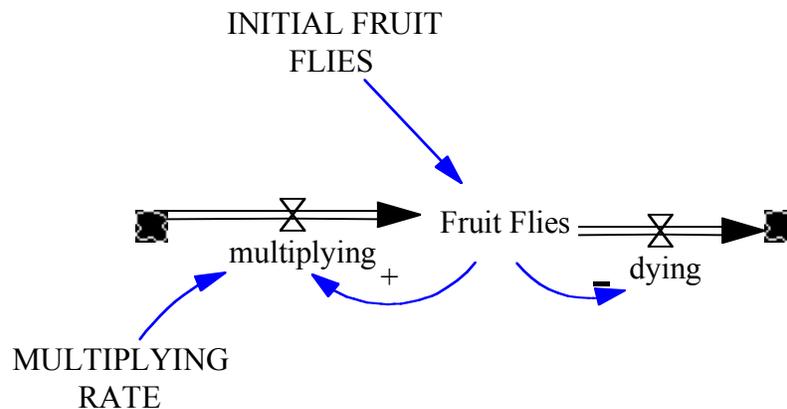


Figure 4. Addition of **MULTIPLYING RATE** to the Fruit Fly system

The **MULTIPLYING RATE** is multiplied by the number of **Fruit Flies** to determine the **multiplying** inflow.

Because the only inflow in the Fruit Fly system is **multiplying**, we have completed formulating the inflows.

4.4 Equating the Outflows

Outflow: In this system, the average lifespan of the flies is 1.5 weeks. Because the death rate is 1/average lifespan, the death rate equals 1/1.5 or 0.67/week (derivation of units is shown below). Thus,

$$\text{dying} = 0.67 * \text{Fruit Flies}$$

The 0.67 constant is named the **DYING RATE**.

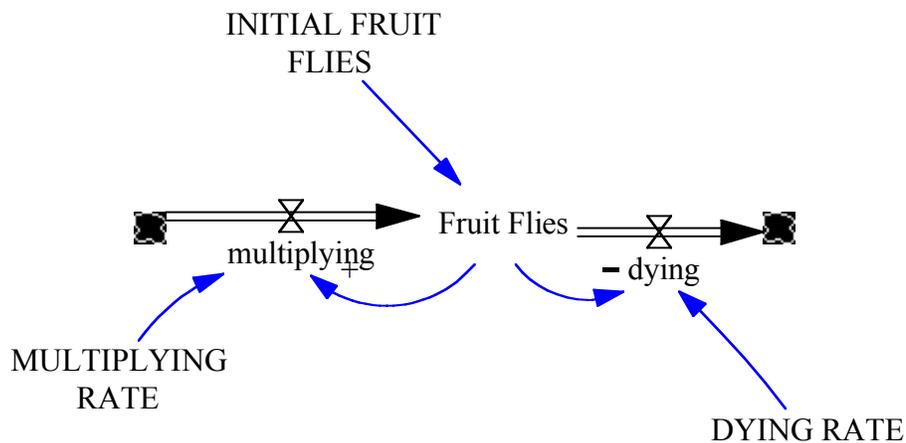


Figure 5. Addition of **DYING RATE** to the Fruit Fly system

The **DYING RATE** is multiplied by the number of **Fruit Flies** to determine the rate at which the fruit fly population shrinks. The **DYING RATE** indicates the proportion of the stock that is multiplied *per week*. Because the **DYING RATE** acts only as a multiplier, its units are 1/week, and when multiplied by **Fruit Flies**, outputs the units fruit flies/week for **dying**.

Because the only outflow in the Fruit Fly system is **dying**, we have completed formulating the outflows.

4.5 Documentation for Model

Documentation is one of the most important and most often neglected modeling practices. By documenting models, one can ensure that the results can be understood, replicated, criticized and extended upon by other modelers.

Below, the documentation for the completed model is presented:

- (01) dying=Fruit Flies*DYING RATE
Units: fruit flies/week
The outflow from fruit flies.
- (02) DYING RATE=0.67
Units: 1/week
The rate at which the flies die. Is equal to the reciprocal of the average lifespan of the flies.
- (03) FINAL TIME = 1.5
Units: week
The final time for the simulation.
- (04) Fruit Flies= INTEG (+multiplying-dying, INITIAL FRUIT FLIES)
Units: fruit flies
The number of fruit flies in the fruit fly system.
- (05) INITIAL FRUIT FLIES=1000
Units: fruit flies
The number of flies initially in the container.
- (06) INITIAL TIME = 0
Units: week
The initial time for the simulation.
- (07) multiplying=Fruit Flies*MULTIPLYING RATE
Units: fruit flies/week
The inflow to fruit flies.
- (08) MULTIPLYING RATE=125
Units: 1/week
The rate at which the flies multiply.
- (09) SAVEPER = TIME STEP
Units: week
The frequency with which output is stored.
- (10) TIME STEP = 0.0625

Units: week
The time step for the simulation.

5. INITIAL SOLUTION TO SYSTEM PROBLEM

According to Figure 5, it appears as though there would be approximately $4.30e+25$ fruit flies in the container after 1.5 weeks. That's 43,000,000,000,000,000,000,000 flies after 1.5 weeks.

The output from the model suggests that a huge number of flies would be in the system after 1.5 weeks.

Although it appears as though the level of **Fruit Flies** remains at or near 0 for the majority of the run, and then begins to exponentially grow near 1.50 weeks, the population is exponentially growing for the duration of the run, but the final values become large at a fast rate toward the end of the run, so that once scaled, the earlier behavior looks linear.

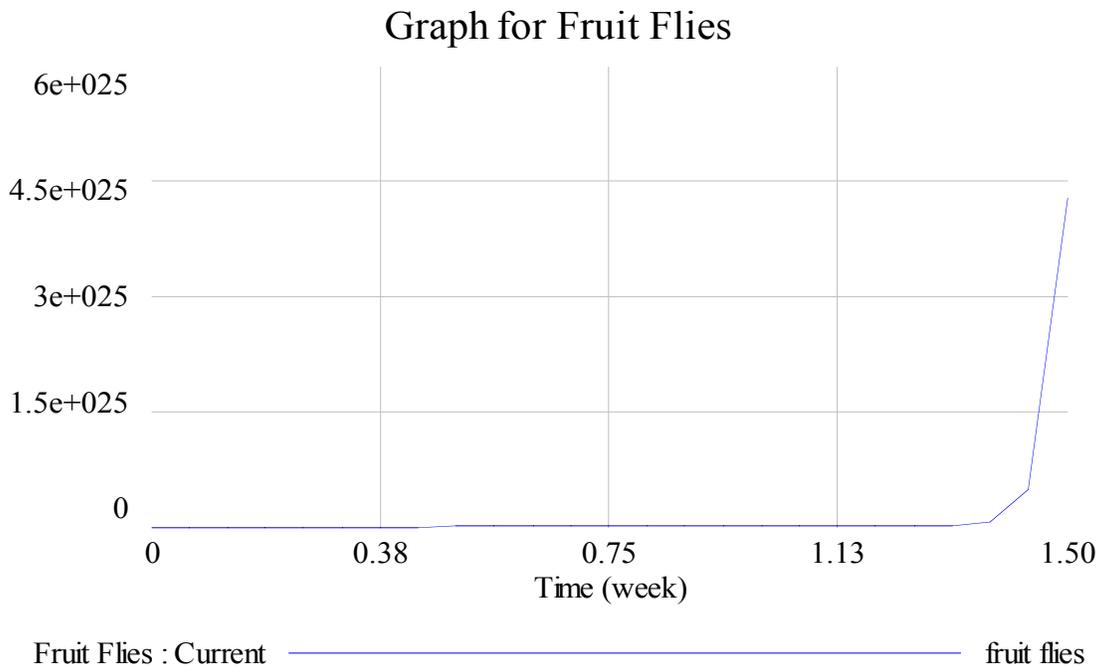


Figure 6. Initial output from the Fruit Fly model

The behavior of the graph for **Fruit Flies** will look the same for any run length. A graph that looks similar to Figure 6, but with different values on the y-axis will result from any run with a **FINAL TIME** different from 1.50 weeks. Figure 7 below shows the fruit fly population growth after half a week.

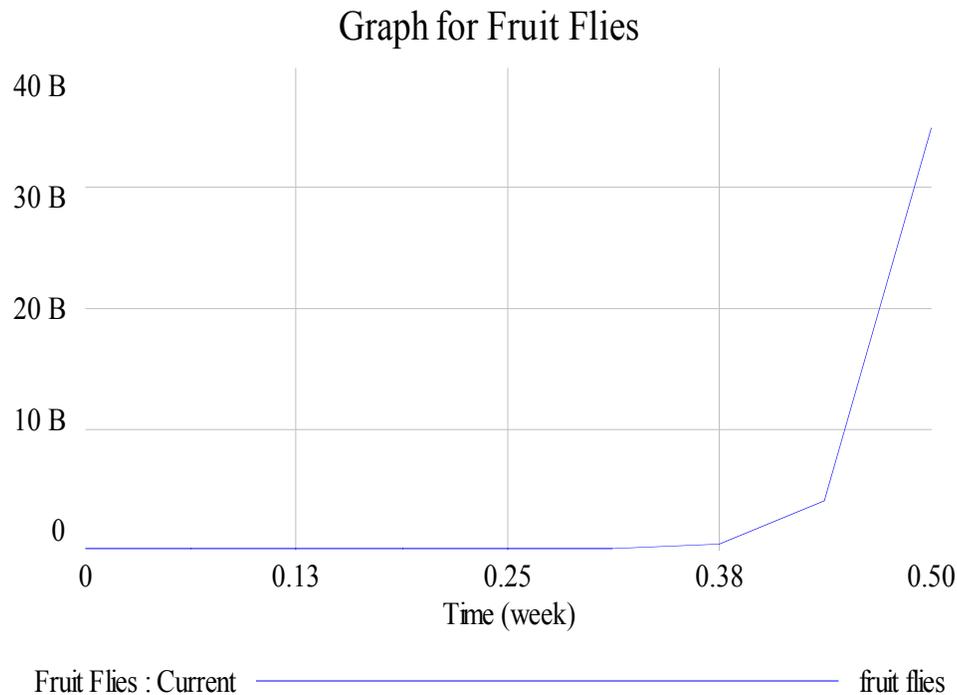


Figure 7. Behavior of Fruit Fly population after 0.5 weeks

6. COMMENTS

The fruit fly stock experiences extreme exponential growth because the positive feedback loop between **Fruit Flies** and the inflow **multiplying** has a greater effect than the feedback loop between **Fruit Flies** and the outflow **dying**.

We calculated the **multiplying** inflow to be $125 * \text{Fruit Flies}$ whereas the **dying** outflow is $0.67 * \text{Fruit Flies}$. Thus, the **Fruit Flies** population will be increasing at a faster rate than they are dying, and because the inflow is dependent on the number of **Fruit Flies** currently in the system, the increase in **Fruit Flies** at each interval will be greater than the previous interval's. Hence, we see the stock experience exponential growth.

Because the **TIME STEP** is long compared to the doubling time of the flies, the resulting behavior of the **Fruit Flies** stock is dependent on the **TIME STEP**, which is not good modeling practice. Following the calculation of the doubling time derived in *Beginner Modeling Exercises Section 2: Mental Simulation of Simple Positive Feedback*². We calculate the doubling time to be $0.693 * \text{TIME CONSTANT}$, or $0.693 * 1 / \text{MULTIPLYING RATE} = 0.005544$ week. Because the doubling time of the **FRUIT FLIES** is 0.005544, we choose a **TIME STEP** of roughly half this amount, or .002772. We can test how reasonable this **TIME STEP** is by making runs and

² Joseph Whelan, 1996. *Beginner Modeling Exercises Section 2: Mental Simulation of Simple Positive Feedback* (D-4487), System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, 15 pp.

computing the final value of the **FRUIT FLIES** stock using runs with **TIME STEPS** that are twice, half, and a quarter of the basic 0.005544 value. The chart below shows the final value of the **FRUIT FLIES** stock that corresponds to each of the described **TIME STEPS** for a run time of 0.5 weeks. We choose this run time because the number of **FRUIT FLIES** computes above the computer register for longer run times.

TIME STEP	FINAL VALUE (Fruit Flies)
0.011088	8.59E+19
0.005544	3.11E+23
0.002772	1.41E+26
0.001386	8.43E+27

Figure 8. Chart for final value given different **TIME STEP**

Figure 8 indicates that as the **TIME STEP** decreases, the final value for **FRUIT FLIES** changes by a smaller and smaller factor. The final value for a **TIME STEP** of 0.001386 is more than 100 times larger than the final value corresponding the next smallest **TIME STEP** listed, but the difference may be of little consequence. Therefore, we can choose a **TIME STEP** around 0.003 for a significant level of accuracy. To integrate using a power of 2 (which the Vensim default features suggest), which is desirable to avoid minor problems that can arise from using other decimal numbers in computers that operate on the binary system, we choose the value 0.0030625.

The Fruit Fly system presented has been greatly simplified in two respects:

1. We assume that the flies, when hatched, can immediately produce new offspring. In essence, we ignore the egg and larva phases of the fly's development.
2. We also ignore any possibility of fruit flies dying from crowding within the container. The system houses a fish-tank sized container, but can this container truly support over $4.30e+25$ fruit flies? If not, the flies should begin to die from crowding within the container.

Actually, the maturation of the **Fruit Flies** occurs a few days after they hatch. The calculation of the **MULTIPLYING RATE** assumes that the fruit flies can immediately reproduce once they hatch.

Crowding should affect the **dying** rate. As the number of **Fruit Flies** increases, they become crowded, and they should begin to die before their natural life span of 1.5 weeks.

In the next section, we adjust the model to reflect how maturing should affect **multiplying** and how crowding should affect **dying**.

7. CORRECTED FORMULATION

In this section, we make adjustments to the inflow **multiplying** to account for the immature stages of the flies' lives and the outflow **dying** to account for the crowding effect in the container.

7.1 Adjusting the Inflow

Because the flies cannot reproduce until they have sexually developed for 3 days, a new stock, **Undeveloped Fruit Flies**, must be created, along with **INITIAL UNDEVELOPED FRUIT FLIES**. The initial value of the stock is 0 fruit flies because we assume the scientist buys flies that are not in larvae stages, but rather are sexually developed. The stock's inflow is **multiplying**, the same original inflow for **Fruit Flies**. The outflow from **Undeveloped Fruit Flies** is **developing**, and it feeds into **Fruit Flies**, because once they are sexually developed (and are not in the larvae state), they have become fruit flies (what the scientist is interested in studying).

developing- Flies develop after 3 days, or .43 weeks. The **DEVELOPING RATE**, therefore, is 1/.43 week, which we round to 2.3/week. The flow is equal to the number of **Fruit Flies** times the **DEVELOPING RATE**.

The updated model is shown below:

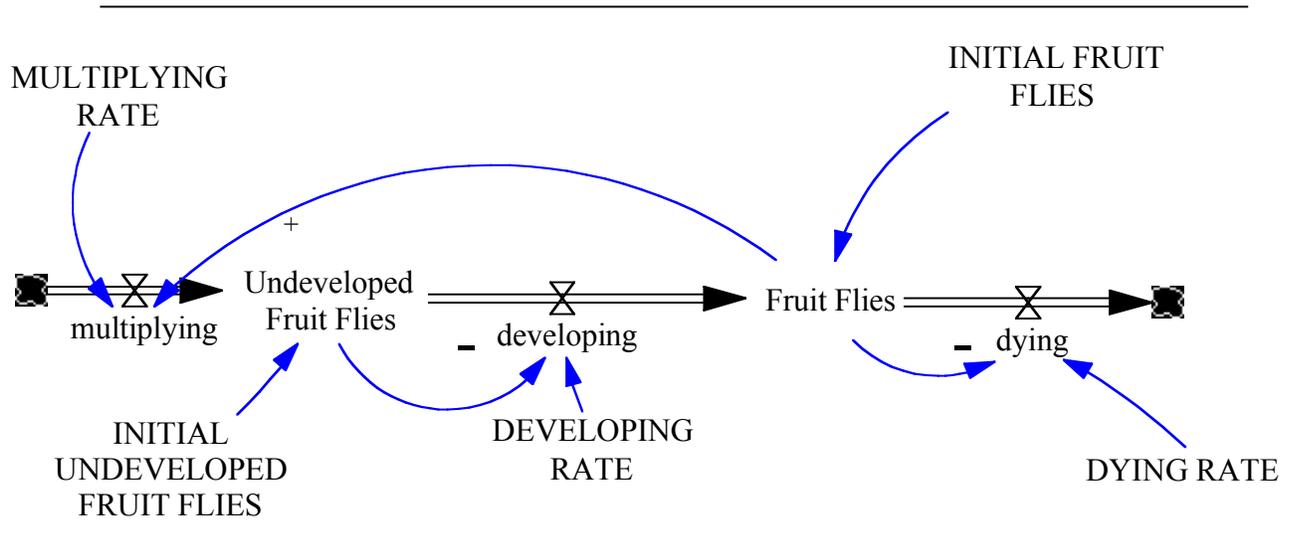


Figure 9. Fruit Fly system with addition of **Undeveloped Fruit Flies** stock and adjustments to inflow

The Documentation for the Fruit Fly system with the addition of **Undeveloped Fruit Flies** is shown below:

- (01) $\text{developing} = \text{Undeveloped Fruit Flies} * \text{DEVELOPING RATE}$
 Units: fruit flies/week
 The outflow from the Undeveloped Fruit Flies and inflow to Fruit Flies.
- (02) $\text{DEVELOPING RATE} = 2.3$
 Units: 1/week
 The rate at which the larva develop into fruit flies. It is equal to the reciprocal of the time it takes for the larva to mature.
- (03) $\text{dying} = \text{Fruit Flies} * \text{DYING RATE}$
 Units: fruit flies/week
 The outflow from the fly flies stock.
- (04) $\text{DYING RATE} = 0.67$
 Units: 1/week
 The rate at which the fruit flies die. It is equal to the reciprocal of their average life span.
- (05) $\text{FINAL TIME} = 1.5$
 Units: week
 The final time for the simulation.
- (06) $\text{Fruit Flies} = \text{INTEG}(\text{developing} - \text{dying}, \text{INITIAL FRUIT FLIES})$
 Units: fruit flies
 The number of fruit flies in the Fruit Fly system.
- (07) $\text{INITIAL FRUIT FLIES} = 1000$
 Units: fruit flies
 The number of fruit flies initially in the container.
- (08) $\text{INITIAL TIME} = 0$
 Units: week
 The initial time for the simulation.
- (09) $\text{INITIAL UNDEVELOPED FRUIT FLIES} = 0$
 Units: fruit flies
 The number of undeveloped fruit flies initially in the container.
- (10) $\text{multiplying} = \text{Fruit Flies} * \text{MULTIPLYING RATE}$
 Units: fruit flies/week
 The inflow to undeveloped fruit flies.

- (11) MULTIPLYING RATE=125
Units: 1/week
The rate at which the fruit flies multiply.
- (12) SAVEPER = TIME STEP
Units: week
The frequency with which output is stored.
- (13) TIME STEP = 0.0625
Units: week
The time step for the simulation.
- (14) Undeveloped Fruit Flies= INTEG (+multiplying-developing,INITIAL UNDEVELOPED FRUIT FLIES)
Units: fruit flies
The number of undeveloped fruit flies in the Fruit Fly system.

The solution to the system problem with the adjusted inflow is presented below. The simulation shows there will be approximately 6 billion fruit flies in the container after 1.5 weeks:

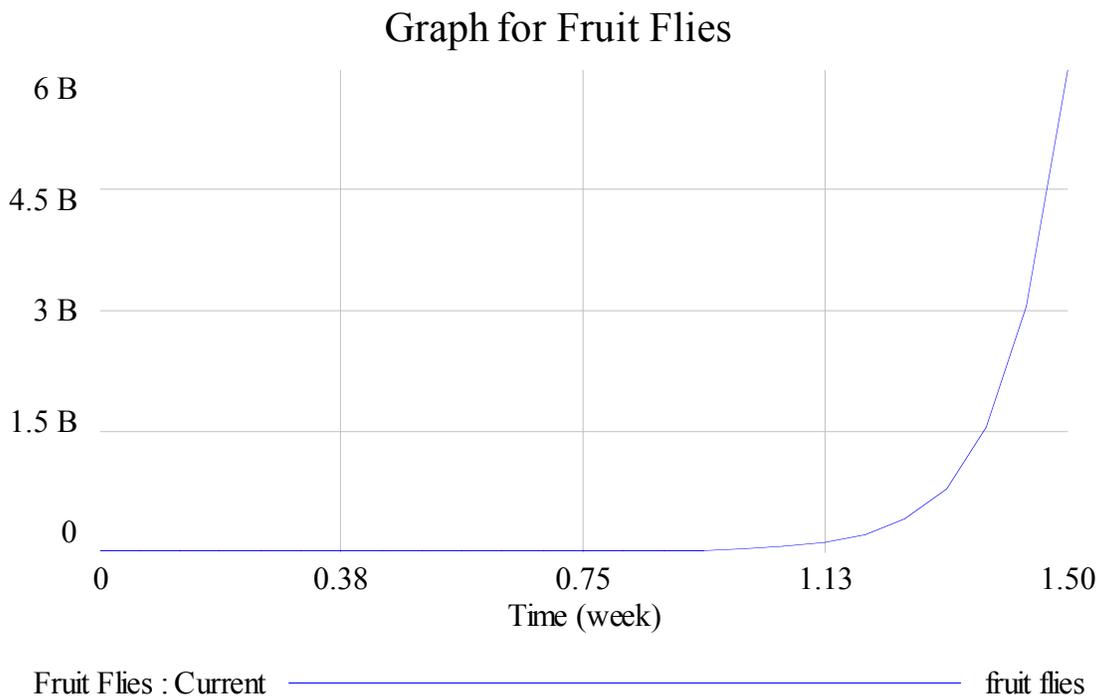


Figure 10. Solution to system problem with adjusted inflow.

The final value of the fruit flies, approximately 6 billion, is much smaller than the previously obtained $4.30e+25$ value. The number of flies is reduced because the flies take time to develop, and the time to develop produces a delay in the system. The delay causes the number of reproducing flies to be smaller at each interval.

In the next section, we adjust the outflow to account for the effect crowding has on the fruit fly dying outflow.

7.2 Adjusting the Outflow

As more and more **Fruit Flies** develop, crowding in the container causes the **Fruit Flies** to die because they must compete for food and space. We assume that crowding affects only the developed flies, and not the undeveloped ones. We also assume the crowding does not affect the maturation process, or **DEVELOPING RATE** of the **Undeveloped Fruit Flies**. To account for crowding, we adjust only the outflow **dying** and not the inflow **multiplying** because we assume that the effect of crowding does not change the reproductive patterns of the fruit fly population. To compute the effect of crowding, we must use a lookup function.

We know that if the flies live in an uncrowded environment, they will die at a natural rate of 0.67/week, which is the value calculated previously as the **DYING RATE**. Because the crowding will affect the **dying** outflow, the rate at which flies die will change. The value 0.67/week thus becomes the **NORMAL DYING RATE** and is multiplied by the **effect of crowding** and the number of **Fruit Flies** to determine the **dying** outflow:

$$\text{dying} = \text{Fruit Flies} * \text{NORMAL DYING RATE} * \text{effect of crowding}$$

The **effect of crowding** is computed by the use of a lookup function, called the **EFFECT OF CROWDING LOOKUP** whose input is the number of **Fruit Flies** divided by the **NORMAL FRUIT FLY POPULATION**. The **NORMAL FRUIT FLY POPULATION** is chosen to be the number of fruit flies that can live in the container without dying from crowding. We choose this value to be 1,000 fruit flies. Thus, if 1,000 or less fruit flies are in the container, the **EFFECT OF CROWDING LOOKUP** will output a value 1, and the **dying** outflow will be equal to **Fruit Flies*NORMAL DYING RATE*1**, or $0.67 * \text{Fruit Flies}$ fruit flies/week. If the proportion of **Fruit Flies** to **NORMAL FRUIT FLY POPULATION** is larger than 1, the **EFFECT OF CROWDING LOOKUP** will output a value larger than 1, at an increasing rate (the function is increasing).

The updated model is shown below:

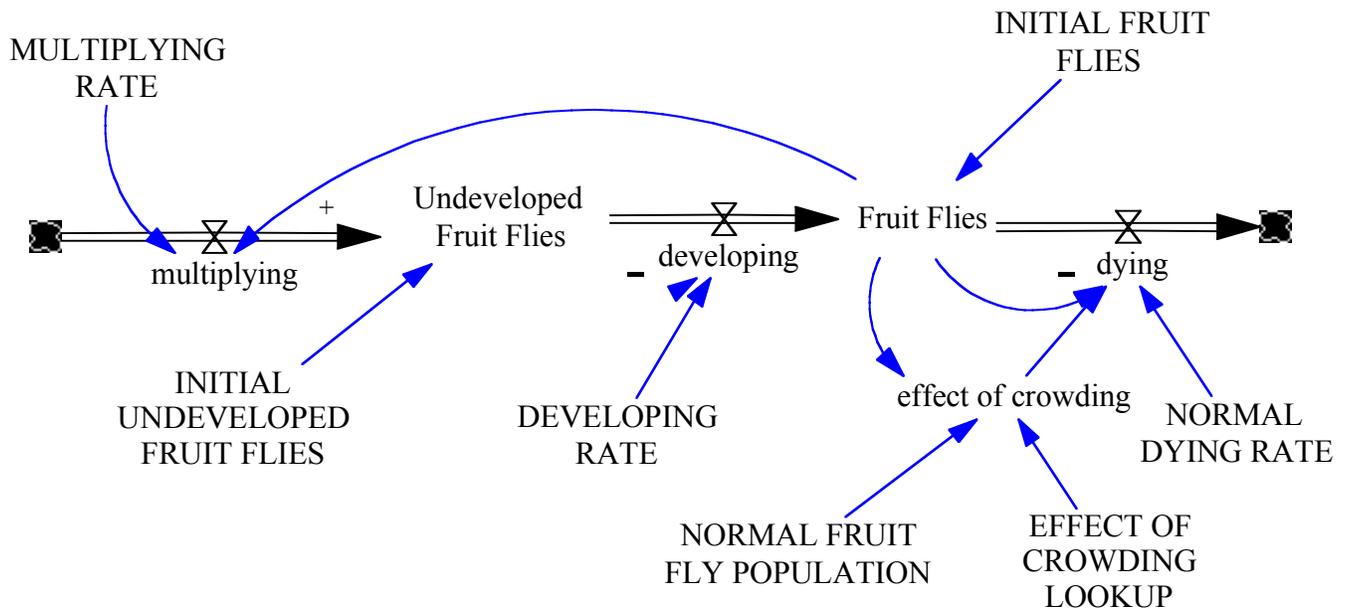


Figure 11. Updated Fruit Fly model

To formulate the **EFFECT OF CROWDING LOOKUP**, we must assess several scenarios. For the X-axis (inputs) in the range from 0-1.0, the corresponding Y-axis (output) values are 1, because there is not enough crowding in the container to affect the normal outflow, **dying**. We can also assume that the tank can hold a maximum number of fruit flies at equilibrium. We reasonably assume that for a **MULTIPLYING RATE** of 125, the tank cannot hold more than 10,000 fruit flies. This assertion implies that when the level of **Fruit Flies** is 10,000 fruit flies, the inflow **multiplying** and the outflow **dying** are equal. We know that the value for **multiplying** is $125/\text{week} * \text{Fruit Flies}$. We also know that the value for **dying** is $0.67/\text{week} * \text{Fruit Flies} * \text{effect of crowding}$. If we equate the flows with 10,000 fruit flies as the value for **Fruit Flies**, we obtain the following:

$$\text{effect of crowding} * 10,000 \text{ fruit flies} * 0.67/\text{week} = 125/\text{week} * 10,000 \text{ fruit flies}$$

or

$$\text{effect of crowding} = 186.6 \text{ (dimensionless units)}$$

Thus, for an input value of 10 (10,000 fruit flies/1,000 fruit flies), the **EFFECT OF CROWDING LOOKUP** should output a value around 186.6/week. However, because we cannot model the system with great precision, we round the value to 200/week. If 1,000 fruit flies are in the container, we expect the output from the **EFFECT OF CROWDING LOOKUP** to be 1, hence we input a (1,1) point in the

lookup function. For inputs lower than 1 (fruit fly populations less than 1,000 fruit flies), the output is 1, hence we input a (0,1) point in the lookup function. The curve connecting the (1,1) point to the (10, 200) point is concave, indicating that at lower levels of crowding, the effect from crowding is smaller, and at higher levels, the effect from crowding becomes more severe. At higher and higher multiples of the **NORMAL FRUIT FLY POPULATION**, the effect from crowding becomes intensified, so the lookup function rises quickly. The **EFFECT OF CROWDING LOOKUP** is shown below:

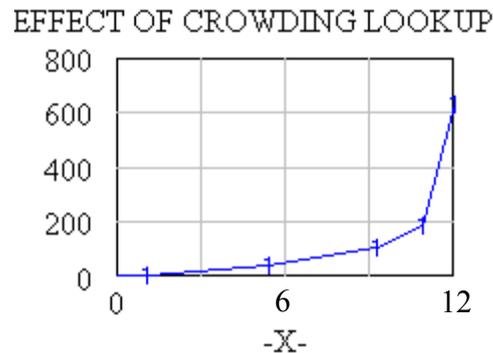


Figure 12. Graph for EFFECT OF CROWDING LOOKUP

The **EFFECT OF CROWDING LOOKUP** is calculated based on what are considered to be the normal multiplying and dying rates. If these rates are changed (or if the **NORMAL DYING RATE** is changed), the equilibrium value of the fruit fly population will change as well.

7.3 Documentation for Adjusted Model

- (01) $\text{developing} = \text{Undeveloped Fruit Flies} * \text{DEVELOPING RATE}$
 Units: fruit flies/week
 The outflow from Undeveloped Fruit Flies and inflow to Fruit Flies.
- (02) $\text{DEVELOPING RATE} = 2.3$
 Units: 1/week
 The rate at which the larva develop into fruit flies. It is equal to the reciprocal of the time it takes for the larva to mature.
- (03) $\text{dying} = \text{Fruit Flies} * \text{NORMAL DYING RATE} * \text{effect of crowding}$
 Units: fruit flies/week
 The outflow from the fruit flies stock.
- (04) $\text{effect of crowding} = \text{EFFECT OF CROWDING LOOKUP}(\text{Fruit Flies} / \text{NORMAL FRUIT FLY POPULATION})$

Units: dmn1

The effect of crowding for a given level of Fruit Flies relative to the NORMAL FRUIT FLY POPULATION is equal to the output from the EFFECT OF CROWDING LOOKUP.

- (05) EFFECT OF CROWDING LOOKUP(
 [(0,0)-(12,800)],(0,1),(1,1),(5,40),(8.5,105),(10,200),(12,650))
 Units: dmn1
 The lookup reflects that at lower levels of crowding, the effect of crowding on dying is small, but at larger levels of crowding, the effect on dying becomes more severe.
- (06) FINAL TIME = 1.5
 Units: week
 The final time for the simulation.
- (07) Fruit Flies= INTEG (developing-dying, INITIAL FRUIT FLIES)
 Units: fruit flies
 The number of fruit flies in the Fruit Fly system.
- (08) INITIAL FRUIT FLIES=1000
 Units: fruit flies
 The number of fruit flies initially in the container.
- (09) INITIAL TIME = 0
 Units: week
 The initial time for the simulation.
- (10) INITIAL UNDEVELOPED FRUIT FLIES=0
 Units: fruit flies
 The number of undeveloped fruit flies initially in the container.
- (11) multiplying=Fruit Flies*MULTIPLYING RATE
 Units: fruit flies/week
 The inflow to Undeveloped Fruit Flies.
- (12) MULTIPLYING RATE=125
 Units: 1/week
 The rate at which the flies multiply.
- (13) NORMAL DYING RATE=0.67
 Units: 1/week
 The rate at which flies die when there is not enough crowding to affect the normal life spans of the fruit flies.
- (14) NORMAL FRUIT FLY POPULATION=1000

Units: fruit flies

The threshold population level where there is not enough crowding to affect the normal life spans of the fruit flies.

(15) $SAVEPER = TIME\ STEP$

Units: week

The frequency with which output is stored.

(16) $TIME\ STEP = 0.0625$

Units: week

The time step for the simulation.

(17) Undeveloped Fruit Flies= INTEG (+multiplying-developing, INITIAL UNDEVELOPED FRUIT FLIES)

Units: fruit flies

The number of undeveloped fruit flies in the Fruit Fly system.

8. REVISED SOLUTION TO SYSTEM PROBLEM

When we first run the simulation, we find that the number of fruit flies becomes an increasingly negative value. However, we know that the **EFFECT OF CROWDING LOOKUP** was constructed so that the flies achieve equilibrium around 10,000 flies. This behavior is caused by the model's sensitivity to the **TIME STEP**, and a **TIME STEP** of 0.0625 is too large to accurately reflect the behavior of the stock. Below, Figure 13 reflects the behavior of the stock using a **TIME STEP** of 0.0625:

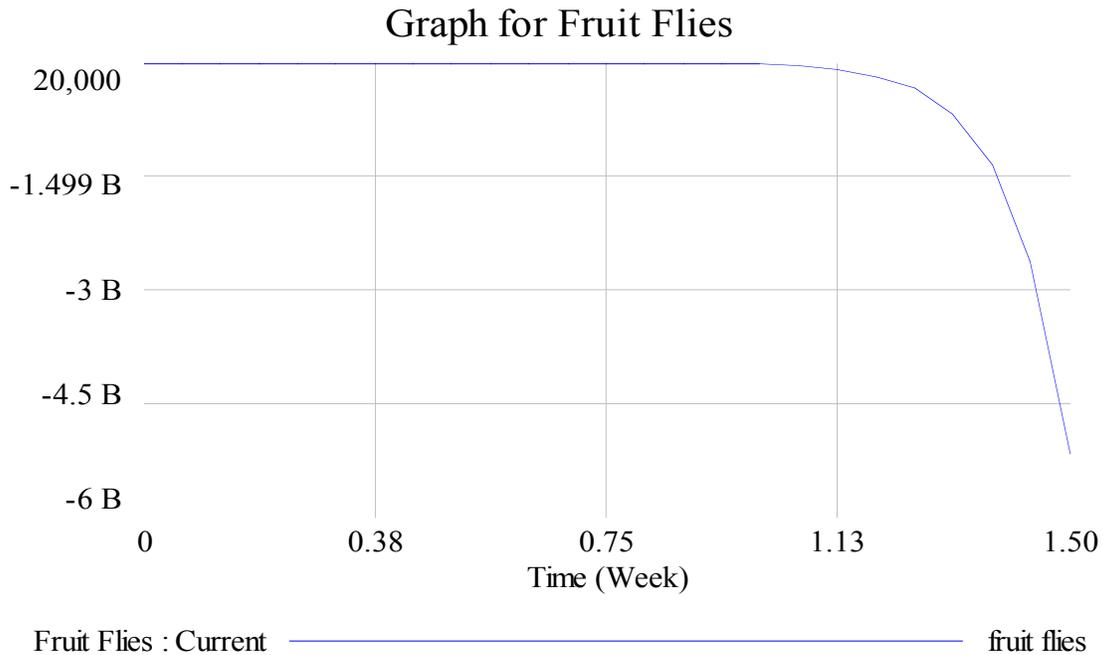


Figure 13. Graph for Fruit Flies with revised model using 0.0625 TIME STEP

Figure 14 presents a table of numbers that show particular values of **Time**, and for each value, the number of **Fruit Flies**, and the values for **developing**, **dying**, and **effect of crowding** are also shown.

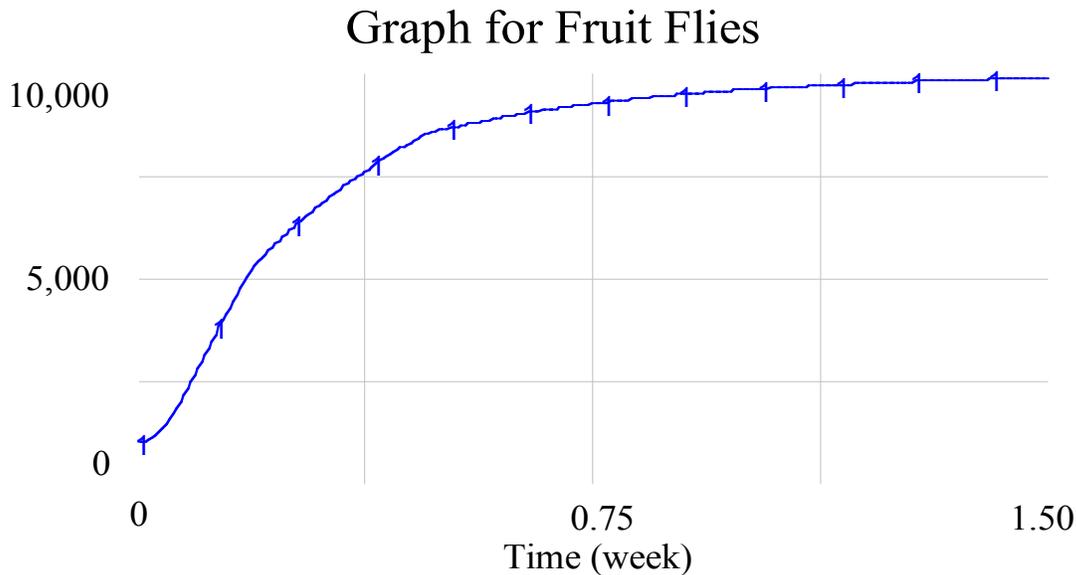
Time (Week)	0.375	0.4375	0.5	0.5625	0.625
Fruit Flies	6239.48	4933.75	16791	-417583	-361841
developing	242556	319805	362486	612091	-6.97933e+006
dying	263448	130089	7.31246e+006	-279780	-242433
effect of crowding	63.019	39.3541	650	1	1

Figure 14. Table of values showing behavior of **Fruit Flies**, **developing**, **dying**, and **effect of crowding** for particular **Times** with a **TIME STEP** of 0.0625.

From Figure 14, we see that at Time 0.5 week, the **effect of crowding** is 650, indicating that the **dying** outflow will be quite large, while the **developing** inflow remains about the same. The **dying** outflow is so large that at the next Time interval, 0.5625, the number of **Fruit Flies** becomes negative. The **effect of crowding** drops back to 1, and the negative multiplying of flies causes exponential growth in the negative direction. For additional information on the instability that can occur in a negative loop with a **TIME STEP** that is too large, consult Section 6.2 of *Principles of Systems*³.

Earlier, we found that the appropriate **TIME STEP** is 0.00390625. Figure 15 shows the results of the simulation using 0.00390625 for a **TIME STEP**.

After running the simulation, we found there will be approximately 10,000 fruit flies in the container after 1.5 weeks.



Fruit Flies : Current fruit flies
 Figure 15. Graph of Fruit Flies with adjusted model using 0.00390625 for a **TIME STEP**

³ Forrester, Jay W. (1968). *Principles of Systems*. Waltham, MA, Pegasus Communications.

The number of **Fruit Flies** begins to grow exponentially, then their growth is slowed as they asymptotically approach the 10,000 fruit flies value. This behavior is known as S-shaped growth.

9. FINAL COMMENTS

In this exercise, we saw that in order to properly formulate the model, certain assumptions had to be made about the maturation processes of the flies and the adequacy of the container. We had to assume that the flies took time to sexually develop. And, for a fish-tank sized container, we found that, given the normal multiplying and dying rates, we cannot expect that more than a certain amount—in this case, 10,000--flies can survive in equilibrium, as they will be competing for food and space. Further, we assumed that 1,000 fruit flies or less could live in the tank without experiencing the effects of crowding.

These assumptions were made with respect to the description of the system. Given that the container can hold only about 10,000 fruit flies at equilibrium, we saw the stock quickly approach this value and remain at it for the duration of the simulation.

We also had to examine the sensitivity of the model to the chosen **TIME STEP**. We found that for large inflows, the behavior or the stock can be very sensitive to the **TIME STEP**, and that by examining the doubling time of the stock, an appropriate **TIME STEP** can be chosen.

Notice that the formulation of this model involved an iterative approach: first, we modeled the system based solely on the description; then we proceeded to run the model; when we found that the results were not reasonable, we proceeded to add assumptions about the system; we modeled these assumptions; and finally retested the model with better results. This iterative approach is an integral part of modeling, and closely links the conceptualization, formulation, and testing techniques together.