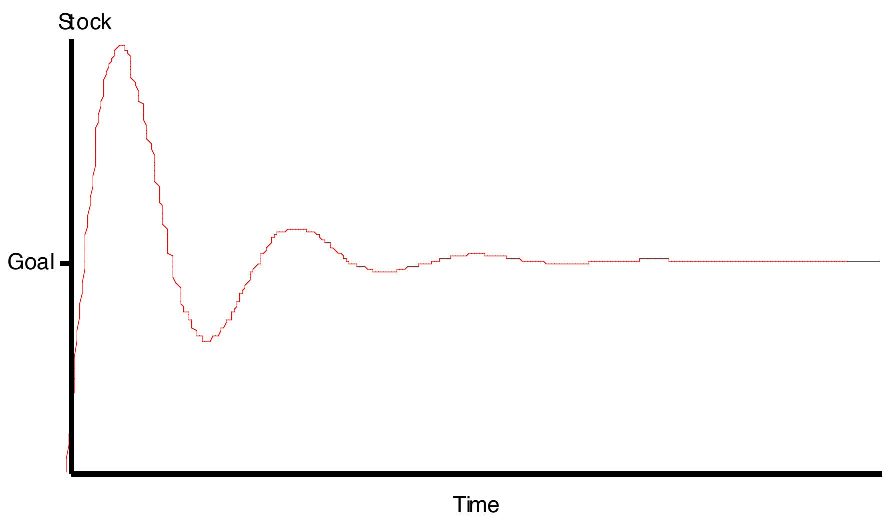
Generic Structures: Damped Oscillations



Prepared for the
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1. ABSTRACT

This paper is the third in a series dedicated to understanding oscillating behavior of systems. The first two papers in this series discussed sustained oscillations[1]. The reader should be familiar with both of these, *Generic Structures in Oscillating Systems I* and *Oscillating Systems II: Sustained Oscillations*, before continuing with this paper.

This paper examines the system structures that generate damped oscillations. Damped oscillation is a particular case of oscillation in which the amplitudes of oscillation decrease over time as the system approaches equilibrium. The paper motivates the study of damped oscillations with several examples and identifies the generic structure common to damped oscillating systems.

[1] See Celeste V. Chung, 1994. *Generic Structures in Oscillating Systems I* (D-4426); and Kevin Agastein, 1997, *Oscillating Systems II: Sustained Oscillations* (D-4602), both published by the System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology.

2. INTRODUCTION

After months of planning and preparing, the much heralded, brand-new Beanie Preschool has just opened. The past few months have been really hectic for the principal, Sandra. After making sure to hire only the best instructors, she worked diligently to convince parents to enroll their children in Beanie. Still, she was only able to convince about 90 couples to enroll their children.

The first day of school in September parents brought hesitant, but curious, children, and then stayed to talk a little with their respective teachers. The parents soon discovered that class sizes were much smaller than at other preschools. Satisfied that their children were going to receive enough attention, the parents left their children to experience their first day of learning and adventure.

Back in her office, Sandra was pleased with how the morning had gone. She took pride in helping to provide great education, so she was particular happy to see the satisfied smiles on the faces of the parents. Nevertheless, instinct told her that the school seemed a little empty. Being an experienced principal, Sandra fully understood that more children would be enrolled as word of the quality of Beanie spread through the community. Tending to her daily tasks, however, she soon forgot about her thoughts that first morning.

Throughout the first year of Beanie's existence, enrollment did increase. In fact, by the next September there were approximately twice as many students enrolled at Beanie as there had been at the start of the previous year. The first day of school the next year once again brought parents intent on meeting the instructors. The preschool was packed with parents and their children. However, Sandra, always a keen observer, noticed that as the parents left they didn't seem nearly as happy as they had the last year.

Sandra's observation had merit. Enrollment was soon on the decline. Though Sandra had expected that enrollment would eventually stop increasing, she was surprised to see enrollment fall. While the decline in enrollment disturbed her, she knew that Beanie still had the best instructors around. Parents were sure to realize the inherent quality of Beanie preschool and start enrolling their children again. Still, Sandra was a little anxious about the declining enrollment. Eventually, Sandra's patience rewarded her: enrollment again started rising.

Through the next couple of years enrollment exhibited similar behavior to what Sandra had experienced her first two years. However, each time enrollment peaked, the number of children in Beanie was not quite as high as in the previous peak. Eventually Sandra noticed enrollment stabilizing around a certain number of students. Being curious, she asked herself a couple of questions. Why did enrollment vary greatly just after Beanie opened? Why was the enrollment fairly stable now? Sandra knew she had made no attempt to influence enrollment. The quality and number of instructors at Beanie had remained essentially the same over the years.

Sandra remembered hearing about a field called System Dynamics a few years before. Frustrated at not being able to explain her enrollment turmoil, she decided to try to utilize system dynamics. After completing Road Maps and formulating a simple model, Sandra felt confident that she could now explain her enrollment woes.

3. The Preschool Model

When Sandra opened Beanie preschool, she had in mind a system like the one represented by the stock and flow structure in Figure 1.[2]

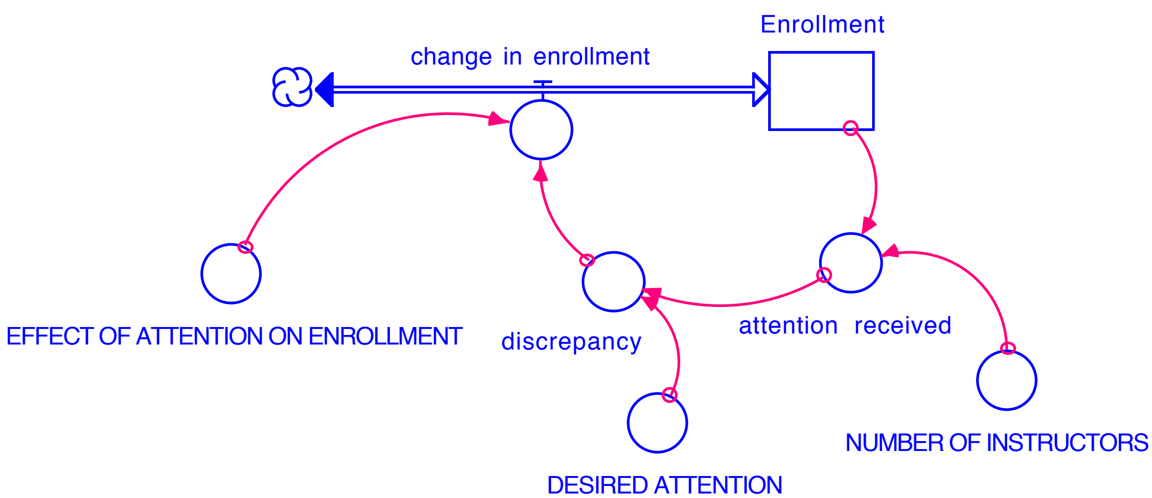

Figure 1: Goal-gap representation of preschool system

In Sandra's mental model, parents simply adjust the enrollment of the school by correcting for the difference between the attention that they desire for their children and the actual attention received from the instructors of Beanie preschool. Sandra expected parents to enroll about fifteen children every month for each unit of discrepancy. Sandra therefore expected to observe the behavior depicted in Figure 2, asymptotic growth of the enrollment of the preschool towards the desired enrollment level, determined by the desired level of attention for each student.

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[2] The equations for this model are not provided in this paper and building the model is left as an excersice to the reader.

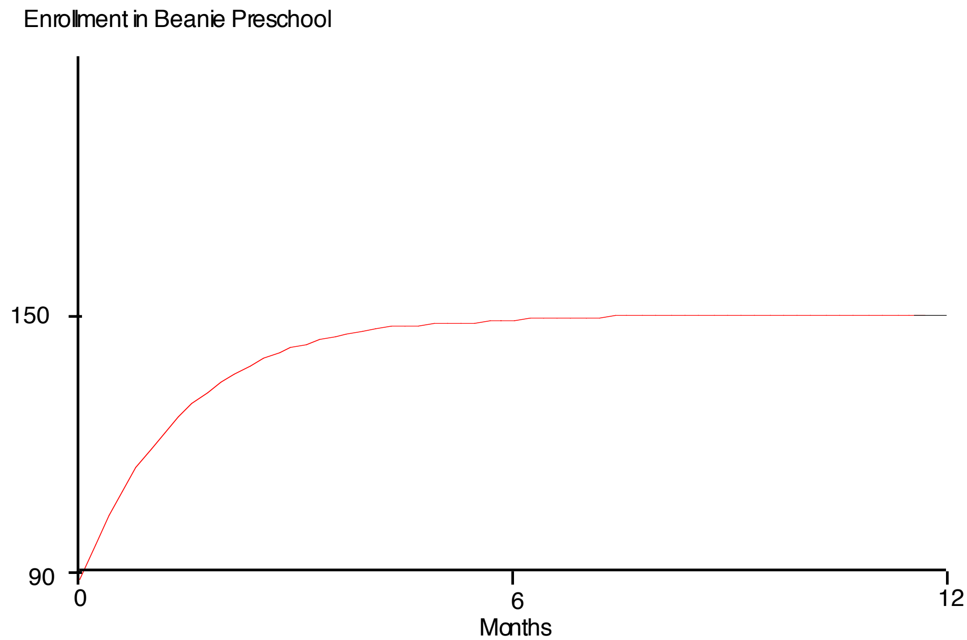

Figure 2: Reference mode for the behavior Sandra was expecting

In real life, however, the enrollment of students at Beanie preschool did not increase asymptotically towards the goal, but demonstrated damped oscillations around the goal. Beanie's enrollment stabilized at the desired enrollment not after nine months but after seventy-five months. Figure 3 shows a reference mode for the behavior that Sandra experienced in the enrollment of the preschool.

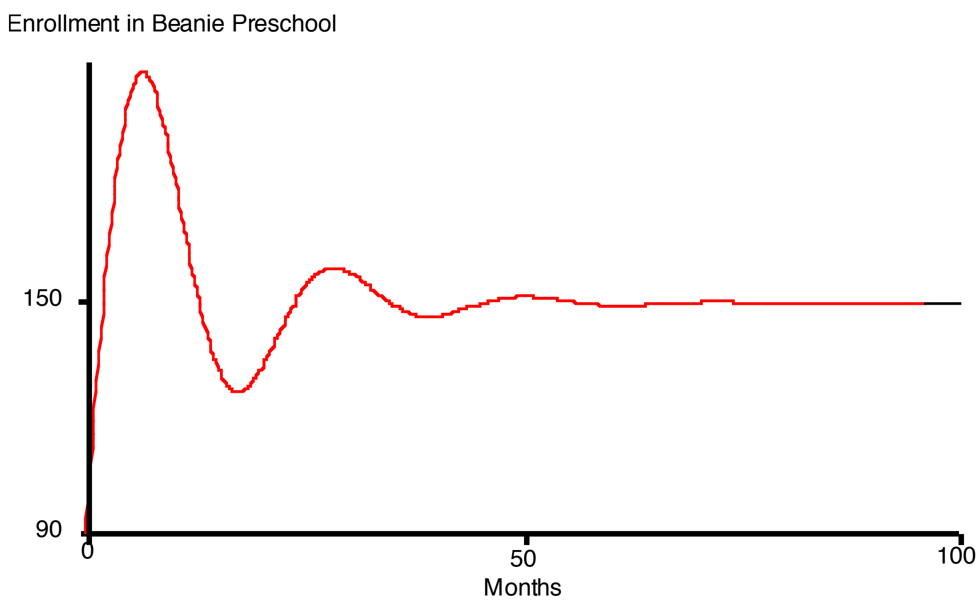

Figure 3: Reference mode for Sandra's experience with Beanie

To improve her mental model, Sandra needs to take into account all of the delays in the system. In addition to the response factor, "EFFECT OF ATTENTION OF ENROLLMENT," there is another delay: the time for parents to change their perceptions of the amount of attention their children are receiving. On average, it takes six months for parents to notice the amount of attention their children are actually receiving at Beanie.

In feedback systems, delays are not additive. If Sandra was to somehow combine "EFFECT OF ATTENTION ON ENROLLMENT" with the time it takes for parents to change their perceptions and simulate the model, she would again see asymptotic behavior—not the actual behavior she experienced with the preschool. To take into account the additional delay, Sandra needs to restructure her model.

The stock and flow diagram in Figure 4 below shows a model of the preschool system that takes into account the delay between the time when enrollment changes and the time when the parents adjust their perceptions of Beanie.

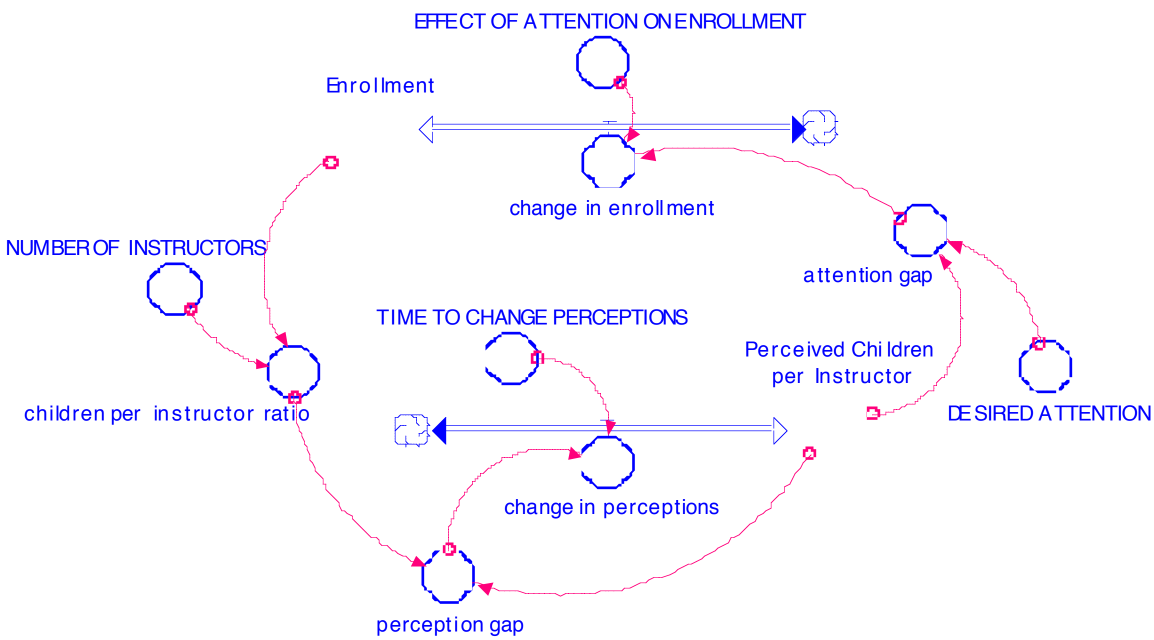

Figure 4: More complete representation of the preschool system

Perceived children per instructor is a stock that represents the attention that parents believe their children are receiving. When parents perceive their children to be receiving a large amount of attention, perceived children per instructor will be low. When parents perceive their children to be receiving a small amount of attention, perceived children per instructor will be high. The amount of attention children receive is inversely proportional to the children per instructor ratio.

The ratio of children to instructors is determined by the enrollment and the number of instructors at Beanie. However, because many parents work during the day, they do not perceive the ratio until an average of six months later. Therefore, Perceived Children per Instructor is the exponential smooth[3] of the actual ratio, represented by the "CHILDREN PER INSTRUCTOR RATIO."

In order to determine whether to enroll their children in Beanie preschool, parents compare their perceptions of attention received with the amount of attention they desire for their children. On average, parents would like to have five children for every instructor. When there is a discrepancy between this level of desired attention and the perceived attention, parents enroll or remove their children accordingly. We assume that a change in the perception gap of one child per instructor results in a change of fifteen children per month, on average, in the enrollment of Beanie.

Figure 5 shows the behavior produced by the improved preschool model.

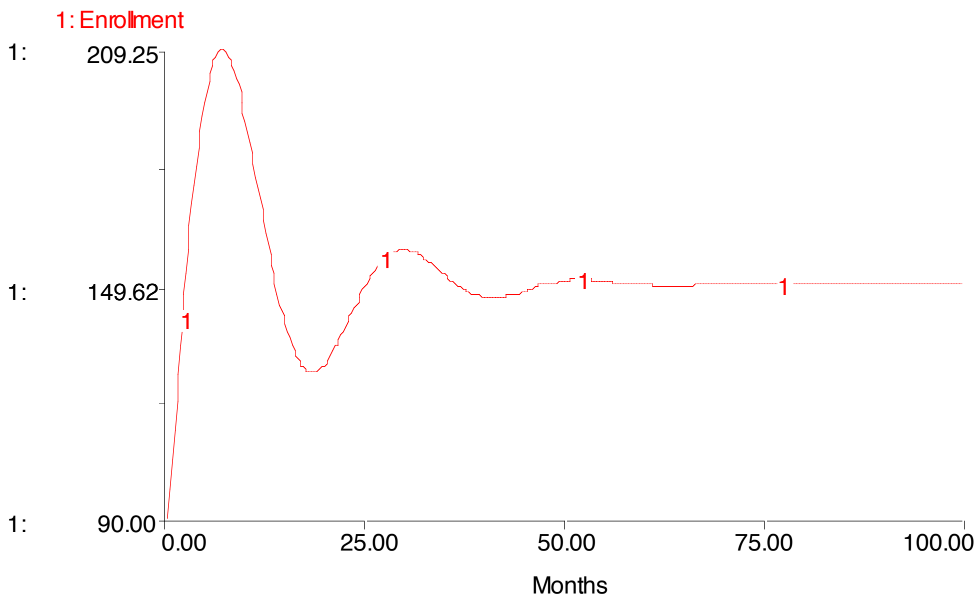

Figure 5: Damped oscillations produced by the complete representation of the preschool model

[3] See Kevin Stange, 1998. *Generic Structures: Exponential Smoothing* (D-4782), published by the System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, November 10, 22pp.

This model is a better representation of the system and shows the oscillatory behavior actually observed, which is caused by the relative delays. Parents respond to changes in their perceptions much faster than they change their perceptions.[4] When the perceived attention is too high or too low, the parents quickly change the enrollment of Beanie. However, because the parents are fairly busy, they do not begin to see the effects of their actions until about six months later. In the meantime, parents perceive changes in attention as a result of prior actions.

The next section takes what we have learned from the preschool model and attempts to generalize it to a generic structure applicable to many systems.

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[4] Relative delays will be discussed further in section 7.1.3.

4. THE GENERIC STRUCTURE

4.1 Sustained Oscillation

Negative feedback loops in second (or higher) order systems are capable of producing oscillatory behavior. Figure 8 displays the generic structure for sustained oscillation, previously discussed in Road Maps.[5] The structure consists of one feedback loop containing two stocks, where the value of each stock determines the value of the flow into the other stock. "PARAMETER ONE" and "PARAMETER TWO" dictate how the value of each stock translates into the value of the flow into the other stock. In order for the feedback loop to be negative, and thus capable of producing oscillatory behavior, the parameters must be of opposite signs.[6] That is, if movement in the value of "Stock One" causes "change in stock two" to move in the opposite direction as "Stock One", then movement in "Stock Two" must cause "change in stock one" to move in the same direction as "Stock Two."[7]

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[5] See footnote 1.

[6] For a discussion of the different (oscillatory and non-oscillatory) behaviors this structure is capable of producing, as determined by combinations of parameter polarities and stock initial values, see Leslie Martin, 1998. *Second Order Systems* (D-4731), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28, 28 pp.

[7] It should be noted that "PARAMETER ONE" and "PARAMETER TWO" aggregate all factors that effect one stock's influence on the change in the other. A model of a specific system could likely contain several converters and/or constants in place of the generic parameters. The important point to realize is that the two stocks must have opposite influences on the rates of change of each other to produce oscillation.

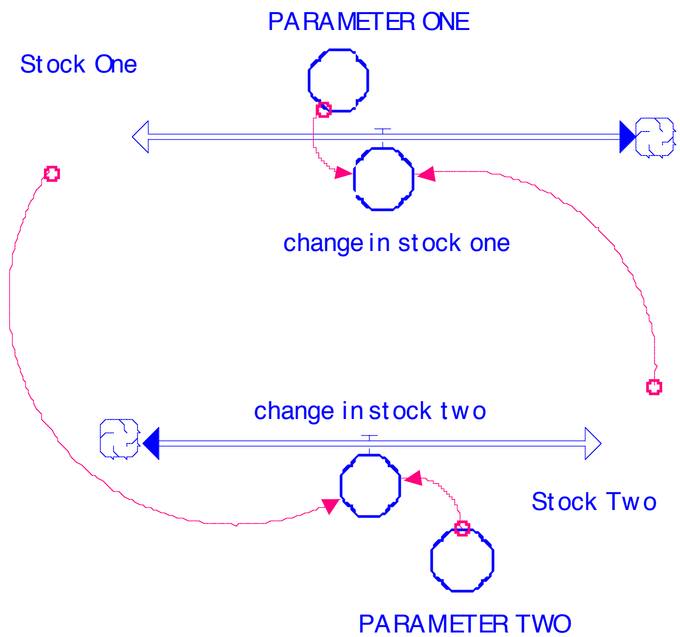

Figure 6: Generic structure for sustained oscillations

The generic structure in Figure 6 produces sustained oscillation as displayed in Figure 7. The negative feedback loop causes each stock to seek a goal that is determined by the model structure. This goal value of each stock is implicitly determined by the aggregate parameters, "PARAMETER ONE" and "PARAMETER TWO". As long as Stock 1 is above its neutral value, it pushes Stock 2 downward. The minimum value of Stock 2 occurs as Stock 1 returns to its neutral value and at that point reverses the influence on Stock 2. Stock 2 has the opposite effect on Stock 1. When Stock 2 is above neutral, it is pushing Stock 1 upward. This behavior is caused by the negative feedback loop in the system. As can be seen in Figure 7, when "Stock One" increases the "change in stock two" decreases and as "Stock Two" decreases the "change in stock one" decreases. This behavior continues indefinitely, causing each stock to oscillate with constant amplitude around its respective goal. For a rigorous study of sustained oscillation refer to previous chapters in Road Maps.[8]

[8] See footnote 1 and Leslie Martin, 1998. *Second Order Systems* (D-4731), System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusetts Institute of Technology, May 28.

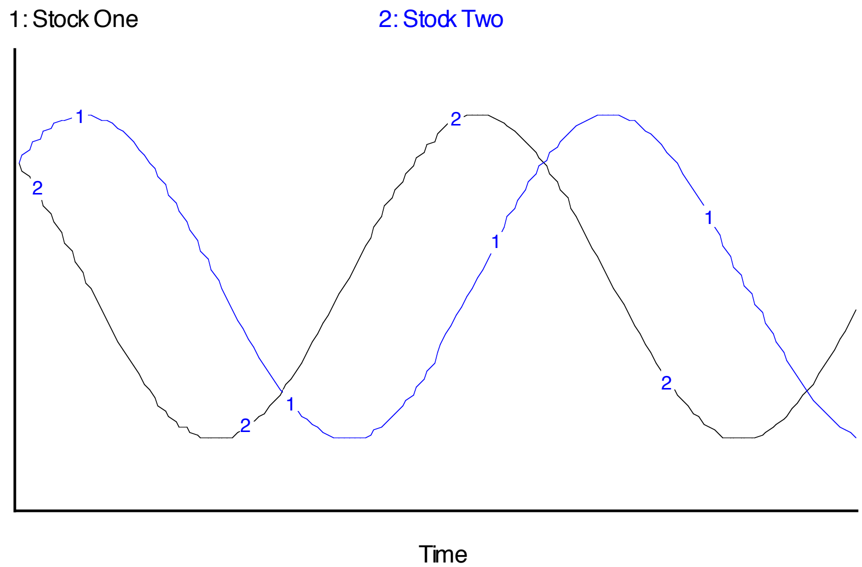

Figure 7: Sustained oscillations

4.2 Damped Oscillation

Compare the sustained oscillatory behavior in Figure 7 with damped oscillatory behavior shown in Figure 8. Figure 8 shows damped oscillation, similar to behavior produced by the preschool model discussed earlier.

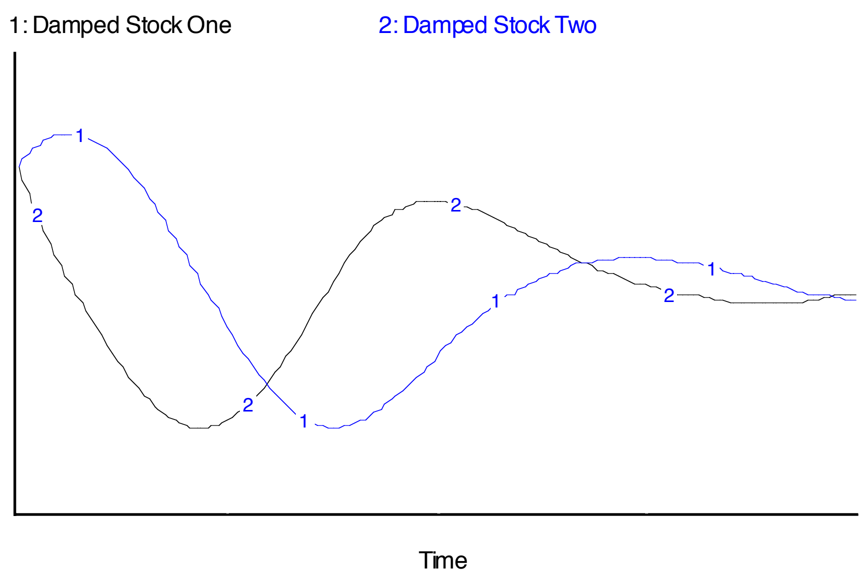

Figure 8: Damped oscillations

In the case of damped oscillations, the amplitude of each subsequent oscillation decreases, and each stock eventually settles to an equilibrium value. What causes each stock to settle and eventually stop oscillating?[9] The answer lies in the difference between the generic structures that produce sustained and damped oscillations. Figure 9 shows the simplest generic structure capable of producing damped oscillations.

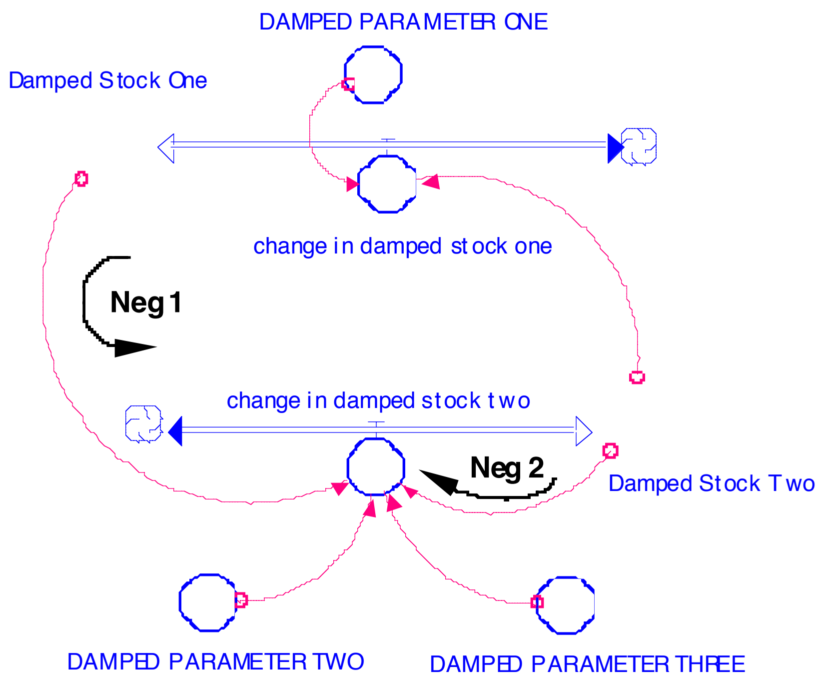

Figure 9: Generic structure for damped oscillations

At its heart, the damped oscillation generic structure consists of the same negative feedback loop found in the sustained oscillation generic structure, here labeled “Neg 1.” “Neg 1” shows that “Damped Stock One” drives “change in damped stock two,” while “Damped Stock Two” drives “change in damped stock one.” With an appropriate combination of parameter values and stock initial conditions, this feedback loop will cause system oscillations similar to the sustained oscillation generic structure discussed earlier.[10]

The damped oscillation generic structure, however, also contains an additional negative feedback loop, labeled in Figure 9 as “Neg 2,” relating "change in damped stock two” to "Damped Stock Two" through “DAMPED PARAMETER THREE.” That is, changes in “Damped Stock Two” are determined by the value of “Damped Stock Two” in

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[9] Though technically the stocks will never stop oscillating, eventually the amplitude of oscillation will become so small that the value of the stocks can be approximated as constant.

[10] See footnote 1.

addition to the value of “Damped Stock One.” This can be seen in the equation for “change in damped stock two.”

change in damped stock two =
*Damped Stock One * DAMPED PARAMETER TWO*
*+ Damped Stock Two * DAMPED PARAMETER THREE*

“DAMPED PARAMETER THREE” dictates how “Damped Stock Two” effects it’s own rate of change. To enable damped oscillatory behavior, this feedback loop must be negative, necessitating a negative “DAMPED PARAMETER THREE.”[11]

Damped Oscillation from Negative Feedback

Why does an additional negative feedback loop dampen the oscillations? Basically, the new loop increases the rate of change of “Stock Two” toward the equilibrium value, which decreases the magnitude of oscillations away from the equilibrium. Examine the equation for “change in damped stock two” again. The last term in the equation, “Damped Stock Two * DAMPED PARAMETER THREE,” represents this additional feedback loop. Since “DAMPED PARAMETER THREE” is negative, this last term causes “change in damped stock two” to decrease (become less positive or more negative) whenever “Damped Stock Two” is above its goal, and to increase (become more positive or less negative) whenever “Damped Stock Two” is below its goal. Therefore, the rate of change of “Damped Stock Two” towards its goal is always greater with the addition of the negative loop “Neg 2,” than without. Similarly, its rate of movement away from its goal is always slower with the additional negative feedback loop, than without.

As a result, “Damped Stock Two” moves closer to its goal with every oscillation, and thus settles faster to its equilibrium. Since the rate of change of “Damped Stock One” is determined by the level of “Damped Stock Two,” the magnitude of oscillations of “Damped Stock One” will also decrease with time.

Another way to think about the effect of the additional negative feedback loop is thinking how “Damped Stock Two” can approach its equilibrium level. In a sustained structure, the only way that “Stock Two” could return to its goal is by further displacing “Damped Stock One,” which in turn causes “Damped Stock Two” to change away from

[11] Again, this parameter is highly aggregated. The value of “Damped Stock Two” will often pass through several converters before eventually feeding back to determine “change in damped stock two.” As long as this feedback is negative (movement in “Damped Stock Two” results in opposite movement of “change in damped stock two”), the structure will produce damped oscillatory behavior.

its equilibrium level. In a damped oscillation structure, “Stock Two” also makes use of the additional negative feedback loop to approach its goal. Due to this additional loop, “Damped Stock Two” affects “Damped Stock One” less adversely when attempting to return to its equilibrium.

Refer back to the preschool enrollment model structure in Figure 4. The structure has two negative feedback loops. The large negative feedback loop through both “Enrollment” and “Perceived Children per Instructor” produces the oscillation. The minor negative feedback loop causes the amplitude of oscillations to decrease with time because the perception of children per instructor approaches the actual value with every successive oscillation, thus closing the "perception gap."

5. EXAMPLE: PENDULUM MODEL

The first generic structure paper on sustained oscillations in Road Maps presented the simplified pendulum model shown in Figure 10.[12]

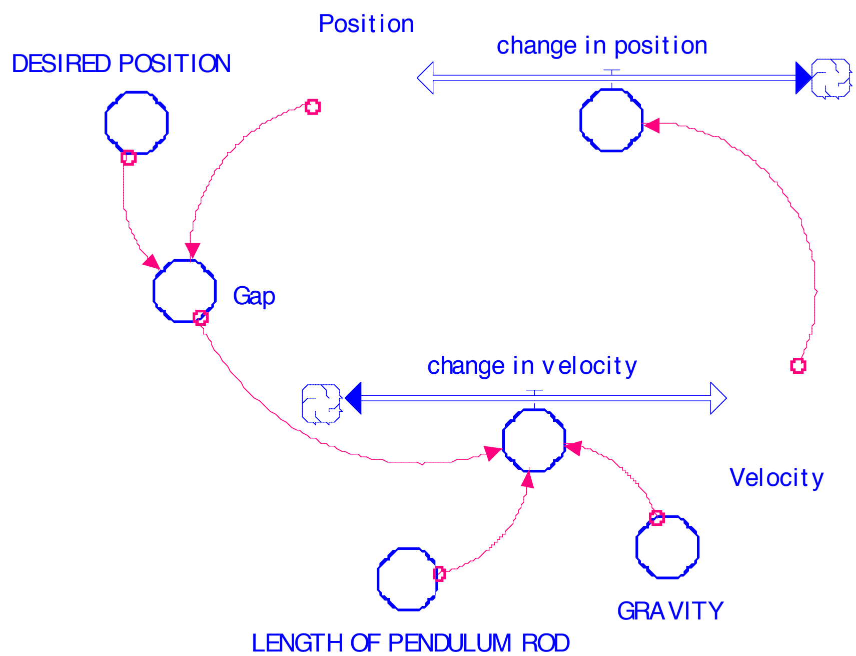

Figure 10: Pendulum model

When released from any point other than its desired position, the pendulum demonstrated sustained oscillatory behavior, as shown in Figure 11.

[12] See Celeste V. Chung, 1994. *Generic Structures in Oscillating Systems I* (D-4426) published by the System Dynamics in Education Project, System Dynamics Group, Sloan School of Management, Massachusettes Institute of Technology.

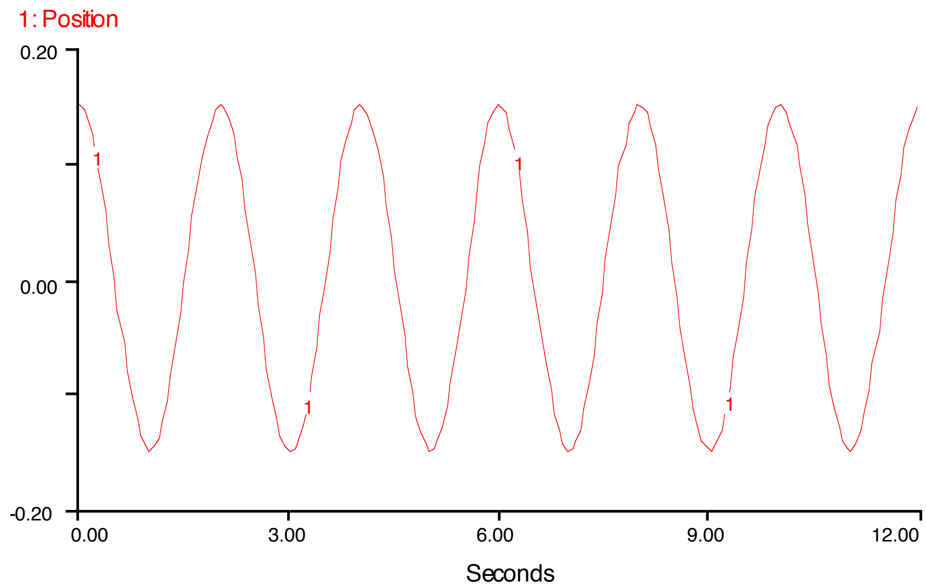

Figure 11: Sustained oscillations produced by pendulum

Several simplifying assumptions were made in this model. One major simplification is the assumption that the pendulum is swinging from a frictionless pivot in a vacuum. In reality, air resistance and friction between the pendulum and its pivot would slow the pendulum motion. These resistive forces are proportional to the velocity of the pendulum. Therefore, friction creates an additional negative feedback loop between "Velocity" and "change in velocity." Large pendulum velocity produces large frictional forces, resulting in a decrease in velocity. A pendulum model incorporating the effects of friction is shown in Figure 12. In the more realistic model, "change in velocity" is also determined by frictional forces in addition to the gravitational forces present in the original model.

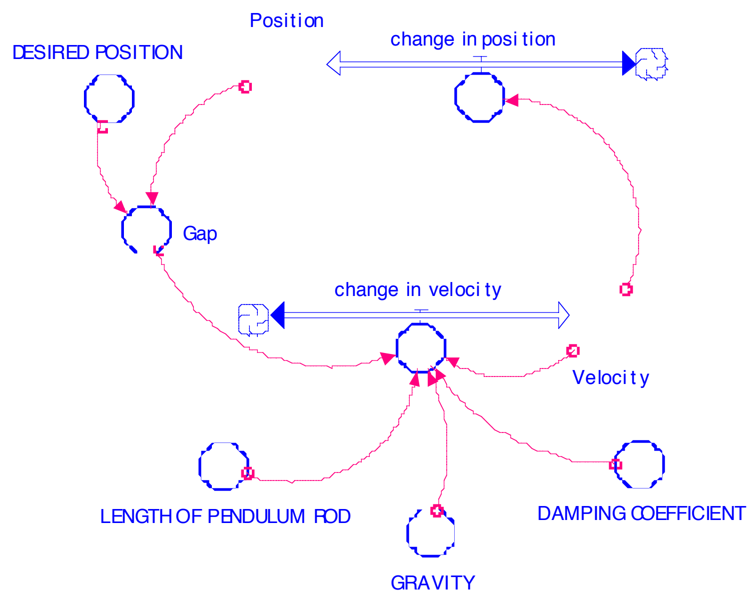

Figure 12: Pendulum model with friction

As Figure 13 shows, the additional negative feedback loop representing frictional effects causes the pendulum to experience damped oscillatory behavior. Intuitively, when the arm of the pendulum is released above its desired position, the pendulum swings back and forth. Due to the frictional forces, some of the momentum of the swinging pendulum is lost every cycle, reducing the height obtained by subsequent swings. Eventually, the pendulum will settle at its "desired" vertical position.

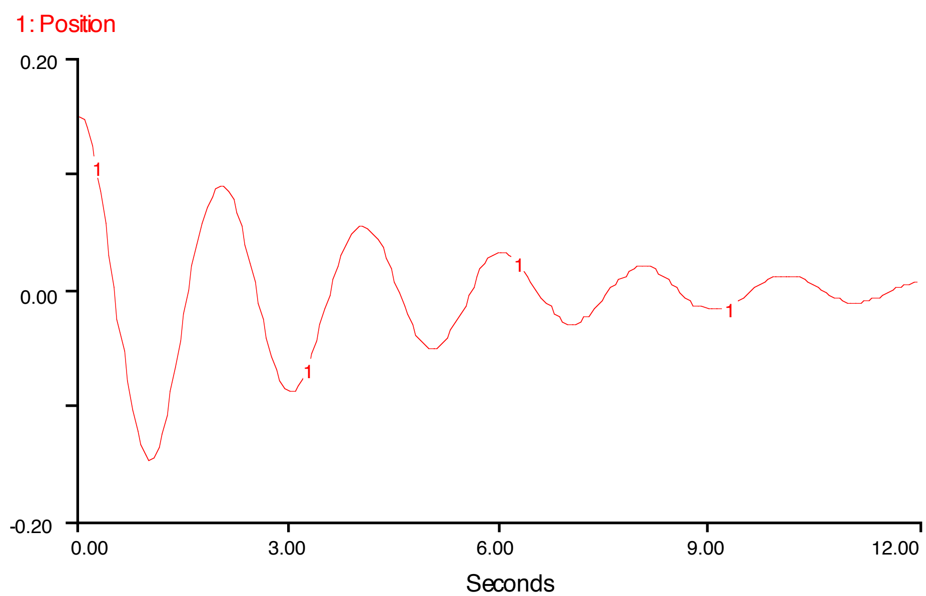

Figure 13: Damped oscillations produced by pendulum with friction

6. EXAMPLE: INVENTORY-WORKFORCE MODEL

Now recall the second example of sustained oscillations presented in *Generic Structures in Oscillating Systems I.*[13] The model describes a manager of a production facility who is having a hard time stabilizing inventory and employment within her company. Figure 14 shows the method currently used by the manager to keep inventory at her desired level.

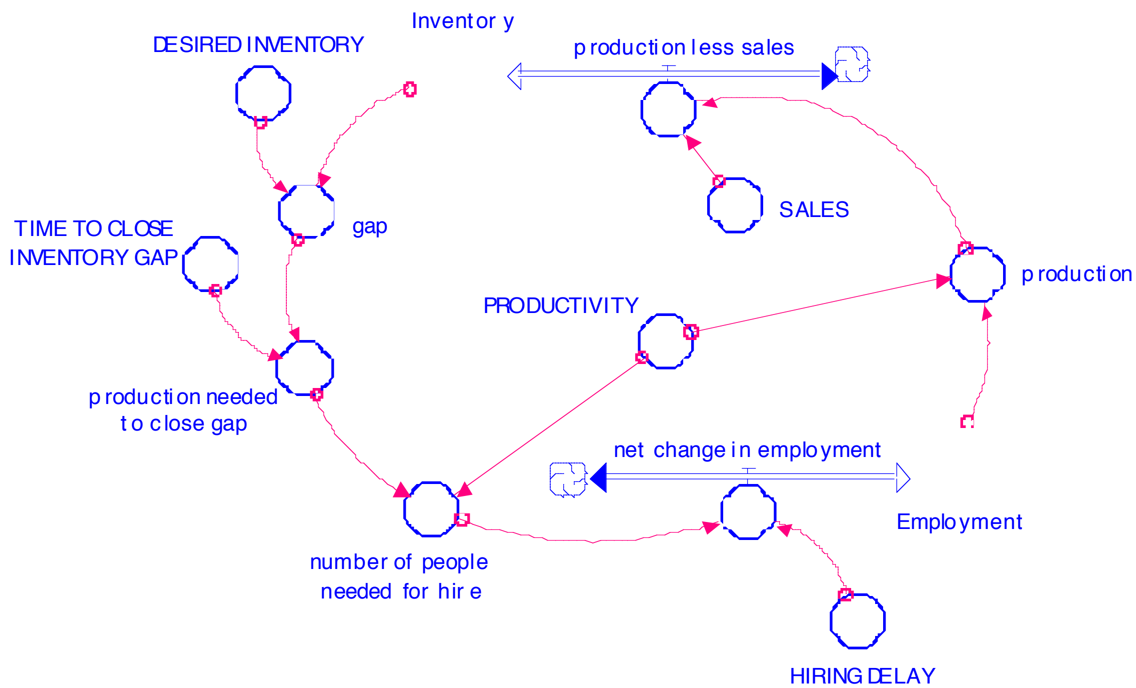

Figure 14: Inventory-Workforce model

Whenever inventory is below “DESIRED INVENTORY” (20,000 widgets), the company hires the number of workers needed to increase production enough to fill the inventory gap at a desired rate. However, by the time the inventory gap is closed, the company now has too many workers and inventory continues to increase beyond its desired level. As a result of the shortsighted hiring practices, the company is never able to have the proper number of workers at the right time, causing never-ending swings between over and under production. Figure 15 shows the sustained oscillatory behavior produced by this model when inventory is initially greater than “DESIRED INVENTORY.”

[13] See footnote 1.

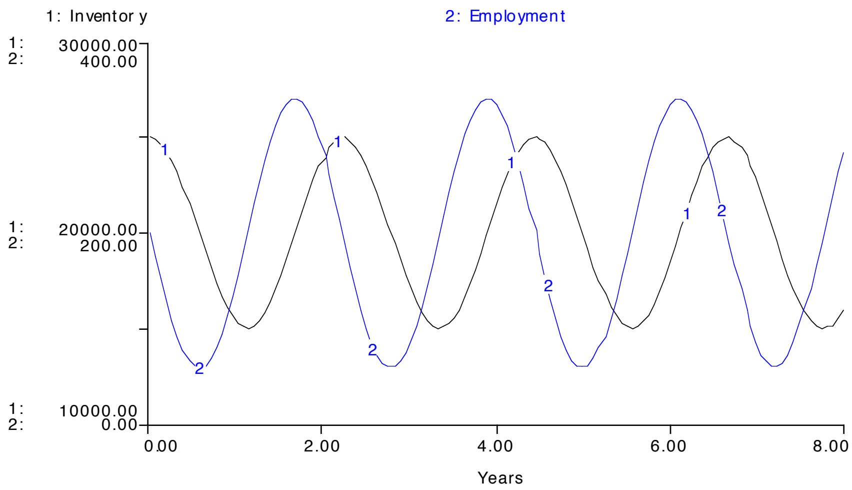

Figure 15: Sustained oscillations produced by Inventory-Workforce model

What new policies should be adopted by the manager to help her better control her inventory and workforce instability? The damped oscillation generic structure suggests that adding an additional negative feedback loop around one of the stocks in the sustained oscillatory structure might help dampen the oscillations. One such policy is shown in Figure 16.

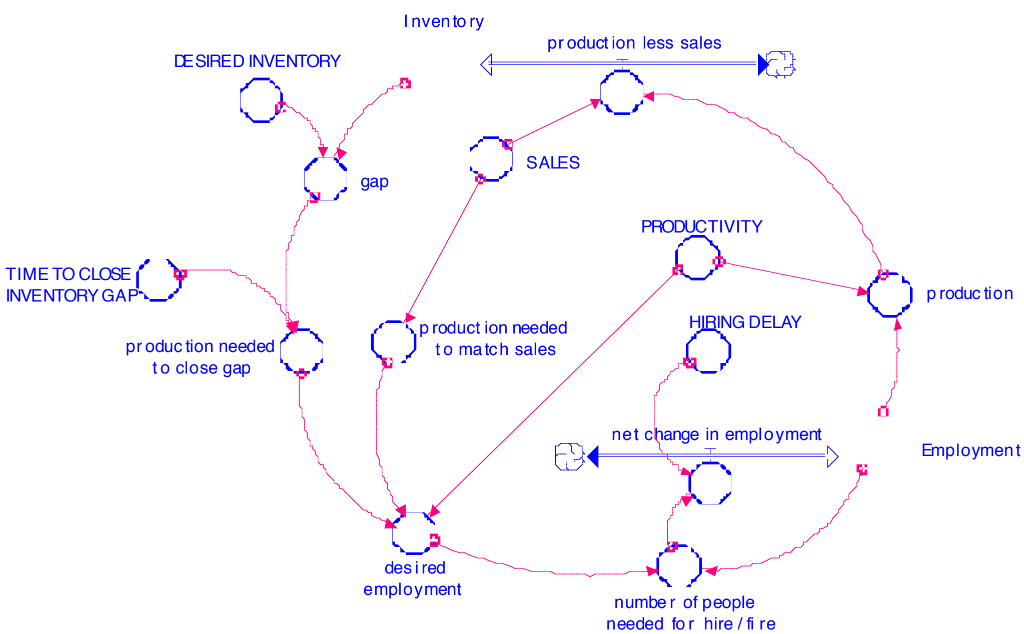

Figure 16: Inventory-Workforce model with additional negative feedback loop

In the revised model, the plant manager now adds the number of workers needed in the long run to match sales and the number of workers needed to close the current inventory gap, to determine "desired employment." She then compares this total to the current employment to determine how many people to hire or fire. Previously, the manager was narrowly focused on closing the inventory gap when making hiring/firing decisions. This caused her not to consider how many workers she would ultimately need in order to match her production to the incoming sales, nor did she account for the number of workers she currently had. As a result, she proceeded to release too many workers (causing underproduction) and then hire too many workers (causing overproduction) that produced sustained oscillations. Figure 17 shows the damped oscillatory behavior, which results from the suggested improvement in the inventory-workforce management policy.

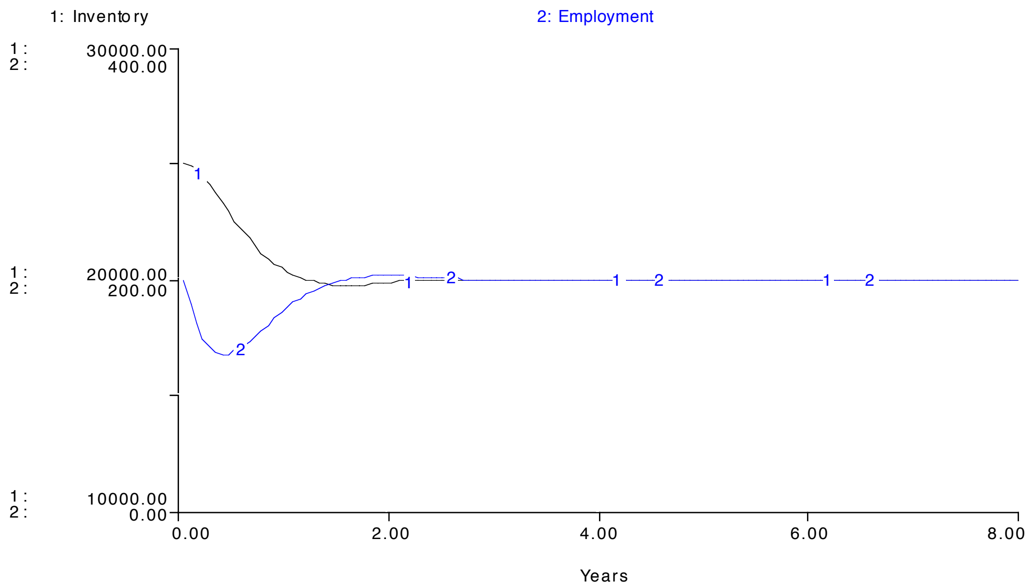

Figure 17: Damped oscillations produced by revised Inventory-Workforce model

Just like before, the manager releases some workers in order to decrease production and reduce her inventory. However, she realizes that in order not to decrease her inventory too much below the DESIRED INVENTORY, she will have to begin hiring workers back even while she still has excess inventory. She knows how many workers are needed in order to match sales once the inventory gap is closed, and she begins to rehire up to that equilibrium level of 200 workers. The damped oscillatory behavior is seen in Figure 17, as the stocks over and undershoot their goal several times before finally coming to rest. The additional negative feedback loop is dominant and thus causes the stocks to quickly attain their equilibrium values.

7. EXERCISES

7.1 Shower Temperature

Todd enters the shower and starts to turn the knob that controls the shower water temperature.

Initially the water is very cold. Todd continues turning the shower knob until the water reaches a comfortable temperature. The water temperature, however, continues to increase. Todd yelps as scalding water splashes onto his back. He jumps from the shower and turns the shower knob in the opposite direction to cool the water. He continues turning the knob until the water cools to his desired temperature. Satisfied with the comfortable temperature, he steps back into the shower. The water, however, continues to cool and soon becomes too cold. Annoyed, Todd again adjusts the shower knob to increases the water temperature. This time, he is very cautious to not increase the temperature too much. The temperature of the water rises until it reaches a comfortable level. Todd relaxes, again, too soon. The temperature of the water continues to increase. Again Todd responds by turning the knob to decrease the temperature.

This cycle continues a few more times until the temperature of the shower water finally stabilizes at a comfortable temperature. In this exercise, you will build a model representing Todd's shower experience to help him understand why he had a hard time obtaining the shower temperature he desired. You will also examine some changes to the system that will enable Todd to control the shower temperature more effectively.

7.1.1 Todd's Mental Model

When Todd first entered the shower in his hotel bathroom in Madrid, he had in mind a system like the one represented by the stock and flow structure in Figure 18.

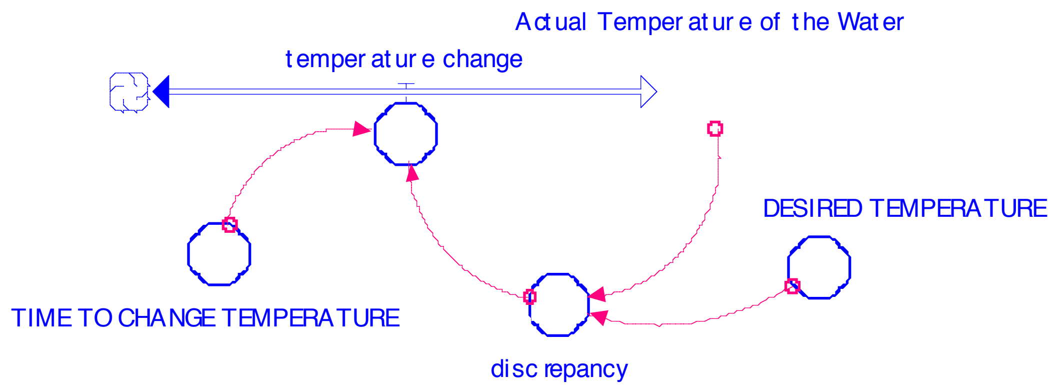

Figure 18: Todd's mental model of shower system

In his mental model, Todd simply feels a discrepancy between the temperature of the shower water and the temperature he desires. Todd then changes the shower temperature to match his "DESIRED TEMPERATURE." Todd needs time to sense a discrepancy in water temperature and to change the temperature accordingly, so he will not instantly adjust temperature. "TIME TO CHANGE TEMPERATURE" determines how quickly Todd adjusts the water to the desired temperature. Todd expected to observe the behavior depicted in Figure 19, asymptotic growth of the temperature of the water towards the temperature that he desired.

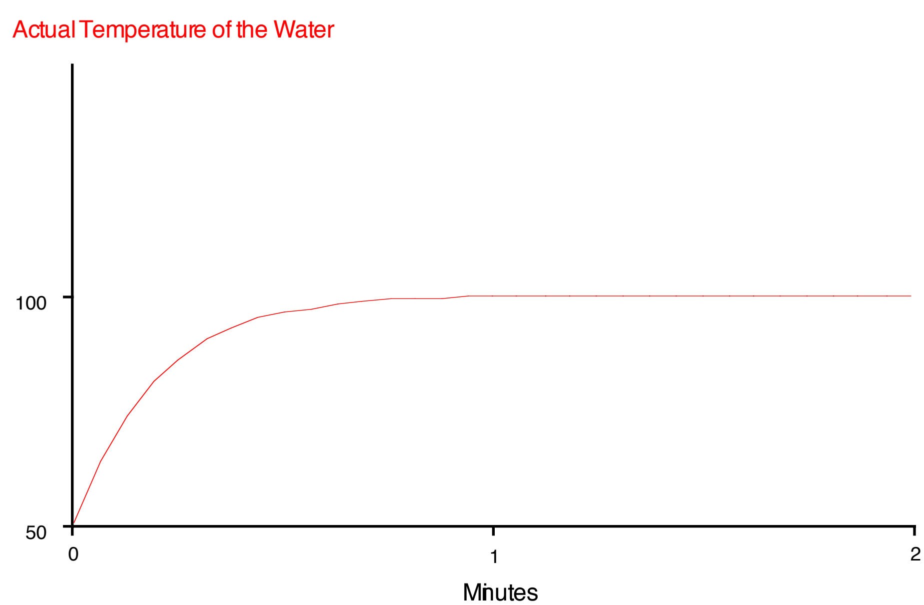

Figure 19: Reference mode for the behavior Todd was expecting

In real life, however, the temperature of the water in Todd's shower did not increase asymptotically towards his goal, but demonstrated damped oscillations around his desired temperature. The temperature of the water stabilized at Todd's desired water temperature not after one minute but after about six minutes. Figure 20 shows a reference mode for the behavior Todd actually experienced in the shower.

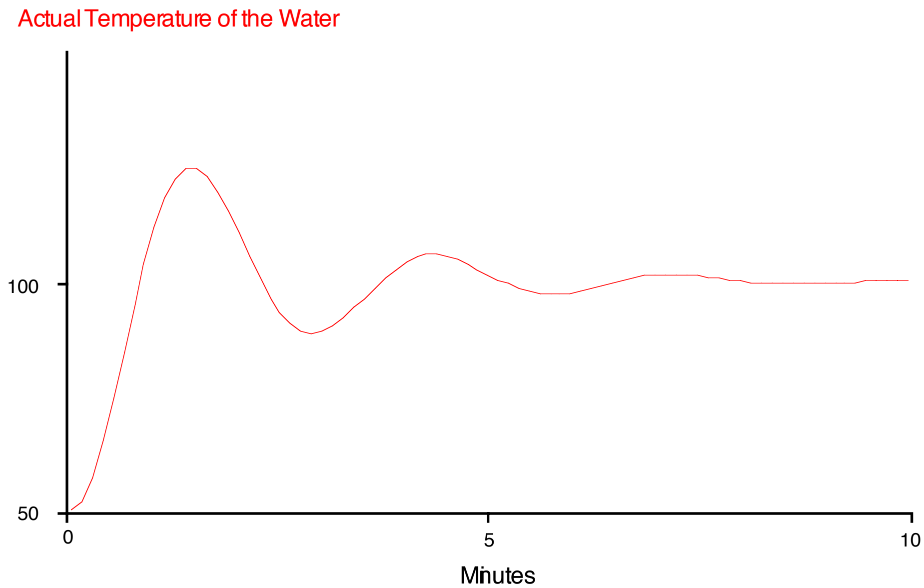

Figure 20: Reference mode for Todd's experience in the shower

What caused Todd's mental model to deceive him? Knowledge of the damped oscillation generic structure will help to investigate this question.

7.1.2 Questioning the Assumptions

An important first step in understanding why Todd's mental model does not produce the behavior he actually experienced, is to question the assumptions his model makes about the real system. Questioning the assumptions often leads a modeler to uncover important system structure that was omitted from the model. This model structure is often vital to producing the observed system behavior. Refer to the system description and Figure 18 to answer the following questions.

1. In the real system, what physical action does Todd take if the water temperature is not what he desires?

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2. Is this action explicitly represented in Todd's mental model?

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3. What does Todd's mental model assume about his ability to change water temperature?

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4. Think about your shower at home. What delays are present (on the part of you and on the part of your shower) that might cause shower temperature to oscillate? Hint: Think about how you might be delayed in responding to temperature and how the shower might be delayed in responding to your actions.

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According to the system description, whenever Todd senses that water temperature is not what he wants, he moves the shower knob until he obtains his desired temperature. This specific action is left out of Todd's mental model. According to his mental model, Todd has the ability to change temperature directly when he senses a temperature discrepancy.

To improve his mental model, Todd needs to take into account all delays present in the system. Todd's reaction time is one delay. The other major delay is the time it takes the shower to actually change temperature once Todd has acted upon it. Todd's mental model essentially lumped both of these delays together into the parameter called "TIME TO CHANGE TEMPERATURE." Explicitly representing both of these delays more accurately describes the forces present in the real system and is necessary to make the model capable of exhibiting the observed damped oscillations.

7.1.3 Revising the Model

Adapt Todd's mental model to be a better representation of the actual system. Keep the following points in mind when building the model.

〈 In order to produce damped oscillations, a model must contain at least two stocks connected by a negative feedback loop (producing oscillations) and an additional minor negative loop causing the oscillations to dampen.

〈 Todd cannot change the temperature of the water directly. Instead, he changes the position of the shower knob when he senses a discrepancy.

〈 The temperature of the water does not change instantly when Todd moves the knob. Instead it smoothly approaches the temperature dictated by the position of the shower knob.

〈 Assume the shower temperature ranges from 50 to 150 degrees, and that each position of the knob corresponds directly to an ultimate temperature within that range.

〈 Todd's desired temperature is 100 degrees.

Compare your model to the one depicted in Figure 21. Model equations for the suggested model can be found in the Appendix.

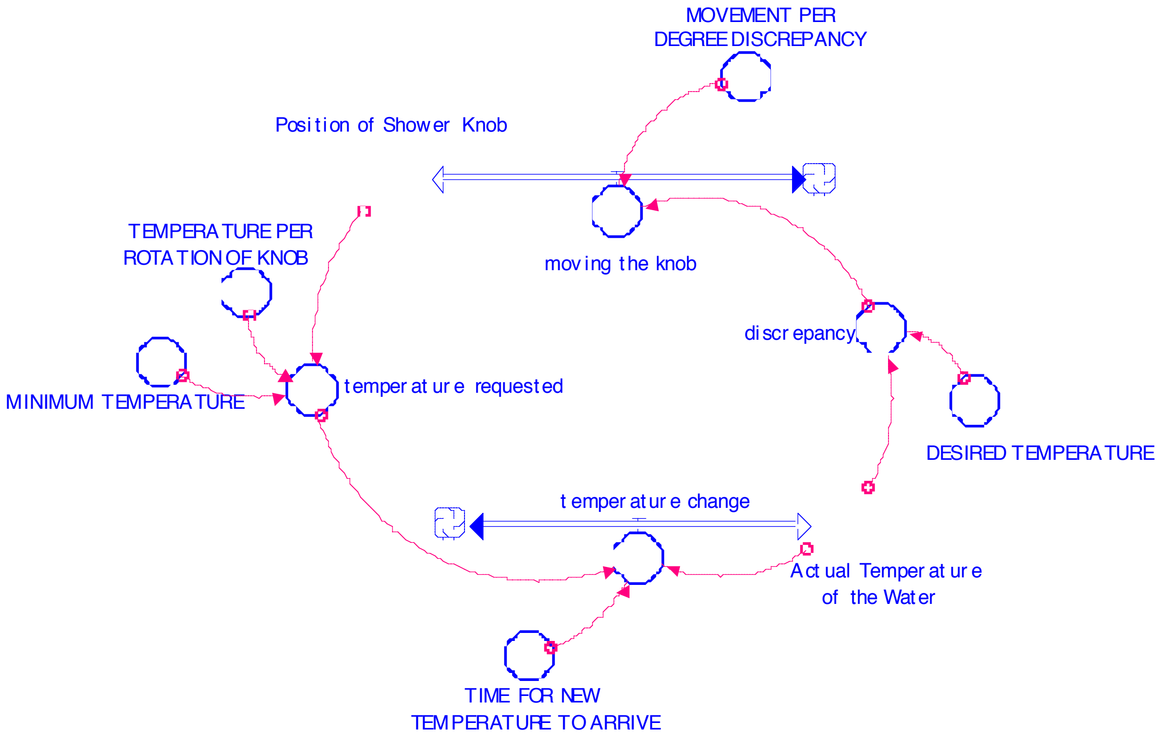

Figure 21: Revised shower system model

The revised shower model has two stocks, "Actual Temperature of the Water" and "Position of Shower Knob." Similar to the oscillating models seen earlier, each stock in this model determines the rate of change of the other. Additionally, the shower temperature stock also influences its own rate of change with a negative goal-seeking loop. This second loop causes the oscillations to dampen.

This model represents both major delays present in the shower system. First, Todd doesn't close the temperature gap instantly. He responds to the discrepancy at a rate determined by "MOVEMENT PER DEGREE DISCREPANCY." This parameter dictates how quickly Todd changes the position of the knob for every degree change in water he wants. Each position of the knob corresponds directly to a shower temperature (here called "temperature requested") ranging from 50 to 150 degrees. Without any other actions by Todd, the shower would eventually settle at "temperature requested." The rate at which "Actual Temperature of the Water" approaches "temperature requested" is determined by the "TIME FOR NEW TEMPERATURE TO ARRIVE." This parameter represents the physical delay in the shower response to a change in the position of the knob. [14] Figure 22 shows the damped oscillatory behavior produced by the revised shower model.

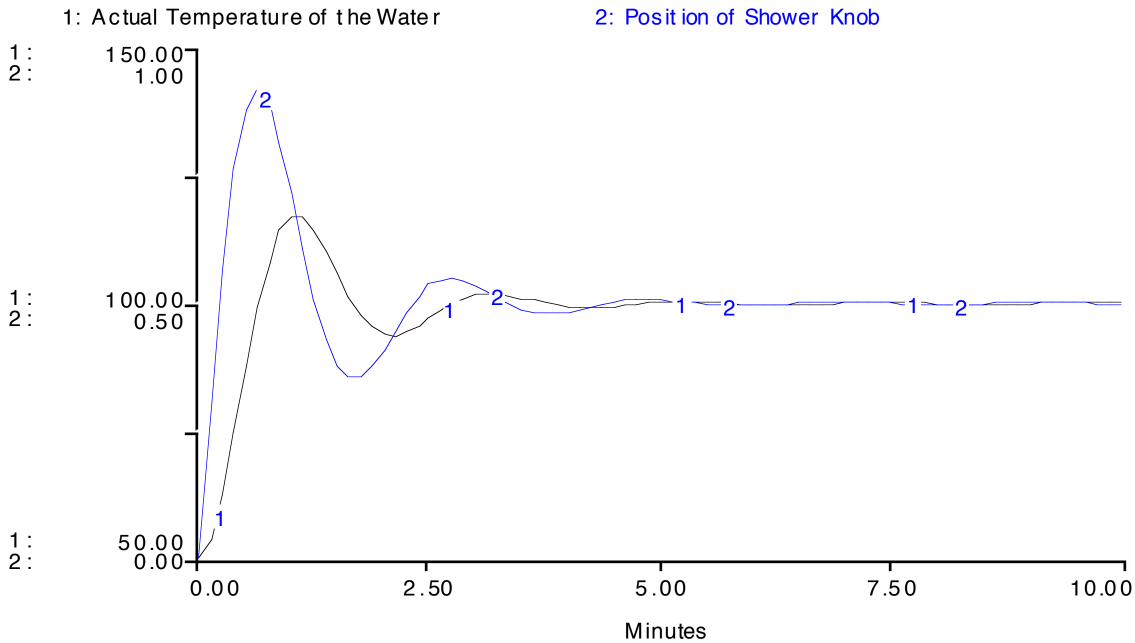

Figure 22: Damped oscillations produced by revised shower model

The system oscillates because of the relative delays. Todd responds to the water temperature much faster than the water temperature responds to Todd. When the water is too hot or too cold, Todd quickly reacts and adjusts the shower knob. However, Todd only feel water at the temperature dictated by the position of the knob after a lengthy delay.

7.1.4 Policy Recommendations

Satisfied that the revised model adequately describes the major forces driving shower system behavior, Todd might wonder what system changes can be made which

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[14] Note that this temperature changing model highly aggregates several physical processes. A more accurate representation of the physical system might have stocks accumulating a change in the combination of hot and cold water streams entering the pipes and leaving the showerhead. Also, the temperature changing process might be better represented by a higher order delay where an amount of time passes before any temperature change occurs. For simplicity, these effects were not included in the model because they would not greatly change the overall mode of behavior of the shower system from damped oscillations. The important characteristic of any shower temperature model is the recognition of the delay between when a specific temperature is "requested" and when the actual temperature has reached the "requested temperature." For a discussion of the differences between single and higher-order material delays, see Stephanie Albin, 1998. *Generic Structures: Exponential Material Delays.* (D-4616), System

would allow him to obtain his desired temperature more rapidly. Changing the parameters “MOVEMENT PER DEGREE DISCREPANCY” and “TIME FOR NEW TEMPERATURE TO ARRIVE” are two such policies that might improve the shower system.

We’ll start by examining the physical delay of the shower adjusting to its new “requested temperature.” As shown in the model, the rate at which the temperature approaches the requested level is determined by dividing the temperature gap by “TIME FOR NEW TEMPERATURE TO ARRIVE.”

1. What effect will reducing “TIME FOR NEW TEMPERATURE TO ARRIVE” have on shower system behavior?

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Check your prediction by simulating the model using different values for “TIME FOR NEW TEMPERATURE TO ARRIVE.” Simulations for several different values of “TIME FOR NEW TEMPERATURE TO ARRIVE” are shown in Figure 23.

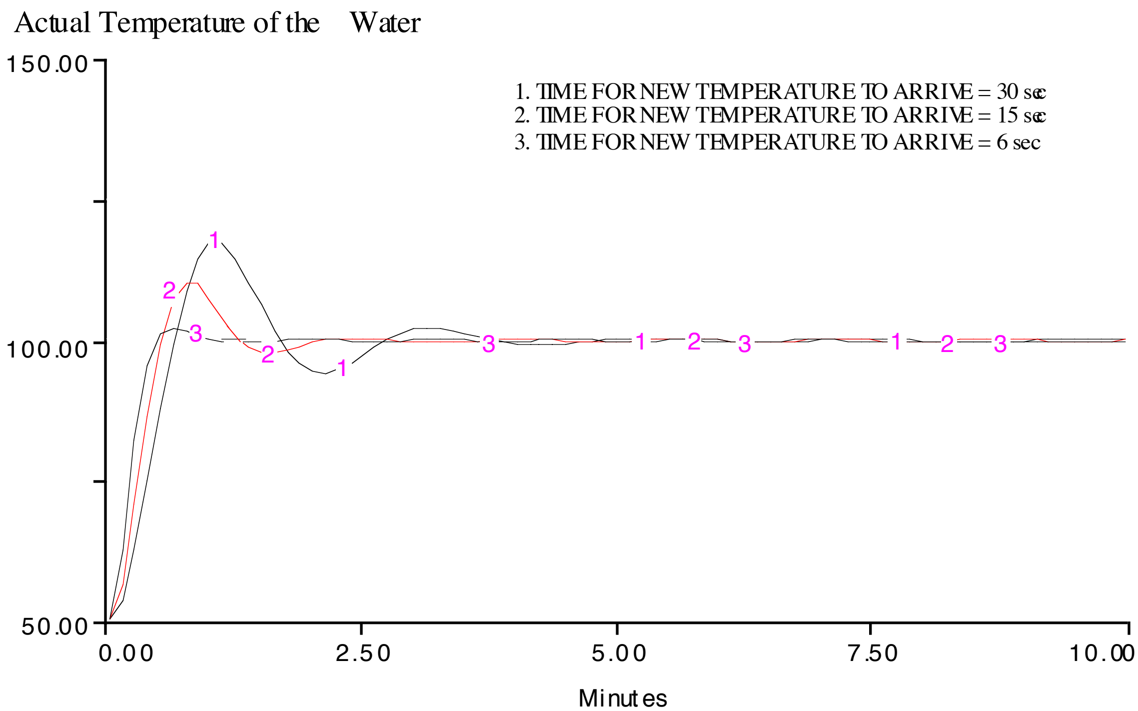

Figure 23: Changing "TIME FOR NEW TEMPERATURE TO ARRIVE"

Changing the length of the temperature delay has an enormous effect on the behavior of the system. In the base simulation run (TFNTTA = 30 sec), shower temperature reaches 120 degrees, then drops to 90 degrees and doesn't settle on the desired temperature until several minutes later. By contrast, a 6 Second temperature delay causes Todd to barely overshoot "DESIRED TEMPERATURE" before settling, after about a minute. Intuitively, the longer it takes for the water to reach an impatient Todd, the further he turns the knob before feeling that the water has become too hot or too cold. A shower that responds quickly to Todd's movements will prevent Todd from moving the knob far past the ultimate desired position.

2. In the physical shower system, how would "TIME FOR NEW TEMPERATURE TO ARRIVE" be reduced?

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3. Is reducing this parameter a viable solution to Todd's problem?

Though reducing "TIME FOR NEW TEMPERATURE TO ARRIVE" helps to solve Todd's shower problem by reducing the magnitude of the oscillations, actually making this change to a real shower would be quite difficult. At the very least, major rework of the piping network in Todd's hotel would be necessary to accomplish the change.

Now we will look at the actions Todd can take to improve his ability to rapidly obtain his desired temperature, which don't require any major construction. "MOVEMENT PER DEGREE DISCREPANCY" reflects the delay in Todd's actions. This parameter determines how quickly Todd attempts to close the discrepancy between "Actual Temperature of the Water" and "DESIRED TEMPERATURE." "MOVEMENT PER DEGREE DISCREPANCY" is the fraction of the complete knob rotation Todd makes per minute for every degree of temperature discrepancy. For example, in the base model, MPDD equals 0.05 (rotations/minute)/(degrees). This means that if the current water temperature is 90 degrees (discrepancy = 10 degrees), Todd will turn the knob at a rate of 0.5 rotations per minute.

4. To reduce the temperature oscillations and settle on 100 degrees more rapidly, should Todd increase or decrease "MOVEMENT PER DEGREE DISCREPANCY?"

Simulate the model using several values for "MOVEMENT PER DEGREE DISCREPANCY" to check your prediction.

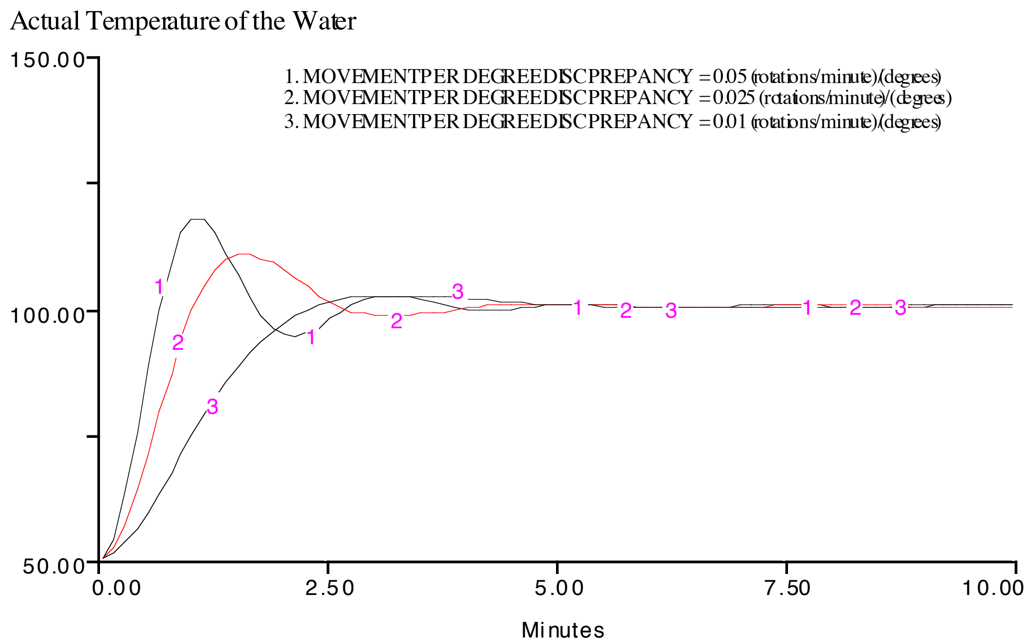

Figure 24: Changing "MOVEMENT PER DEGREE DISCREPANCY"

As shown in Figure 24, reducing "MOVEMENT PER DEGREE DISCREPANCY" (increasing the delay in Todd's actions) reduces the amplitude of the temperature oscillations. Hence, the more impatient Todd is, the more he overcorrects for variations in the temperature of the water.

We've seen two ways by which Todd can greatly reduce the oscillations (or increase the damping of the oscillations). Since changing the pipe structure of the hotel may prove to be difficult, Todd may simply want to wait for the shower temperature to adjust before moving the shower knob.

7.2 Physical Fitness

Leslie lives on the forth floor of a dormitory that, unfortunately, has no elevators. When she used to come home at night she would always run out of breath climbing the stairs leading to her room. As a result, she decided to exercise more. After months of being in and out of shape, she finally found that after consistently jogging 8 hours a week, she is eventually able to climb the 80 stairs that lead up to her room without wheezing.

When she arrived at school in September, Leslie was only able to make it up 20 stairs before wheezing. At the time she was running 2 hours a week. Upset at her lousy fitness, she decided to start exercising more in order to make it up 80 stairs without wheezing. It took her about a week to change her exercising habits and consistently run more. It took her about six weeks, however, to feel the results of her new exercising habits. What happened to her physical fitness over the course of the year? To answer this question, build a model demonstrating the relationship between Leslie's level of fitness (measured by the number of stairs she can climb without wheezing) and her level of hours a week spent exercising. Try to use your knowledge of generic structures to help you predict the type of behavior produced by your model before you simulate it. One possible model is given in Figure 25 on the following page.

7.3 Solution to Physical Fitness Modeling Exercise

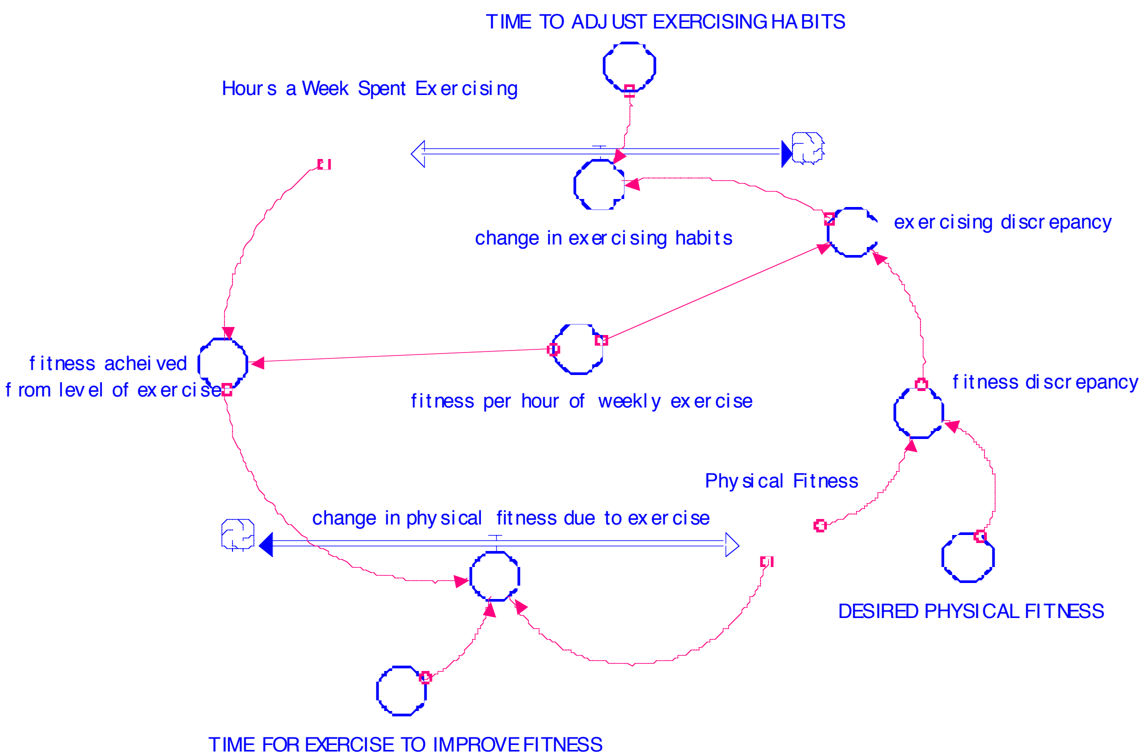

Figure 25: Physical fitness model

This model has two stocks, “Hours a Week Spent Exercising” and “Physical Fitness.” Similar to previous oscillating structures we’ve seen, the rate of change of each stock is determined by the value of the other stock. That is, the difference between Leslie’s desired and actual physical fitness determines how she changes her exercising habits. Similarly, her level of exercise determines her ultimate level of fitness, which influences her rate of movement towards that ultimate level. Because of this negative loop traversing through both stocks, we will expect Leslie’s fitness to oscillate.

Additionally, it takes several weeks for Leslie’s fitness to reach the ultimate level determined by her exercising habits. The negative loop between “Physical Fitness” and its own rate of change causes “Physical Fitness” to slowly approach its goal of “fitness achieved from level of exercise.” As shown in the damped oscillation generic structure, this additional negative loop causes the fitness oscillations to dampen and eventually settle to the equilibrium level of 80 stairs, as shown in Figure 26.

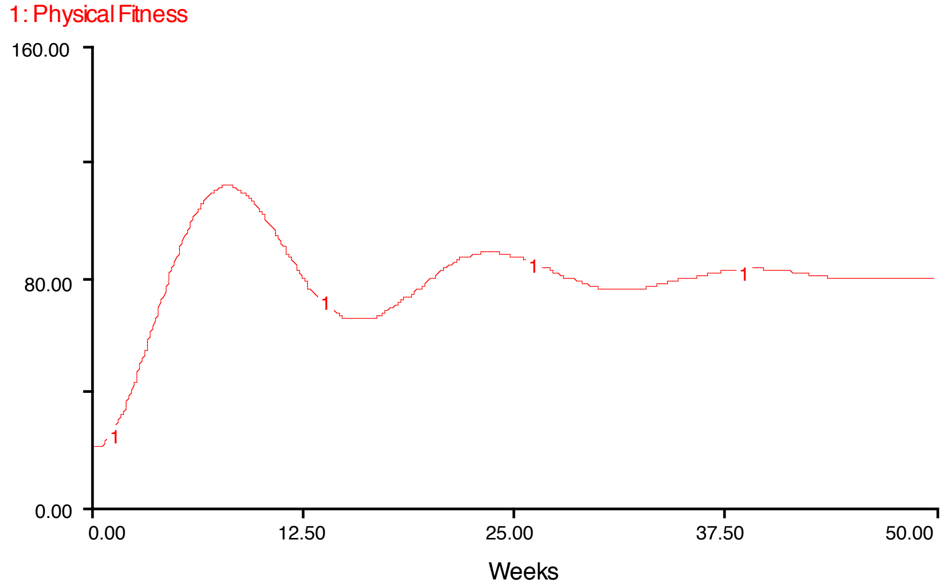

Figure 26: Damped oscillations produced by physical fitness model

8. APPENDIX

8.1 Documentation for Pendulum Model

Position(t) = Position(t - dt) + (change_in_position) * dt
INIT Position = .15

change_in_position = Velocity

Velocity(t) = Velocity(t - dt) + (change_in_velocity) * dt
INIT Velocity = 0

change_in_velocity = (GRAVITY/LENGTH_OF_PENDULUM_ROD)*Gap

DESIRED_POSITION = 0

Gap = DESIRED_POSITION-Position

GRAVITY = 9.8

LENGTH_OF_PENDULUM_ROD = 1

8.2 Documentation for Inventory-Workforce Model

Employment(t) = Employment(t - dt) + (net_change_in_employment) * dt
INIT Employment = 200

net_change_in_employment = (total_number_of_people_needed-Employment)/HIRING_DELAY

Inventory(t) = Inventory(t - dt) + (production_less_sales) * dt
INIT Inventory = 25000

production_less_sales = production-SALES

DESIRED_INVENTORY = 20000

gap = DESIRED_INVENTORY-Inventory

HIRING_DELAY = .25

production = Employment*PRODUCTIVITY

production_needed_to_close_gap = gap/TIME_TO_CLOSE_INVENTORY_GAP

PRODUCTIVITY = 100

SALES = 20000

TIME_TO_CLOSE_INVENTORY_GAP = .5

total_number_of_people_needed = (production_needed_to_close_gap+sales)/
PRODUCTIVITY

8.3 Documentation for Shower Model

Actual_Temperature_of_the_Water(t) = Actual_Temperature_of_the_Water(t - dt) +
(changing_the_temperature) * dt
INIT Actual_Temperature_of_the_Water = 50

changing_the_temperature = (temperature_requested-Actual_Temperature_of_the_
Water)/TIME_TO_CHANGE_TEMPERATURE
DOCUMENT: Units: degrees Fahrenheit/minute

Position_of_Shower_Knob(t) = Position_of_Shower_Knob(t - dt) + (moving_the_knob)
* dt
INIT Position_of_Shower_Knob = 0
DOCUMENT: Units: degrees of rotation
The shower knob can rotate 180 degrees. It starts off to the right, ☐ at 0 degrees of rotation. There the knob is set for cold water at the minimum temperature of 50 degrees farenheit. When the pointer of the knob is vertical (at 90 degrees of rotation) the temperature of the water will be set to be a comfortable 95 degrees farenheit. When the knob is rotated all the way to the left, at 180 degrees of rotation, the temperature is set to a scalding 140 degrees farenheit.

moving_the_knob = (discrepancy/TEMPERATURE_PER_ DEGREE_ROTATION_
OF_KNOB)/TIME_TO_RESPOND_TO_THE_DISCREPANCY
DOCUMENT: UNITS: degrees of rotation/minute

DESIRED_TEMPERATURE = 100

discrepancy = DESIRED_TEMPERATURE-Actual_Temperature_of_the_Water

MINIMUM_TEMPERATURE = 50

TEMPERATURE_PER_DEGREE_ROTATION_OF_KNOB = 0.5
DOCUMENT: UNITS: degree farenheit/degree of rotation

temperature_requested = MINIMUM_TEMPERATURE+Position_of_Shower_Knob*
TEMPERATURE_PER_DEGREE_ROTATION_OF_KNOB
DOCUMENT: UNITS: degrees farenheit

TIME_TO_CHANGE_TEMPERATURE = 2
DOCUMENT: UNITS: minutes
The time for the water to travel through the pipes

TIME_TO_RESPOND_TO_THE_DISCREPANCY = .2
DOCUMENT: UNITS: minutes
This is your average reaction time to the temperature of the water.

8.4 Documentation for Physical Fitness Model

Hours a week spent exercising = INTEG (change in exercising habits, 2)
Units: hours/week
The number of hours every week that I jog along the Esplanade.

change in exercising habits = exercising discrepancy/TIME TO ADJUST EXERCISING HABITS
Units: hours/week/week
Adjusting my exercise habits.

Physical Fitness = INTEG (change in physical fitness due to exercise, 20)
Units: stairs
The number of stairs that I can climb without wheezing.

change in physical fitness due to exercise = (physical fitness goal-Physical Fitness)/TIME FOR EXERCISE TO IMPROVE FITNESS
Units: stairs/week
A goal-gap structure here.

DESIRED PHYSICAL FITNESS = 80
Units: stairs
I would like to be able to climb the 80 stairs that lead to my room without wheezing.

exercising discrepancy = fitness discrepancy/fitness per hour of weekly exercise
Units: hours/week
How much more I should exercise.

fitness discrepancy = DESIRED PHYSICAL FITNESS-Physical Fitness
Units: stairs
How many stairs I still have to climb after I start wheezing.

fitness per hour of weekly exercise = 10
Units: stairs/(hour/week)
For every hour a week I consistently exercise, I can climb this number of stairs without wheezing.

physical fitness goal = Hours a week spent exercising * fitness per hour of weekly exercise
Units: stairs
If I develop a habit of exercising at my current rate, I will be able to climb this many stairs without wheezing.

TIME FOR EXERCISE TO IMPROVE FITNESS = 6
Units: weeks
After six weeks my physical fitness fully reflects my weekly exercise.

TIME TO ADJUST EXERCISING HABITS = 1
Units: week
It takes me a week to get into a new jogging schedule.