

Mistakes and Misunderstandings: Examining Dimensional Inconsistency

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Introduction

Dimensional inconsistencies within equations result from incorrect assumptions about relationships between variables. This paper examines one case where a dimensional inconsistency invalidated a model of a thermostat-controlled furnace. The error resulted from an unclear image of heat and temperature. Scrutinizing equations to ensure dimensional accuracy can certainly help identify errors resulting from an unclear understanding of variables. In addition, keeping in mind that all rates have units of Level / Time can help avoid serious modeling errors.

This issue of Mistakes and Misunderstandings was written by Michael S. Gary, a junior at MIT in the Management Science Department. He has been working on the System Dynamics In Education Project since September 1990.

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The System

People often have unclear mental models of systems that are part of their everyday life. Many times, this is a result of not understanding the relationships between the variables within the system. These misunderstandings often lead to dimensional inconsistencies that invalidate models. As an example, let us consider a "simple" physical system familiar to almost everyone: the thermostat-controlled furnace in a house.

The mechanics of the system are simple- the furnace turns on and off depending on the temperature within the house. The furnace remains off as long as the temperature in the house stays above the goal temperature set on the thermostat. Unfortunately, heat escapes most houses through windows, walls, and doorways and, as a result, the temperature in the house falls below the goal temperature. When this happens, the furnace turns on and remains running until the temperature in the house increases to the goal temperature set on the thermostat. At that point, the furnace shuts off again until reactivated by another temperature drop. In short, the difference between the temperature within the house and the temperature set on the thermostat represents a temperature gap that controls whether the furnace is on or off.

The Model

Given the information we have about the system, one might begin with the following model:

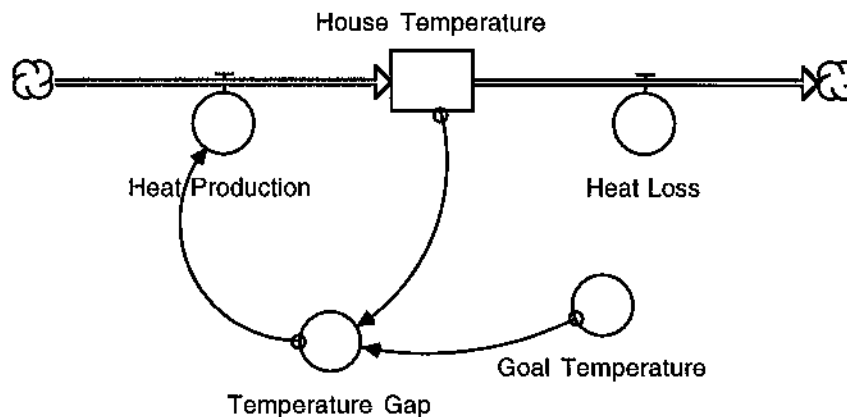


Figure 1. Model of a thermostat-controlled furnace in a house.

The explanation for the model follows:

"Heat Production increases the House Temperature when the furnace is on. As the House Temperature increases, the Temperature Gap (difference between the Goal Temperature and the actual House Temperature) decreases. This continues until the Temperature Gap decreases to zero. The Temperature Gap influences Heat Production by keeping the furnace on until the House Temperature equals the Goal Temperature set on the thermostat. The furnace remains off until the Heat Loss brings the temperature of the house below the goal."

The assumptions in any model should be explicitly stated in order to correctly document the hypothesis the model represents. In this model, the following assumptions apply: 1) The goal temperature is set at a comfortable room temperature which is above the temperature outside the house, 2) Heat Production is a constant rate that is either on or off; on if the gap is greater than zero and off if the gap is zero (this implicitly means that House Temperature can never be greater than the goal temperature), and 3) the Heat Loss rate is assumed to be a constant for purposes of this model.¹ For this paper, the assumption of a constant heat loss will be accepted, because the Heat Production Rate serves to illustrate the concepts.

Mistakes are Exposed

A few questions often serve to identify mistakes made when modeling. Experience and patience teach the right questions to ask.

"So, Heat Production is a rate that flows into a stock of House Temperature. If that is the case then what are the units of House Temperature?"

"Degrees, of course, are the units of temperature. Degrees Celsius or Fahrenheit."

"Well then, what are the units of Heat Production?"

"BTU's per unit time or Calories per unit time; those are the standard units of Heat Production."

¹ Modeling the heat loss rate as a constant is incorrect for the real physical system, but it will remain a simplification for this model.

The dimensions of the Heat Production equation follow.

Heat / Time or Calories per minute or BTU's per minute.

The dimensions of the stock, House Temperature are:

Degrees or Degrees Celsius or Degrees Fahrenheit.

At this point, it should be clear that a mistake has been made. Defining a stock and a flow in units that are not consistent, immediately serves as a sign that an error has been made. Using Temperature as a stock and Heat Production as an inflow is incorrect, because Heat Production must flow into a stock of BTU's or Calories, not Degrees. Remember that a stock is something that accumulates in time, and the flows into and out of the stock are defined in *stock units per time*. Heat Production, in units of BTU's per unit time or Calories per unit time, cannot be directly summed over time into units of Degrees. There must be a conversion from BTU's or Calories to Degrees. The dimensions must be consistent for the equations to be correct, or the model is entirely wrong.

Now, we move to another question.

"Could temperature be used as a stock?"

No, a stock is something that accumulates over time, and temperature does not accumulate. In the physical world, degrees do not accumulate into an *amount* of temperature. In the model in Figure 1, we have tried to define a flow of heat into a stock of temperature. The nature of the mistake can be more easily grasped by examining a comparable analogy.

Suppose you had an ordinary bucket that you wanted to fill to a certain height with water. The model in Figure 2 might represent this system.

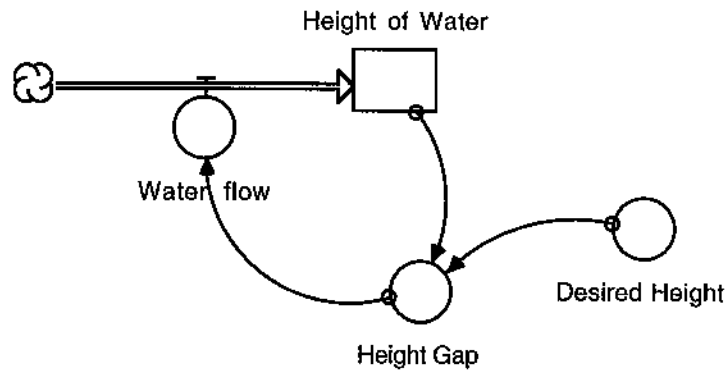


Figure 2. Model of filling a bucket up to a certain height with water.

This model is very similar to the model initially constructed for the thermostat controlled furnace. Height, instead of temperature, is the variable that is important to us. In the above model, Water flow represents pouring water into the bucket. As can be seen from the model, water would be poured into a height of water. Can that really be possible? Does water flow into a height, or does it flow into an amount of water?

The dimensions of Water flow follow.

Amount of water per time or **Volume per minute** (Liters, gallons, etc.).

The dimensions of Height of Water follow.

Length (inches, centimeters, meters, etc.)

In this example, the distinction should be clear. Pouring water into a bucket, results in a bucket with some amount of water in it. The height or depth of the water in the bucket depends on the dimensions of the bucket.² The same amount of water in a glass with different dimensions, results in a markedly different height of the water in the glass than the height of the water in the bucket. The correct way to keep track of the water flowing into the bucket, is to define the stock as water amount or water volume. The

² Dimensions here, refers to the exact measurements of the volume of the bucket and the average surface area of the bucket.

height of the water can be found by using the relationship between the volume of water in the bucket and the height of the water in the bucket.

Similarly, in the thermostat-controlled furnace example, heat flows into an amount of heat, not an amount of temperature. In the analogy of the bucket being filled with water, the height of the water in the bucket is simply a function of the amount of water in a bucket. If you know the volume of the bucket and the average surface area within the bucket, then you can calculate the height of the water. In the same way, the temperature of a space that heat flows into, is a function of the amount of heat in that space. The specific function that relates heat and temperature consists of dividing the amount of heat in the house by the product of the specific heat of air, the density of air, and the volume of the house. In the heat equation, the volume of the house remains constant, the heat capacity of air remains constant, the density of air remains constant (assuming a constant altitude), but the heat within the house changes over time. It is the change of heat within the house that creates the dynamic behavior of the house temperature.

Mistakes are Corrected

Below is the corrected formulation, with the information given so far, of the thermostat-controlled furnace System.³

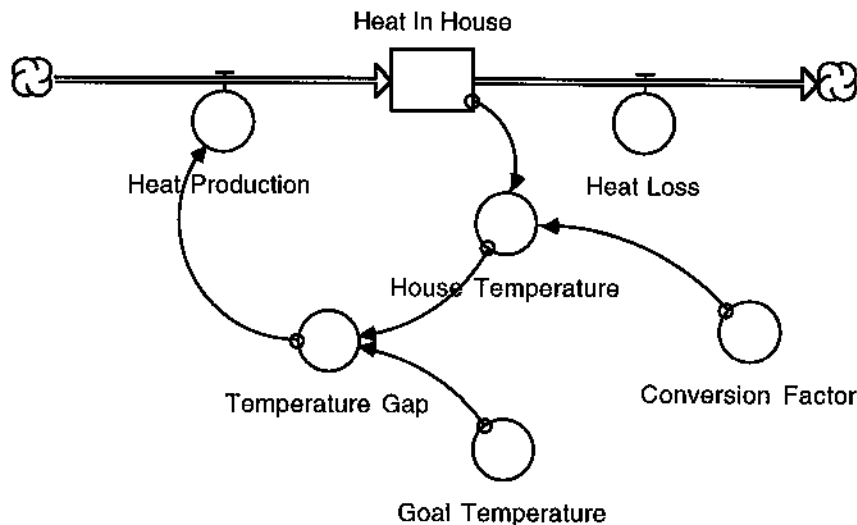


Figure 3. Corrected model of a thermostat-controlled furnace in a house. The change to the model defines the physical situation more accurately: a furnace produces heat, which fills the house with heat, and the amount of heat in the house can be used to calculate the temperature in the house.

Heat Production flows into a stock of Heat In House. The House Temperature is determined by the Heat In House. The computation of House Temperature from the Heat In House is made by converting the Heat In House into degrees through a conversion factor.⁴ The conversion factor is a term that incorporates the specific heat capacity of air, the volume of the house, and the density of air.⁵ As before, Heat Production is controlled by the

³ There is no need to determine an initial value of Heat In House for the purposes of this paper, but a value could be determined by experiment and research to gain a further grasp on the concepts of heat and temperature.

⁴ House Temperature = Heat In House / Conversion Factor

⁵ Conversion Factor = specific heat capacity of air * volume of the house * density of air

Temperature Gap.⁶ When the Gap is greater than zero, the furnace is on. When the Gap is equal to zero, then the furnace is off. Heat Loss, defined as a constant outflow, does not represent reality, but remains a simplification in order to discuss other issues.

Discussion of the Mistakes

Without a clear understanding of the relationship between heat and temperature, the model was formulated incorrectly. It is easy to confuse temperature for a stock when just beginning to model; however, temperature is not a measurable *quantity* as defined in the laws of thermodynamics. Temperature scales are designed to measure the *amount of heat* in an object or space. In the scientific world, temperature is recognized as a property which must be carefully distinguished from the quantity of heat. One way to grasp the distinction between heat and temperature, is to think about the two variables together.

Imagine a huge lake filled with water. The temperature of the water is 50 degrees. In order for the temperature to be 50 degrees, there must be a certain amount of heat within the water. Now, imagine that you are standing next to the lake with a glass of water in your hand. There is obviously only a small fraction of the volume of water in your glass as there is in the lake. The temperature of your glass of water is also 50 degrees, but the amount of heat in your glass of water is a lot smaller than the amount of heat in the lake. It takes less heat to keep your glass of water at 50 degrees than it takes to keep the lake at the same temperature. The idea is that different volumes of a substance can be at the same temperature, but that implies that the larger volume of the substance contains more heat.

The final result of the distinction is this: the property of an object or space, identified as temperature, does not constitute a physical measurable quantity. Heat is the quantity that remains to be measured and that can flow and accumulate.

⁶ Heat Production = If Temperature Gap > 0 Then {a value of heat per unit time}
Else 0
The value of heat per unit time varies according to the specifications for each furnace.

The mental model discussed for the thermostat-controlled furnace followed a line of reasoning that seemed correct at first glance, but was shown to be mistaken. The model correctly defined Heat Production in terms of the Temperature Gap, but incorrectly defined Temperature as a stock. The mental model broke down when confronted with the task of clearly distinguishing between heat and temperature. As stated before, checking the units is a way to catch the error after the incorrect line of reasoning has led to a dimensionally incorrect equation. Preferably, the research stage is the point at which all of the misconceptions and unknown relationships among the key variables and flows can be defined correctly in the modeler's own mind.

Key Lesson

All equations must be defined in units that maintain dimensional consistency. In the model discussed, a rate of heat per time must flow into a stock of heat (not degrees as was incorrectly formulated). The importance of dimensional consistency cannot be over emphasized, because without consistent units the model is invalid from the start. When designing models, be very meticulous about the units for each variable. Do not let an unclear mental model lead you into defining relationships that are dimensionally impossible.