DRAWING AND READING BEHAVIOR OVER TIME GRAPHS

Four Math Lessons to Build Graphing Skills

Prepared
With the Support of
The Gordon Stanley Brown Fund

Based on Work Supported by
The Waters Foundation

By
Rob Quaden
With **Debra Lyneis**

May 9, 2000

Available from
The Creative Learning Exchange
1 Keefe Road
Acton, MA 01720
(978) 287-0070
http://www.clexchange.org/

INTRODUCTION

Understanding behavior over time graphs is a very important system dynamics skill. Students must know how to read and interpret the graphs. They must also be able to draw graphs to specify their ideas and explain them to others. In effect, they need to develop the skills of thinking and communicating in terms of graphs. In this series of exercises, eighth grade math students hone these skills.

BACKGROUND

Rob Quaden is an eighth grade algebra teacher, math coordinator and Waters Foundation systems mentor at the Carlisle Public Schools in Carlisle, MA. As the use of system dynamics spreads into all levels and subjects of the Carlisle K-8 curriculum, Rob and fellow Waters mentor Alan Ticotsky are establishing a matrix of developmentally appropriate system dynamics skills throughout the grades. Right now, the following graphing lessons are part of the eighth grade math curriculum because they complement a physics of motion modeling unit in science. (See "Let It Roll" by Quaden, Trierweiler and Lyneis, available on-line from the Creative Learning Exchange at http://www.clexchange.org or http://sysdyn.mit.edu/cle/.) However, as students appear able to do modeling at younger ages, perhaps some of these graphing lessons should be started earlier too. Stay tuned.

PLANNING THE LESSONS

Curriculum Area

- Eighth grade mathematics

Time Requirements

- Lesson 1: One class period for worksheet and class discussion.
- Lesson 2: Homework assignment followed by one class period of discussion.
- Lesson 3: Classroom activity taking one class period.
- Lesson 4: Half a period followed by a homework assignment.
- Note: These lessons can be done on consecutive days or separately as time allows.

Materials

- Copies of the attached worksheets for each student. Pencils and paper.
- An overhead projector with transparencies and colored markers, or a blackboard and chalk.

LESSON 1

Use this lesson to gauge students' initial graphing skills. Distribute the following worksheet in class and give students a few minutes to work on it. It is self-explanatory.

Name_____

BEHAVIOR OVER TIME GRAPHS
LESSON 1

The Money Section headline from *USA Today* on December 3, 1999 states:

"REPORT: GROWTH OF NEW WEB USERS SINKS"

The story goes on to bemoan that fewer people are signing up for on-line service suggesting problems for the industry.

- From the headline, write a paragraph explaining what is happening to the number of *web users* over time.

- Draw and label a behavior over time graph of the number of web users.

Lesson 1 Teacher Notes

After students have written their paragraphs and drawn their graphs, collect their papers. These will give you an idea of the students' initial graphing skills. Then, discuss the exercise together as a class.

Expect a wide range of responses from the students. Many students will be able to state in their paragraphs that although the number of *new* web subscribers is declining for a variety of reasons, the *total* number of web users is still growing, only at a slower rate than before. The headline does not say that the number of *users* is sinking, only that the growth of *new* web users is sinking. (If they can get this, they are more discerning than most newspaper readers!) Students who are familiar with stocks and flows may use those terms to explain what they mean.

Graphing their stories is a bit more difficult, however. In teams and through class discussion, help students draw graphs that accurately express what they mean to say. Most often, this will be S-shaped growth if they believe that the number of web users started off small and grew more rapidly until the market became saturated and growth slowed. Students can propose other scenarios as long as their graphs match their stories and the total number of users is not declining.

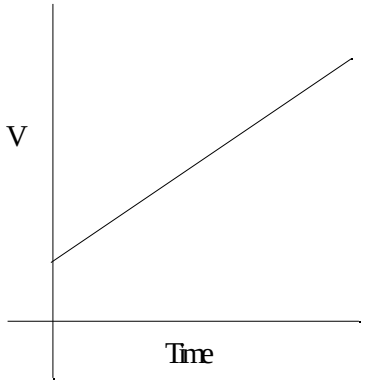
LESSON 2

Distribute the following worksheet for homework. There are two versions. The first uses the volume of water in a bathtub as an example; the second uses the velocity of a car. Use either one, or both.

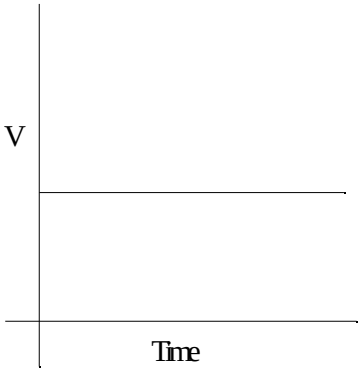
BEHAVIOR OVER TIME GRAPHS Name_____
LESSON 2 (Bathtub)

Each of the following graphs shows the volume of water in a bathtub (V) over time. On another sheet of paper, describe in words how the amount of water is changing in each case.

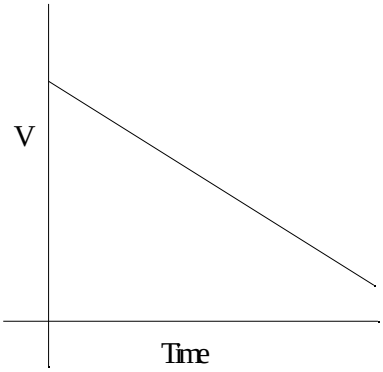
a.



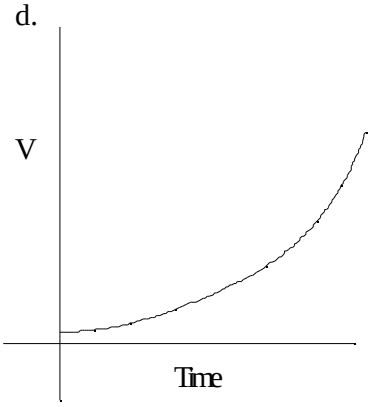
b.



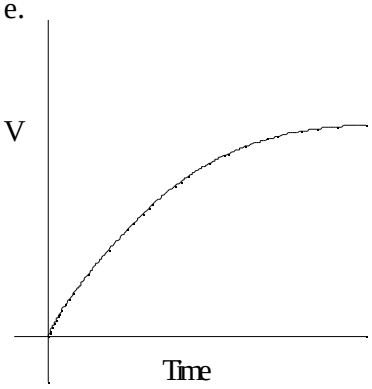
c.



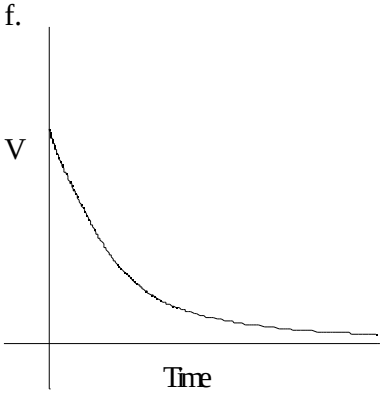
d.



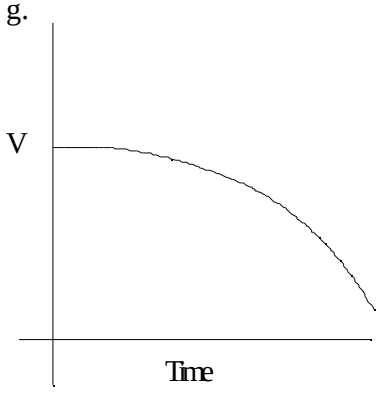
e.



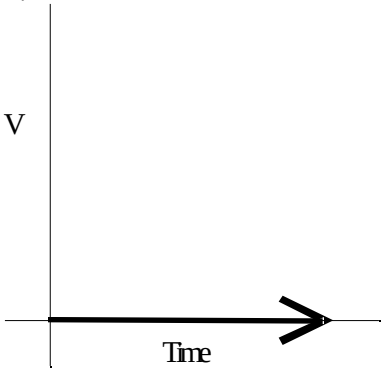
f.



g.



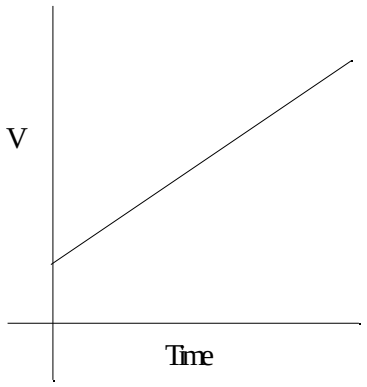
h.



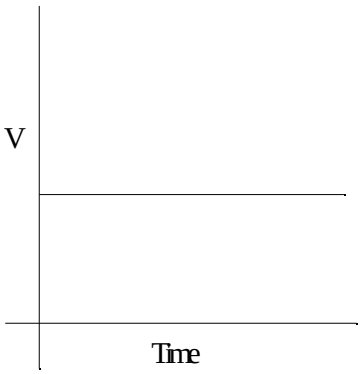
BEHAVIOR OVER TIME GRAPHS Name_____
LESSON 2 (Car)

Each of the following graphs shows the velocity of a car (V) over time. On another sheet of paper, describe in words how the car is moving in each case.

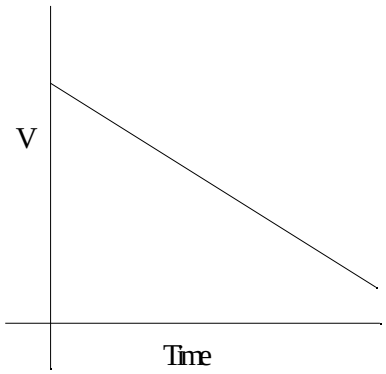
a.



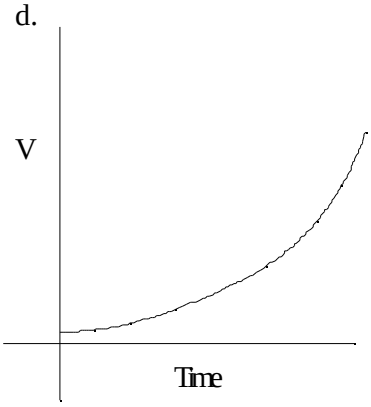
b.



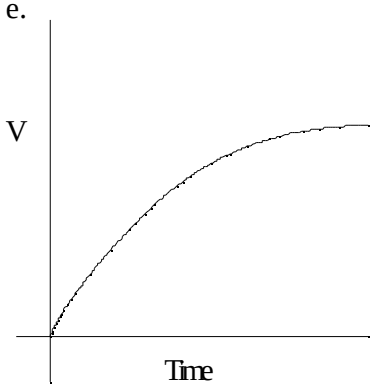
c.



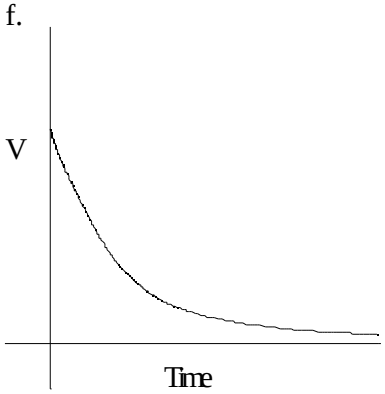
d.



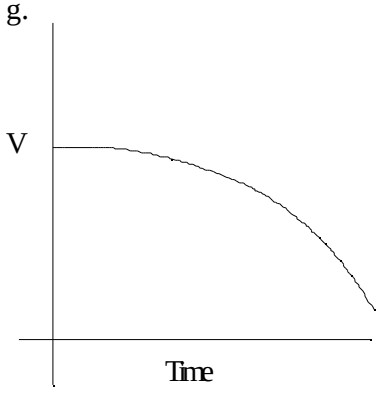
e.



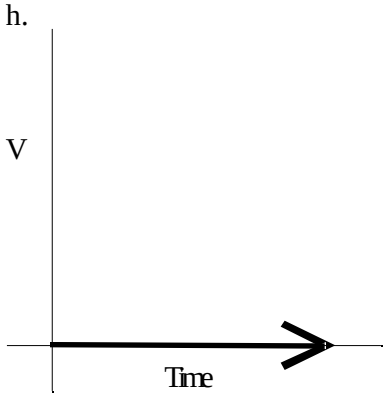
f.



g.



h.



Lesson 2 Notes

Ask students to discuss their graphs first in teams and then as a class. Students will have more elaborate and colorful stories about how and why the changes occur, but these are the basic ideas:

a. The water is filling at a constant rate. The faucet is open.
b. The water level stays the same. The stopper is in.
c. The water is draining at a constant rate. The drain is open.
d. The water is filling at a constantly increasing rate as the faucet is opened wider and wider.
e. The water fills rapidly at first with the faucet wide open, but it fills more slowly as the faucet is turned down.
f. The water drains rapidly at first, but it drains more slowly at the end. It is draining at a constantly decreasing rate.
g. The full tub drains very slowly at first but it begins to drain faster as the drain is opened up more until the tub is empty.
h. There is no water in the tub.

Similarly, students will have more elaborate stories about the car's velocity and the driver's intentions, but these are the basic ideas:

a. The speed of the car is increasing at a constant rate (acceleration).
b. The car is travelling at a constant speed.
c. The car is slowing down at a constant rate.
d. The car is moving faster and faster.
e. The car is accelerating fast at first, but gaining speed more slowly as it approaches cruising speed.
f. The car is traveling fast. It slows down very quickly at first and then more and more slowly as it approaches a stop.
g. The car is travelling fast and slowly starts to slow down. It slows down more and more quickly (pressing harder on the brakes) until it stops.
h. The car is not moving.

Eventually, students will learn the stock/flow structures that create these behaviors. For now, they just need to read the graphs and understand that each represents a very different real-world behavior. Also, they see that the same graphs can describe different phenomena, from bathtubs to cars and more.

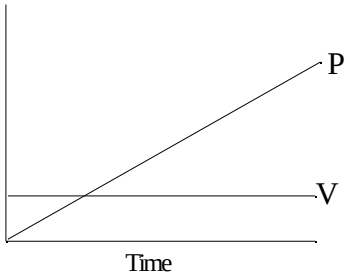
LESSON 3

In this lesson, students graph behavior that they observe in the classroom.

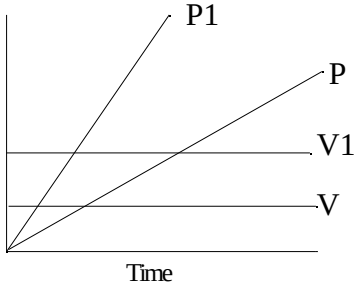
1. Ask students to very carefully observe your behavior as you walk across the front of the classroom at a constant pace. Ask them to graph your velocity on a behavior over time graph. Explain that you are walking at a perfectly even pace. (To avoid confusion about acceleration at the start and finish, ask students to start the graphing at a certain spot and start your walking a few steps before that so that the graph shows only the constant pace.) Ask students to work in their teams and reach agreement on their answers. (These graphs are all drawn free-hand on plain paper.)

2. With some discussion, students should be able to produce a graph of the constant velocity. Discuss all answers and the reasoning behind them, allowing students to learn from one another. Draw the graph on the blackboard or overhead projector.

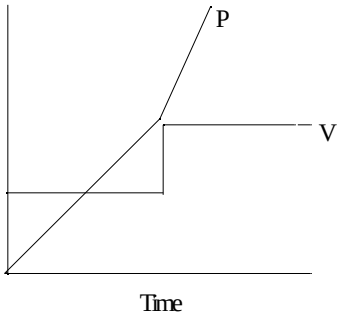
3. Walk across the room again at the same constant pace, only this time ask students to graph your distance from the start on the same graph. Call your distance Position (P). Again let students work in their groups and, as a class, discuss the reasoning behind *all* answers.



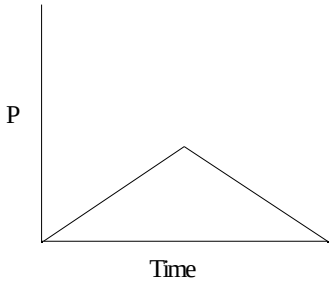
4. Ask students to observe as you walk across the room again but at a faster constant pace. Ask them to graph your velocity on the same graph. Repeat the steps and ask them to also graph the new distance over time. By now, this is easy.



5. Now try something a little bit more tricky. Ask them to observe as you walk halfway across the room at a slow constant pace and the rest of the way at a faster pace. Have them graph your velocity first and discuss it before asking them to also graph your position.

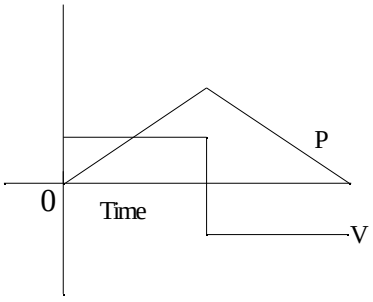


6. The final problem is a good challenge and students enjoy it. It provokes very animated discussions in their groups. Put the following graph up on the overhead projector (or blackboard.)



Ask students how you should walk to produce this distance (position) graph. If it is possible, they should tell you how; if it is not possible, they should tell you why. Specify that you need your walking instructions in the form of a graph. Give each team a blank transparency and a marker. They will draw a behavior over time graph of the velocity and you will walk according to each graph's instructions to see if it produces the same distance graph.

Let students struggle with this in their groups and they will come up with a variety of graphs and lots of justifying reasoning. Most of their graphs will be way off! Still, students will watch in suspense as you enact each one and discuss the position graphs that they actually produce. Someone will figure out that velocity is speed *in a certain direction* and eventually one student will come up with the right graph, surprising everyone. The velocity must go negative to return the position to 0. You walk half way across the room and back to the starting point.



7. A possible follow-up to this lesson would be to have teams of students create their own position graphs with walking instructions on velocity graphs. Then, have teams exchange position graphs to see if the other teams can also produce the correct velocity graphs and walk according to them.

LESSON 4

Following steps similar to those in Lesson 3, ask students to graph your velocity and position using actual numbers and the graphs on the following page.

These worksheet graphs are blank STELLA graphs which will prepare students for reading behavior over time graphs on models in the future. Make a transparency of the page to use on an overhead projector. Show students step by step how to graph your velocity and position.

1. Use the following problem as an example: What does the graph of your velocity look like if you walk across the room at a constant rate of 1 foot/second? Demonstrate by walking across the room at this rate.

2. Label the horizontal time axis "Seconds" from 0 to 10. Label the scale on the *left* axis "Velocity" from 0 to 4. Graph the velocity together as a class.

3. Next graph your position on the same graph. This time label the "Position" scale on the *right* axis from 0 to 20. Starting at time 0, graph your increasing position together as a class. (This introductory step accustoms students to reading two different vertical scales. Later, all scales will be labeled on the left axis.)

4. Let students try a few more problems in their teams and discuss their results together as a class. Remind students to set their own reasonable scales and label them for each problem. Examples:

    - Try the same problem with a walking rate of 2 feet/second.
    - Try it with a rate of 3 feet/second.
    - Try graphing a rate of 1 foot/second for 5 seconds, changing to 2 feet/second for another 5 seconds.
    - Try 2 feet/second for 3 seconds, changing to 1 foot/second for 5 seconds.

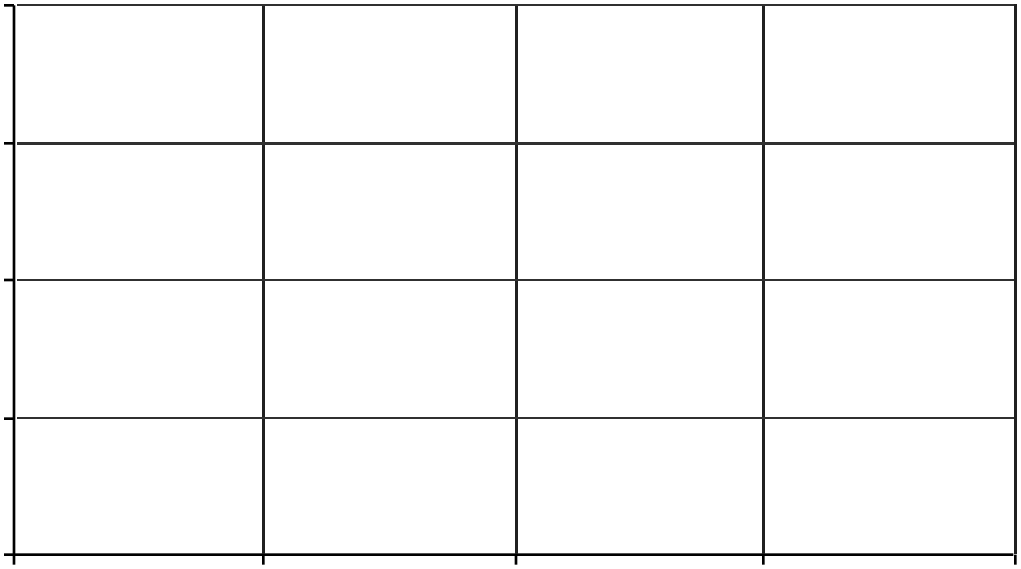
YOUR FEEDBACK

We welcome your feedback as you try these graphing lessons with students. Please send us your comments and suggestions for improvement through the Creative Learning Exchange at LyneisD@clexchange.org. Thank you.

BEHAVIOR OVER TIME GRAPHS Name_____

Label all scales for each graph.

Graph 1



Graph 2