Thinking Systemically About Common Core Mathematical Practice Standards: "Mathematically Proficient Students Can…"

Jeff Potash

As system thinkers and dynamic modelers, we strive to apply our unique tools and perspective to foster in ourselves and in our students a deeper understanding of how to perceive and manage change. Behavior-over-time graphs, stock/flow diagrams, causal/feedback loops and computer models combine to nurture a powerful set of critical thinking skills.[1] Effectively used, systems tools and concepts build skills and habits of mind over time that deepen learning, challenge preconceptions, build new conceptual frames, and enhance student confidence to translate learning into action in becoming system citizens.

While we all share that vision, for any systems educator unfamiliar with the machinations of mathematics education in K-12, the Common Core standards for K-12 Mathematics present a daunting challenge: seventy pages of text list hundreds of detailed standards across what is described as a "sequence of topics and performances." Jay Forrester reminds us that "laundry list" thinking, contrasted with systems thinking, can inhibit understanding. Consider the mundane illustration of a grocery trip: Will I remember a list of ten items I need (flour, eggs, vanilla, butter, cocoa….), without an overarching conceptual frame (a recipe) and an understanding how the parts create the whole (chocolate cake)? It's a system, not just a haphazard list.

In trying to connect the *Dollars and Sense* personal finance curriculum with grade relevant Common Core Mathematics Standards, I discovered that the "sequence" of common core standards in the text did not match up with the progression of mathematical skills developed in our lessons. References to money are, for the most part, focused on the lower elementary level, where they are applied to learning about numbers and arithmetic. Some references appear at the middle school, typically dealing with linear algebra and exponential growth. The mathematics of balancing multiple financial instruments over time is far outside what the standards specify. In sum, our approach seems altogether out of sync.

Hence the question: *Can the mathematics standards be organized to support systems thinking and learning?*

Here I would call attention to a three-page section of the standards at the beginning of the document (pp. 6-8) labeled "Standards for Mathematical Practice." This section focuses upon "what is known about how students learn," followed by an important question, "What does mathematical understanding look like?"

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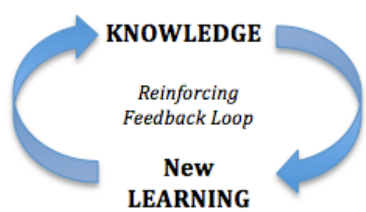
[1] See the Creative Learning Exchange, *Critical Thinking: Using Systems Thinking & Dynamic Modeling* (2012), for an elaboration on Barry Richmond's pioneering efforts to describe eight types of systems thinking skills.

What follows is an overarching or systemic set of learning skills that operates across specific content areas. The "Standards for Mathematical Practice" identify familiar National Council of Teachers of Mathematics[2] "processes" (problem solving, reasoning and proof, communication, representation, and connections) along with "proficiencies" described in the National Research Council's report, *Adding it Up*[3] (adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition), that support long-term learning. Interestingly, the eight Common Core Practice Standards are presented in the same manner as the content standards, in list form:

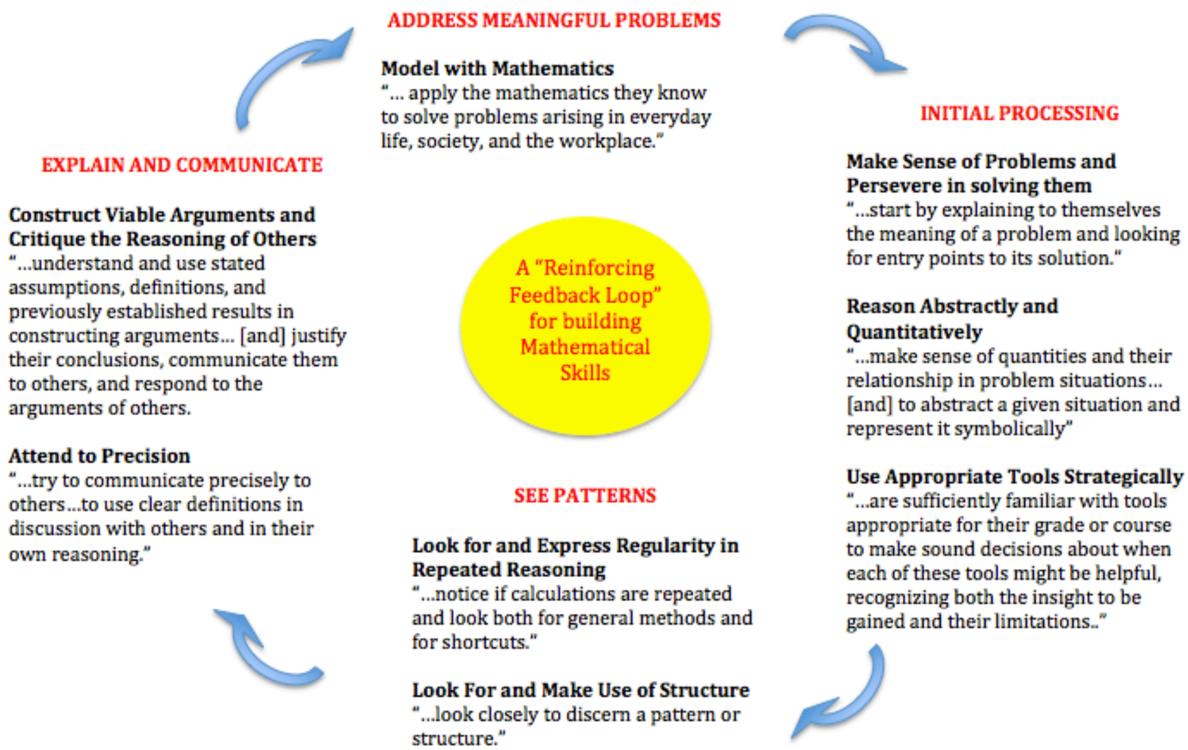
1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

But there is something in these practices that speaks to systems educators. Over time, the document explains, these practices strive to develop in students an increasing ability "to engage with subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle, and high school years." That describes a learning curve, or, in behavior over time terms, an ever growing rate in student capacity, interest, and ability to apply mathematical skills in meaningful ways. As systems thinkers, we recognize that the structure underlying this behavior is a reinforcing feedback loop. Over time, as student knowledge, capacity, and confidence grow, so, too, do rates of future learning. Represented in the language of cognitive science, systemic learning opens new doors to discover what one doesn't yet know, provides a conceptual basis upon which to grow that knowledge and, most critically, apply that knowledge to issues of personal import. When students "own" and apply what they've learned, the discoveries foster curiosity and new questions upon which future learning can then occur.



The key here rests with conceptually converting the list of eight key practices into a systemic process. As such, what I have proposed—largely for conversational purposes as a first step in fostering systems "ownership" of the standards—is a consciously "virtuous" learning loop (recognizing, of course, that it can quickly reverse itself and become a vicious cycle).

[2] www.nctm.org
[3] http://www.nap.edu/catalog.php?record_id=9822



The diagram I've constructed serves two purposes. The first is to rearrange the "list" so that it more accurately and effectively illustrates a network of causal relationships. Imagine the overarching goal is to "Model with Mathematics," specifically to "apply the mathematics they know to solve problems arising in everyday life, society, and the workplace." For that to occur, there needs to be a set of initial processing skills (moving clockwise in the diagram). These involve taking a mathematical problem, deciphering how to approach it and which tools to use.

The second purpose is to inform patterns and structure, both of which are at the center of systems thinking. These then allow for progression into the next level of thinking, which involves comparing and communicating one's own understanding with others. That is where one learns most deeply, both in describing what one knows and in learning from others. All of that leads to the ultimate goal, which is to empower individuals to solve meaningful problems. And, again, from a systems perspective, learning is demonstrated by asking "better questions," relating what one knows to what one would like to know. Questions may seek to extrapolate or apply knowledge in other areas, or deepen understanding in the same area. All of this cycles back to "modeling with mathematics," which reinitiates the cycle. It is, over time, in the repeated iterations of this learning cycle that deeper learning occurs.

I’ve experimented with this cycle using the *Dollars and Sense* curriculum. In contrast with the scattered and disparate content standards, the D&S curriculum operates in a purposeful way to foster a powerful understanding for where and how mathematical concepts and tools inform meaningful real-world problems. There is a logical progression in mathematical concepts, illuminated through the use of equations, tables, and graphs offering multiple perspectives for how to represent patterns of change over time (eg., adding $, subtracting $, adding and subtracting simultaneously, adding compound interest, subtracting compound interest, then adding income, subtracting expenses, while adding and/or subtracting compounding interest). Equally as important, the simulations emphasize the value of undertaking multiple runs: in addition to challenging preconceptions about what factors matter most over time, comparative runs inform recurring patterns. These, in turn, help inform the next set of questions.

Again, I offer this not as an “answer,” but as a starting point for asking better questions relating to where and how we systems educators can own these standards, recognizing that true learning involves putting pieces together to solve meaningful problems. The Common Core Practice Content Standards in Mathematics provide a powerful framework to foster learning in mathematics. And *Dollars and Sense* provides a set of tools with which to foster the Practice Content Standards.

What do you think?